

Natural frequency changes due to damage in composite beams

I Negru¹, G R Gillich^{1*}, Z I Praisach¹, M Tufoi¹, N Gillich²

¹Department of Mechanical Engineering, “Eftimie Murgu” University of Resita,
P-ta Traian Vuia 1-4, 320085 Resita, Romania

²Department of Electrical Engineering, “Eftimie Murgu” University of Resita,
P-ta Traian Vuia 1-4, 320085 Resita, Romania

E-mail: gr.gillich@uem.ro

Abstract. Transversal cracks in structures affect their stiffness as well as the natural frequency values. This paper presents a research performed to find the way how frequencies of sandwich beams change by the occurrence of damage. The influence of the locally stored energy, for ten transverse vibration modes, on the frequency shifts is derived from a study regarding the effect of stiffness decrease, realized by means of the finite element analysis. The relation between the local value of the bending moment and the frequency drop is exemplified by a concrete case. It is demonstrated that a reference curve representing the damage severity exists whence any frequency shift is derivable in respect to damage depth and location. This curve is obtained, for isotropic and multi-layer beams as well, from the stored energy (i.e. stiffness decrease), and is similar to that attained using the stress intensity factor in fracture mechanics. Also, it is proved that, for a given crack, irrespective to its depth, the frequency drop ratio of any two transverse modes is similar. This permitted separating the effect of damage location from that of its severity and to define a Damage Location Indicator as a sequence of squared of the normalized mode shape curvatures.

1. Introduction

Proper maintenance of engineering structures implies their periodic inspection and control, in order to observe the occurrence of faults and assure proper rehabilitation operations. This leads to reduced operational costs, by interventions in the incipient damage state, or can justify replacing structural elements or even the whole structure, in order to avoid collapse accidents. Numerous global evaluation methods were developed in last decades, the concern of researchers being oriented to find algorithms applicable to a large range of structures in terms of configuration and material properties.

The role of composite materials increases continuously because of their high mechanical characteristics to weight ratio. An advantage, of using these materials, consists in the possibility to elaborate them to fulfil predefined needs, permitting new approaches in structural design. Sandwich structures are a special class of composites; they are composed of sheets with diverse mechanical and physical properties, spatially distributed in a way to provide desired static and dynamical behaviour. This class of composites has been studied from the mid of the twentieth century, the main attempts being focused on bending and buckling [1-2], shock-resistance [3-4] and vibrational behaviour [5-7]. Works devoted to detection of various types of damages in sandwich structures [7-10] shown the complexity and difficulty of this performance, the authors resolving specific cases in regard to the damage type, boundary conditions and structures' mechanical and geometrical structure characteristics. The difficulty exists because actual vibration-base damage detection methods make use



of the stress intensity factor to quantify the crack extend, this being difficult to be performed for layered structures. In previous researches, the authors contrived a mathematical relation that predicts the natural frequency values for isotropic beams with transversal cracks of any depth and location [11-14]. Using this relation, a two-step damage assessment algorithm was developed; in the first step the crack location is precisely found, afterwards the damage severity is evaluated. Since it is possible to separate the problem of location from that of severity evaluation, the method was tested on layered structures. In this paper, the damage detection method tailored for multi-layered beams is present.

2. Vibration of multi-layered beams

For isotropic beams the natural frequencies of bending modes are derived as:

$$f_i = \frac{(\alpha_i L)^2}{2\pi} \sqrt{\frac{EI_z}{mL^3}} \quad (1)$$

where f_i is the natural frequency and $\lambda_i = \alpha_i L$ is the wave number of mode i for a beam with length L , width B and height H . E denotes the Young's modulus, ρ the volumetric mass density, $I_z = B \cdot H^3 / 12$ is the moment of inertia regarding the weak axis of the prismatic beam and $A = B \cdot H$ is the beam cross-sectional area.

For multilayered beams, the natural frequencies of the bending modes can be derived if an appropriate expression of the rigidity EI is used. Since the interest is to attain high rigidity and low weight, the outer faces are manufactured from metals (usually steel or aluminum) and the inner layers of lightweight material; thus, an odd number of layers results. In this paper, a beam having 5 layers is considered. It has the total height of 5 mm, the layers being defined as follows:

- the outer layers 1 and 5 are made of steel (OL) of thickness $s_1 = s_5 = 1\text{ mm}$;
- the intermediate layers 2 and 4 layers are of PVC, with the thickness $s_2 = s_4 = 1\text{ mm}$;
- the central layer 3 is also of steel and has the thickness $s_3 = 1\text{ mm}$,

the configuration being shown in Figure 1.

