Frequency Domain Decomposition performed on the strain data obtained from the aluminium model of an offshore support structure

M Mieloszyk[1], **S Opoka**[1] **and W Ostachowicz**[1,2]

[1] Mechanics of Intelligent Structures Department, Institute of Fluid-Flow Machinery, Polish Academy of Sciences, 14 Fiszera Street, 80-231, Gdansk, POLAND.
[2] Faculty of Automotive and Construction Machinery,
Warsaw University of Technology, 84 Narbutta Street, 02-524, Warsaw, POLAND

E-mail: mmieloszyk@imp.gda.pl

Abstract. This paper presents an application of Fibre Bragg Grating (FBG) sensors for Structural Health Monitoring (SHM) of offshore wind energy support structure model. The analysed structure is a tripod equipped with 16 FBG sensors.
From a wide variety of Operational Modal Analysis (OMA) methods Frequency Domain Decomposition (FDD) technique is used in this paper under assumption that the input loading is similar to a white noise excitation. The FDD method can be applied using different sets of sensors, i.e. the one which contains all FBG sensors and the other set of sensors localised only on a particular tripod's leg. The cases considered during investigation were as follows: damaged and undamaged scenarios, different support conditions. The damage was simulated as an dismantled flange on an upper brace in one of the tripod legs. First the model was fixed to an antishaker table and investigated in the air under impulse excitations. Next the tripod was submerged into water basin in order to check the quality of the measurement set-up in different environmental condition. In this case the model was excited by regular waves.

1. Introduction

This paper presents an application of Fibre Bragg Grating (FBG) sensors for Structural Health Monitoring (SHM) of offshore support structure. FBG sensors have small size and weight, multiplexing capabilities, immunity to electromagnetic fields, high corrosion resistance and require no calibration [5]. These sensors can be used for measuring strain on structures and, in particular, for damage detection and localization. The advantages of FBG sensors make them a very interesting tool in SHM systems used for civil engineering structures (bridges, [3], buildings [4]) and marine structures (marine vessels [12], offshore structures [9]).

In the paper the results of an operational modal analysis using Frequency Domain Decomposition (FDD) method applied to strain measurements on tripod model are presented. The FDD method has been applied for different sets of sensors, i.e. the one which contains all FBG sensors and the other sets of sensors localised only on a particular leg of the tripod. The analysis was performed on thermo-mechanical strain ε_{cs} calculated from single axis FBG sensors measurements. All signals before FDD decomposition were standardised to have zero mean and unit sample standard deviation. The strain change due to the bonding procedure and the influence of change in the support conditions on static

strain readings of selected sensors for the same tripod model was earlier presented in [7]. The FDD method allows to determine differences in natural frequencies for undamaged and damaged tripod structure in the air as well as to distinguish excited wave frequency during measurements performed in the water environment.

2. FDD method applied to scale model of tripod
Modal analysis methods can be divided into two categories: experimental modal analysis (EMA) and Operational Modal Analysis (OMA). In EMA both input (excitation of structure) and output (structural response) signals are used while OMA is based only on structural response [11]. OMA can determine modal properties by the structure provided that the excitation is broadband and stochastic (e.g. white noise). In such case excitations are from environmental loadings like wind or waves. OMA methods can be divided into parametric in time domain and non-parametric in frequency domain [2]. In parametric methods, a certain numerical model of the system is assumed, then its selected uncertain parameters are optimizing (updating) based on the measured structural response signals. Non-parametric methods extract the dynamic parameters of a structure by performing set of mathematical operations using measurement data [10]. Parametric methods are preferred in determination of damping parameters while non-parametric ones are used for determination of frequencies and mode shapes [2].

2.1. FDD method description
FDD method is one of non-parametric OMA methods applied especially for analysing of large civil engineering structures like bridges or buildings [1], [11]. This method allows extraction of modal parameters (mode shapes with their frequencies and damping ratios) under ambient excitation caused by wind [6]. The main advantage of FDD method is possibility of discrimination of close mode shapes and its ability to analyze signals with high noise to signal ratio.
The FDD method uses the power spectral density (PSD) of the signals instead of the Fourier transform of the impulse response function (i.e. frequency response function) [10].
The relationship between unknown input signal vector *x(t)* and measured response signal vector *y(t)* can be described as

