

# Quantum relativistic transport theory for asymmetric nuclear matter: collective modes

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**Abstract.** Isospin, density and electromagnetic waves in asymmetric nuclear matter with the addition of electrons (ANM) are studied. The use of a semi classical relativistic approach based on the Wigner function and its Vlasov like equation to study ANM allowed to obtain the dispersion relations for small amplitude oscillations. The small longitudinal wave case results were equivalent with a previous work that used a somewhat different formalism called generating function. The small transverse wave results, that cannot be obtained directly using the generating function formalism, were also obtained.

## 1. Introduction

Relativistic phenomenological models, such as the quantum hadrodynamical model (QHD) [1], are important tools to investigate and describe nuclei and nuclear matter. With the adequate parameters, these models are effective to study systems that vary from simple nuclei, both stable and unstable [2, 3], to astrophysical objects, e.g. neutron stars and supernovae [4, 5]. Due to this success, it is evident that a full investigation of the models properties is necessary. In particular, the quantum relativistic kinetic theory, and its quantum relativistic transport equations, play an important role to obtain the non-equilibrium behavior of these models.

In order to achieve a complete quantum relativistic kinetic theory, it is necessary to use a quantum relativistic distribution function, also known as the covariant Wigner function [6]. This approach allows us to develop a more robust theory, capable of investigating phenomena in non-equilibrium systems. Besides, its deduction is very similar to the classical one, allowing a deeper understanding of the quantum effects on the theory.

There are previous works already done on instabilities, phase transitions and collective modes in asymmetric nuclear matter (ANM) where the results were obtained using various methods. For example in Ref. [7] the author used a non relativistic Landau-Fermi liquid theory. In Refs. [8, 9], the Wigner function was used in a semi classical approach for the symmetric nuclear matter and the Vlasov equation (transport equation) was obtained, but the way to manipulate the Vlasov equation was through the generating function method. The same formalism was used in Refs. [10, 11] for ANM.

The present work aims to develop a different, more straightforward, formalism. Instead of using the generating functions to solve the Vlasov equation, we use the Vlasov equation and the system's equations of state. In this way we are able to reobtain the results from Ref. [11] for longitudinal small amplitude oscillations around a equilibrium state in nuclear matter.



Furthermore, we were able to obtain results for transverse oscillations, which to our knowledge was never made before.

## 2. Theory Overview

We consider a system of baryons of mass  $M$  interacting with and through a isoscalar-scalar field  $\phi$  with mass  $m_s$ , a isoscalar-vector field  $\mathcal{V}^\mu$  with mass  $m_v$  and an isovector-vector field  $\vec{b}^\mu$  with mass  $m_\rho$ . Furthermore we include electrons with mass  $m_e$ . The electrons and the protons interact through the electromagnetic field  $A^\mu$ . The Lagrangian density reads:

$$\mathcal{L} = \sum_{i=p,n} \mathcal{L}_i + \mathcal{L}_e + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_A, \quad (1)$$

where:

$$\begin{aligned} \mathcal{L}_i &= \bar{\psi}_i [i\gamma_\mu D^\mu - M^*] \psi_i; & \mathcal{L}_\omega &= -\frac{1}{4} \Omega_{*\mu\nu} \Omega^{*\mu\nu} + \frac{1}{2} m_v^2 \mathcal{V}_\mu \mathcal{V}^\mu; \\ D^\mu &= i\partial^\mu - g_v \mathcal{V}^\mu - \frac{g_\rho}{2} \vec{\tau} \cdot \vec{b}^\mu - e \frac{1+\tau_3}{2} A^\mu; & \mathcal{L}_\rho &= -\frac{1}{4} \vec{B}_{\mu\nu} \vec{B}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{b}^\mu \vec{b}_\mu; \\ M^* &= M - g_s \phi; & \mathcal{L}_A &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}; \\ \mathcal{L}_e &= \bar{\psi}_e [\gamma_\mu (i\partial^\mu + eA^\mu) - m_e] \psi_e; & \Omega^{*\mu\nu} &= \partial^\mu \mathcal{V}^\nu - \partial^\nu \mathcal{V}^\mu; \\ \mathcal{L}_\sigma &= \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 - \frac{1}{3} \kappa \phi^3 - \frac{1}{4!} \lambda \phi^4 \right); & \vec{B}^{\mu\nu} &= \partial^\mu \vec{b}^\nu - \partial^\nu \vec{b}^\mu - g_\rho (\vec{b}^\mu \times \vec{b}^\nu); \\ & & F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu; \end{aligned}$$

We also need the Wigner function definition:

$$F(x, p)_\alpha^\beta = \frac{1}{(2\pi)^4} \int d^4 R e^{-ip \cdot R} \left\langle \bar{\psi}^\beta(x + R/2) \psi_\alpha(x - R/2) \right\rangle, \quad (2)$$

where the so called Wigner function actually is a  $4 \times 4$  matrix.