$$G_{yy}(j\omega) = \bar{H}(j\omega)G_{xx}(j\omega)H(j\omega)^T \qquad (1)$$

where matrix $G_{xx}(j\omega)$ with a size (*r x r*) is the Power Spectral Density matrix (PSD) of input signals, *r* – number of input signals, matrix $G_{yy}(j\omega)$ with a size (*m x m*) - the Power Spectral Density matrix of the system response, *m* - number of output signals, *H(jω)* with a size (*m x r*) is Frequency Response Function (FRF) of the system. The dash over the letter *H* denotes complex conjugate while the index *T* – transposition [1].
According to (1) when the PSD matrix of input signals is constant matrix ($G_{xx}(j\omega)=C$) the system response matrix can be determined based on measured output signals. This assumption is satisfied for broadband stochastic excitation like white noise [1]. In special circumstances other environmental excitations (e.g. wind) can be also interpreted as broadband stochastic excitation.
FDD algorithm can be divided into several steps. First, PSD matrix of output signals $G_{yy}(j\omega)$ for discrete frequencies ($\omega=\omega_i$) is calculated. The power spectral density matrix was calculated using cpsd procedure in Matlab environment with Welch's averaged, modified periodogram method for spectral estimation:

$$G_{xy}(\omega) = \sum_{m=-\infty}^{\infty} R_{xy}(m)e^{-j\omega m}, \qquad R_{xy}(m) = E\{x_{n+m}y*_n\} = E\{x_n y*_{n-m}\}, \qquad (2)$$

where R_{xy} means cross-correlation sequence and x_n and y_n are jointly stationary random processes, $-\infty<n<\infty,$ and *E* is the expected value operator. During the procedure the measurement signal is divided into sections with a length of 4096 points. A Hann window is applied to each measurement section before spectral density estimation. In next step singular value decomposition (SVD) method is performed on this matrix [1]:

$$G_{yy}(\omega_i) = U_i S_i U_i^H \tag{3}$$

where matrix $U_i = [u_{i1}, u_{i2}, ..., u_{im}]$ is an unitary matrix including singular vectors u_{ij} and S_i is a diagonal matrix including scalar singular values s_i, index H denotes conjugate transpose.

The outcome of FDD method is a decomposition of PSD matrix to a set of PSD functions. Each PSD function is related to single degree of freedom (SDOF) of the system [1]. Peaks of the first singular $s_1(\omega)$ are equal to the natural frequency of the system and singular vectors $u_{i1}(\omega)$ corresponding to the peaks of the first singular values approximate mode shape vectors [10].

The FDD method performs best if the structure is excited by white noise, the damping is relatively small and the natural mode shapes are geometrically orthogonal. In other cases the decomposition process into the set SDOF systems is approximate, but still yields better results than classical analysis using Fast Fourier Transformation (FFT) [1].

2.2. Experimental investigation

The analysed offshore support structure is a 2 m height tripod model equipped with 15 single os3120 Micron Optics FBG sensors together with os4100 Micron Optics temperature compensation sensor, see Figure 1(a). The si425-500 Micron Optics interrogator with sampling frequency 250 Hz was used in experiments.

FDD method was applied to measurements performed on tripod structure for two experimental cases. In the first case, the structure was investigated in the air, under impulse excitations (repeated hammer impacts in chosen points on the tripod structure). In the second case the investigation was performed in the water basin (Figure 1(b)) in the Ship Design and Research Centre (CTO S.A.) laboratory and the structure fixed to the support base and partially submerged up to the water level about 150 cm from tripod's bottom flanges. The partially submerged model was excited by artificial waves originating from wave generator installed on one side of the basin.

For every case FDD method was applied for four different set of sensors. The first one denoted as CC contains all FBG sensors, while the others (S1, S2, S3) contains only sensors located on a particular side of the leg, SX denotes set of sensors ubX 1, ubX 2, pgX, tsX, ts Xa for X=1,2,3, compare Figure 1(a).
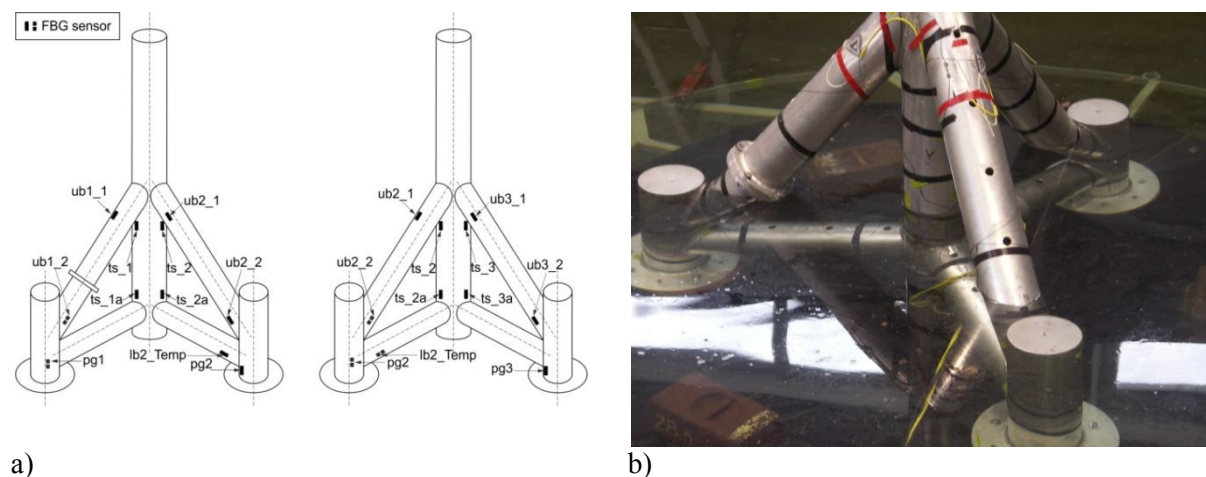

a)   b)

Figure 1. Tripod scheme with marked FBG sensors localisations (a). Partially submerged model in the water basin in CTO S.A. laboratory (b).

For the tripod standing on the antishaker table different structural states were considered: the tripod with fixed flange on an upper brace in one of the legs was considered as intact one (A1), while unbolted flange (visible on the left in Figure 1b) on the upper brace (A2) simulated occurrence of damage in the model. The structure was fixed directly to the table (B3) or relatively thick foams were

inserted under the tripod legs which were further fixed by bolts to the table (B4) in order to simulate different cases of boundary conditions. Additional masses 54 kg (C1) and 35 kg (C2) were put on main tube of the model. The case C3 denotes tripod with no additional mass on the top of the structure.
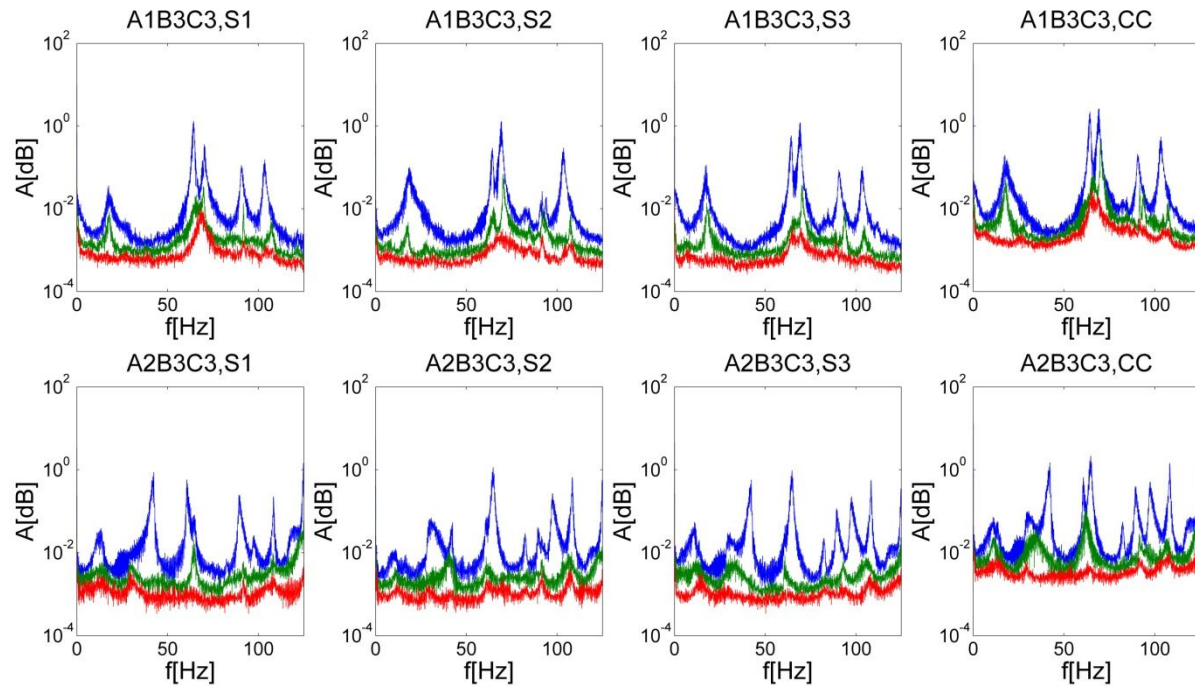

Figure 2. Comparison of first three spectral eigenvalues calculated using FDD decomposition (CC, S1, S2, S3) for tests A1B3C3 and A2B3C3.