Using the field equations together with the definition of the Wigner function, and assuming the ansatz  $f(x, \Pi) = 2\Theta(\Pi_0)\delta(\Pi^2 - M^{*2})\tilde{f}(x, \vec{\Pi})$  for the scalar Wigner function component, which is in complete agreement with the stationary case, it is possible to obtain the Vlasov equation (transport equation) for the system, in  $O(\hbar)$ :

$$\left[ \Pi^\mu \partial_\mu - \Pi^\mu \Omega_{\mu k} \partial_\Pi^k + M^* (\partial^k M^*) \partial_k^\Pi \right] \tilde{f}(x, \vec{\Pi}) = 0, \quad (3)$$

where  $\Omega^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$ ;  $V^\mu = g_v \mathcal{V}^\mu + \frac{g_\rho}{2} \vec{\tau} \cdot \vec{b}^\mu + e \frac{1+\tau_3}{2} A^\mu$ ,  $\Pi^\mu$  is the kinetic four-momentum, the greek indices represents four-vectors and the latin indices spatial three-vectors.

The process to obtain this equation is straightforward, and details can be found in Ref [12]. We can rewrite this equation as:

$$\frac{d}{dt} f(x, \vec{\Pi}) = \frac{\partial f}{\partial t} + \{f, H\} = 0, \quad (4)$$

where  $H$  is the system's Hamiltonian. This is the conservation of the absolute particle number (particles-antiparticles). Thus, the Vlasov equation represents one of the most basic conservation law in physics.

### 3. Small Oscillations

Solving the Vlasov equation for the most general case is impossible, but as we are interested in studying the collective modes, we should approach the equilibrium state through small oscillations. These small deviations can be described by:

$$f_i = f_{i(0)} + \delta f_i, \quad \phi = \phi_0 + \delta \phi, \quad V_i^\mu = V_{i(0)}^\mu + \delta V_i^\mu,$$

where  $f_{i(0)}$ ,  $\phi_0$  and  $V_{i(0)}^\mu$  are the equilibrium quantities.

When we apply these near equilibrium conditions in the Vlasov equation and in the field equations, assuming again  $O(\hbar)$  and considering the Fourier transform of all the quantities considered, we obtain the following set of equations:

$$\left\{ \begin{array}{l} \delta f = \left[ \delta \vec{V} - \frac{\left( \delta V^0 - \frac{\vec{\Pi}}{E_\Pi^{(0)}} \cdot \delta \vec{V} + \frac{M^{(0)*} g_s}{E_\Pi^{(0)}} \delta \phi \right)}{\omega - \frac{\vec{\Pi} \cdot \vec{q}}{E_\Pi^{(0)}}} \vec{q} \right] \cdot \nabla_{\vec{\Pi}} f(\vec{\Pi}), \\ \delta j_i^\mu(x) = \begin{cases} \frac{2}{(2\pi)^3} \int d^3 \vec{\Pi} \delta f_i, & \mu = 0 \\ \frac{2}{(2\pi)^3} \int d^3 \vec{\Pi} \frac{\Pi^k}{E_\Pi^{(0)}} \delta f_i, & \mu = k \end{cases}, \\ \delta \rho_s^i(x) = \frac{2}{(2\pi)^3} \int d^3 \vec{\Pi} \left[ \frac{M^{(0)*}}{E_\Pi^{(0)}} \delta f_i - g_s d\rho_s^{(0)i} \delta \phi \right], \\ \left[ -\omega^2 + \vec{q}^2 + \tilde{m}_s^2 \right] \delta \phi(\vec{q}, \omega) = g_s \sum_{i=p,n} \frac{2}{(2\pi)^3} \int d^3 \vec{\Pi} \frac{M^{(0)*}}{E_\Pi^{(0)}} \delta f_i(\vec{q}, \omega, \vec{\Pi}), \\ \delta V_i^\mu(\omega, \vec{q}) = \sum_{j=p,n} \left[ \frac{g_\omega^2}{-\omega^2 + \vec{q}^2 + m_\omega^2} + \frac{(g_\rho/2)^2 \tau_i \tau_j}{-\omega^2 + \vec{q}^2 + m_\rho^2} \right] \delta j_j^\mu(\omega, \vec{q}), \end{array} \right. \quad (5)$$

where  $M^{*(0)} = M - g_s \phi^{(0)}$ ,  $E_\pi^{(0)} = \sqrt{\vec{\Pi}^2 + (M^{*(0)})^2}$ ,  $d\rho_s^{(0)i} = \frac{\partial}{\partial M^{*(0)}} \rho_s^{(0)i}$ ,  $\tilde{m}_s^2 = m_s^2 + k\phi^{(0)} + \frac{\lambda}{2} \phi^{(0)2} + \frac{2}{(2\pi)^3} g_s^2 \sum_{i=p,n} \int d^3 \vec{\Pi} \frac{\partial}{\partial M^{(0)*}} \rho_s^{(0)i}$  and  $i = p, n$ . Again, obtaining these equations is straightforward and a more detailed derivation can be found in Ref [13].