In Figure 2 the comparison of spectral eigenvalues received after decomposition using FDD method (CC, S1, S2, S3) for tests denoted as A1B3C3 (intact structure, fixed to the table, without additional mass) and A2B3C3 (damaged structure, fixed to the table, without additional mass) are presented. All curves denoted by blue colour (first eigenvalues of decompositions) are similar in shape. Not all peaks in Figure 2 correspond to vibration modes of the tripod. Numerical simulations [8] performed for intact structure revealed the following eigenfrequencies of tripod structure: 61.75 Hz, 61.99 Hz, 94.28 Hz, 95.36 Hz. The first numerical frequency value is 61.75 Hz, so any frequency peak below such value cannot be treated as indication of vibration mode of the structure. For a structure without any damage (A1) the results for set of sensors S2 differs the most from the others. In such case instead of one frequency peak (91 Hz) two smaller frequency peaks (91 Hz and 94 Hz) are visible. Also small differences in amplitudes between frequency peaks (64 Hz and 70 Hz) can be noticed for SX and CC sets of sensors. The occurrence of damage (A2) results in appearance of additional peaks (82.4 Hz and 97.5 Hz). Moreover for set of sensors S2 and S3 instead of two frequencies (60 Hz and 64 Hz) as in A1 case only one frequency is visible (64 Hz). For a set of sensors S1 calculated for sensors from leg with flange in a range of frequencies above 80 Hz only three peaks are visible, instead of four like for the other sensor sets. Comparing the above results with the numerical natural frequencies [8] the main differences in the number of frequency peaks are visible in A2 case (about 90 Hz) where only three natural frequencies are visible. For all sensors sets (S1, S2, S3, CC) the differences between A1 and A2 case are noticeable in frequency domain especially in the overall number of frequency peaks. This feature can be useful in the future for damage detection and localisation.

The energy distribution of the measurement results presented above were compared with Le & Tamura [2] analysis performed for a building with velocity sensors. In their investigation the energy

distribution for the first four eigenvalues and eigenvectors was as follow: 99,9%, 0,07%, 0,01%, 0,00%. That means that the first three eigenvalues contain almost whole information about the excited dynamic behaviour of the structure. For tripod structure the energy in first eigenvalue strongly depends on number and localisation of sensors as well as on structural state (Table 2). The highest energy value (91,5%) is obtained for S3 sensor set for an intact structure, while the lowest (81,0%) for CC sensor set case for a damaged structure. For every case the first three eigenvalues contains more than 90% of whole vibration energy. The first-order spectral eigenvalues contain most of the energy and they possess almost all information about dominating natural frequencies (see Figure 2).

Table 1. Comparison of percentage energy distribution among three first spectral eigenvalues after FDD decomposition.

	sensor set for A1B3C3				sensor set for A2B3C3			
Eigenvalue	CC	S1	S2	S3	CC	S1	S2	S3
1	83,7%	88,1%	89,7%	91,5%	81,0%	86,1%	87,8%	87,7%
2	8,5%	7,0%	6,3%	5,2%	9,4%	9,2%	7,5%	7,3%
3	2,8%	2,7%	2,1%	2,0%	3,4%	3,0%	2,7%	2,9%

The application of FDD method for partially submerged model (up to 1.5 m) are presented in Figure 3. The results are presented for intact structure. The leg with flange is parallel to the direction of travelling waves in the basin. The experimental investigation was performed for regular wave excitation with height (peak to peak amplitude) H and frequency f. The measured signal length was long enough to register the strain values on the structure for undisturbed water, influence of regular wave on the tripod structure and the waves after the wave exciter was turned off. In this case tripod vibration amplitude was about 4 $\mu\varepsilon$, while the interrogator strain resolution is 1-2 $\mu\varepsilon$. For tripod in the water the highest energy values were obtained for sensor set S1, while the lowest – for CC.
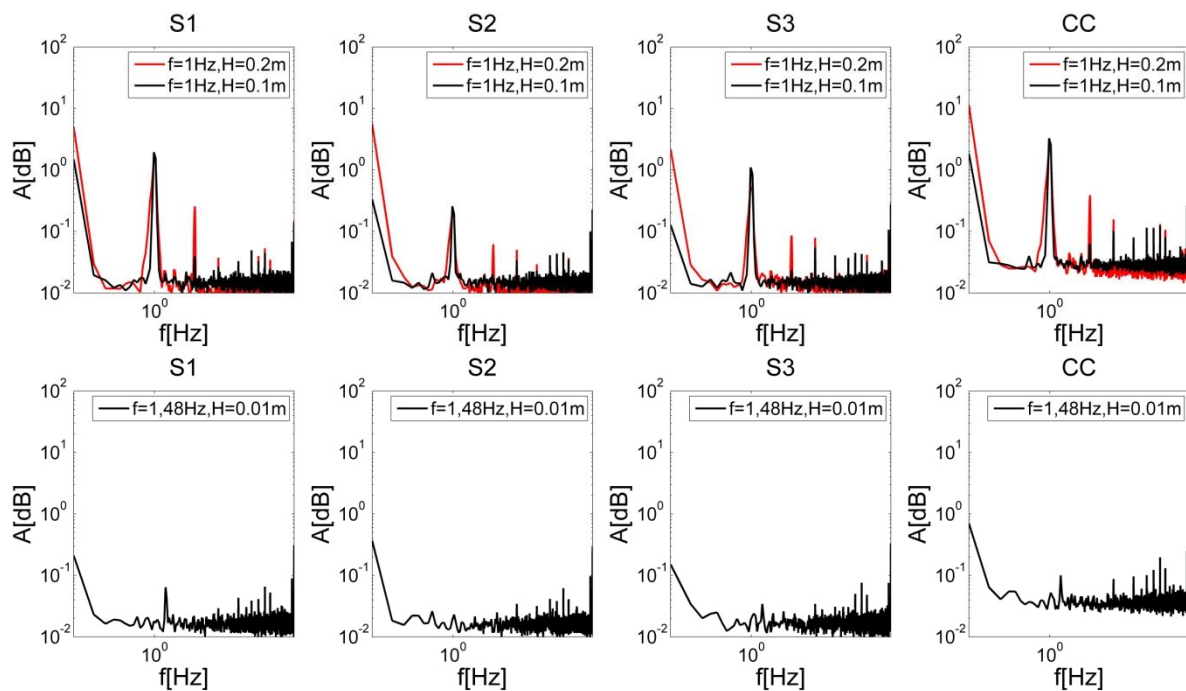

Figure 3. Comparison of first spectral eigenvalues received after FDD decomposition (CC, S1, S2, S3) for different wave excitation.

The preliminary purpose of tripod submersion was experimental verification of underwater strain measurements performed by FBG sensors. The measurement system survived mounting activities performed on the tripod in the basin as well as no deterioration of the quality of submerged sensors was observed. The results for FDD decomposition of strain signals under different wave excitations are presented in Figure 3. The final spectrum of the regular waves is too narrow to excite natural frequencies of the structure due to frequency mismatch between excitation and first natural frequencies of the model, but the sensitivity of the sensors was strong enough to allow determination of excited waves frequencies. For the wave height 0.2 m and 0.1 m the excited wave frequency 1 Hz is visible for all sensor sets while for wave height 0.01 m the excited wave frequency 1.48 Hz is observable only for sensors set S1, S3 and CC.

3. Conclusions

In this paper an application of FDD decomposition method for strain measurements using FBG sensors on offshore support structure are presented and discussed. The results allow to confirm the usefulness of FDD method for determination of natural frequencies of an offshore structure in the air for different structural conditions. In particular the frequency content of the set of measurement signals varies depending on case considered but not all results are fully consistent for the number of peaks in considered frequency interval. These results could be used in the future for damage detection procedures but more sophisticated analysis is needed. The investigation performed in the basin verify the usefulness of FBG sensors as a part of SHM system permanently installed on underwater elements of offshore structure. They also show the possibility of the FDD method to determination of exciting waves frequencies even when the wave height is very small (0.01 m) but the discrimination between origins of different peaks (response or excitation) need to be developed if possible.

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