Strictly speaking the problem is solved. This set is compatible and determined. But, as we can see, for the most general case the solution is too complex. For this reason we will work with two cases in which the system can be analytically solved up to a certain point. The electron equations can be obtained from (5) by putting  $g_s = g_\omega = g_\rho = 0$ , since the electronic Lagrangian does not have such terms.

#### 3.1. Longitudinal Waves

Let us first consider the longitudinal waves for two main reasons: it is the simplest problem and it was already solved in Ref. [11], so we are able to compare results. In our case, a longitudinal wave is described by the following set:

$$\delta \vec{V}_i = \delta V_{i3} \hat{e}_3, \quad \vec{q} = q \hat{e}_3, \quad \omega \delta V_i^0 = q \delta V_i^3,$$

where the last equation comes from the potential conservation law and  $i = p, n$ . Inputting this new information on the set obtained previously and after long algebraic manipulations, we obtain the following equation:

$$\begin{pmatrix} 1 + F^{pp} L_p & F^{pn} L_p & C_e^{pe} L_p \\ F^{np} L_n & 1 + F^{nn} L_n & 0 \\ C_e^{ep} L_e & 0 & 1 - C_e^{ee} L_e \end{pmatrix} \begin{pmatrix} A_{\omega p} \\ A_{\omega n} \\ A_{\omega e} \end{pmatrix} = 0 \quad (6)$$

where the  $F$ 's,  $C$ 's are functions depending on the equilibrium densities and fields and the  $A$ 's are the variables. This is the called dispersion equation for the system. This matrix is equivalent

to the matrix obtained in Ref [11]. Thus the method used is equivalent to the generating function method used in that reference.

If we want non-trivial solutions for this equation, we should make the determinant of the first matrix zero. With this condition we are able to get  $F$ 's,  $C$ 's values and using these we get information about the system properties, such as the collective modes amplitude for both protons, neutrons and electrons. This was already investigated in [11].

### 3.2. Transverse Waves

We are now heading into new territory. These calculations are harder to be done due to the fact that the generating function formalism is not well adapted to work in this condition. But as we will show the formalism used in the present work proved to be effective in this case. Let us begin with the transverse wave condition:

$$\delta\vec{V}_i = \delta V_{i\perp} \hat{e}_\perp, \quad \vec{q} = q\hat{e}_3, \quad \vec{q} \cdot \delta\vec{V}_i = 0$$

Again, using these equations in the set obtained previously and after long algebraic manipulations, we obtain:

$$\begin{pmatrix} 1 + F^{pp} & F^{pn} & C_e^{pp*} \\ F^{np} & 1 + F^{nn} & C_e^{np*} \\ C_e^{ep*} & 0 & 1 - C_e^{ee*} \end{pmatrix} \begin{pmatrix} B_{\omega p} \\ B_{\omega n} \\ B_{\omega e} \end{pmatrix} = 0 \quad (7)$$

where the  $F$ 's,  $C$ 's are functions depending on the equilibrium densities and fields and the  $B$ 's are the variables.

Again if we want non-trivial solutions for this equation, we should make the determinant of the first matrix zero. Of course the problem is solved and the next step are numerical simulations. We are implementing the algorithms and we expect to have results soon.

## 4. Conclusions

In order to validate our formalism to solve the Vlasov Equation we were able to rederive results obtained in [11] for longitudinal waves. This shows that the formalism used in this work is equivalent to the generating functions method.

Furthermore the formalism developed here was able to solve the transverse wave collective mode, which was not treated before as seen in Ref. [11]. This couldn't be done using the generating function formalism, at least not in a direct way. Using this result we could investigate how an electromagnetic wave propagates through the nuclear matter described by the ANM Lagrangian.

Also we see that the Wigner function is well suited to work in near equilibrium  $O(\hbar)$  conditions. There are various works in the literature studying near equilibrium systems using a multitude of methods and approximations, but using the covariant Wigner function we are able to get more detailed and reliable results. Besides, the formalism used in this work gives results for the mesonic collective modes that are ignored in other methods, e.g. the Landau-Fermi Liquid Theory.

The next step in this research is the numerical solution of the transverse wave modes. After we have the numerical solution, we will be able to draw conclusions from the theory and investigate if there is any regions of instability in the npe matter. We expect to solve the same problem, but including a strong background magnetic field that are speculated to exist in the core of some neutron stars, named magnetars.

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