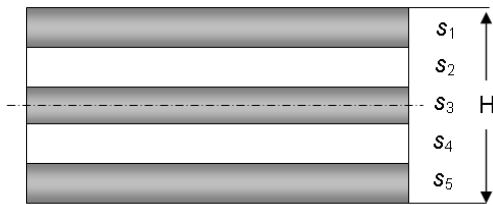


Figure 1. Cross-section in the composite beam showing the layer distribution

For this composite, the rigidity EI of the individual layers is derived, in concordance to the parallel axis theorem, as:

$$EI_{1,5} = E_{OL} \left[\frac{Bs_{1,5}^3}{12} + d_{1,5}^2 (Bs_{1,5}) \right] = E_{OL} \left[\frac{Bs^3}{12} + 4Bs^3 \right] \quad (2)$$

$$EI_{2,4} = E_{PVC} \left[\frac{Bs_{2,4}^3}{12} + d_{2,4}^2 (Bs_{2,4}) \right] = E_{PVC} \left[\frac{Bs^3}{12} + Bs^3 \right] \quad (3)$$

$$EI_3 = \frac{EBs_3^3}{12} = \frac{EBs^3}{12} \quad (4)$$

where, $d_{1,5}$ is the distance from the centre of gravity of the external layers (1 and 5) to the neutral axis and $d_{2,4}$ the distance from the centre of gravity of layers 2 and 4 to the neutral axis.

Because all layers have the same thickness s , and the distances between the centre of gravity and the neutral axis are $d_1 = d_5 = 2s$ and $d_2 = d_4 = s$, the bending rigidity is:

$$EI_{comp} = 2EI_1 + 2EI_2 + EI_3 = E_{OL} \left[3 \frac{Bs^3}{12} + 8Bs^3 \right] + E_{PVC} \left[2 \frac{Bs^3}{12} + 2Bs^3 \right] =$$

$$= (8.25E_{OL} + 2.167E_{PVC}) \cdot Bs^3 \quad (5)$$

Regarding the 5-layer composite structure weight, appearing in the frequency equation (1), it is derived as:

$$m_{comp} = 3\rho_{OL}BsL + 2\rho_{PVC}BsL = (3\rho_{OL} + 2\rho_{PVC})BsL \quad (6)$$

By introducing equations (5) and (7) in the equation (1), the frequency of the analysed composite beam results as:

$$f_i = \frac{(\alpha_i L)^2}{2\pi} \sqrt{\frac{(8.25E_{OL} + 2.167E_{PVC}) \cdot Bs^3}{(3\rho_{OL} + 2\rho_{PVC})BsL^4}} \quad (7)$$

Note that the modal shapes are similar with those determined for isotropic materials as long as the stiffness-mass ratio does not exceed a critical value. Else, the Euler-Bernoulli model is no longer adequate and more complex models are necessary to be used, such as Shear or Timoshenko [4-5].

In the numerical example, two beams are used. The first is a composite beam, as described above, in which the steel sheets have the Young's modulus $E_{OL} = 2.06 \cdot 10^{11} \text{ N/m}^2$ and the mass density $\rho_{OL} = 7850 \text{ kg/m}^3$, while the PVC sheets have $E_{PVC} = 2.41 \cdot 10^9 \text{ N/m}^2$ and $\rho_{PVC} = 1300 \text{ kg/m}^3$. Each layer has the length $L = 1 \text{ m}$ and the cross-section edges $B = 0.02 \text{ m}$ and $H = 0.005 \text{ m}$. The results related to the stiffness and the mass, are shown in Table 1 together with the results achieved for the same beam geometry entirely made of steel. The steel beam has similar global dimensions as the composite one.

Table 1. Stiffness and mass values

Beam type	EI_{OL} (N/m ²)	EI_{PVC} (N/m ²)	EI_{total} (N/m ²)	m_{OL} (kg)	m_{PVC} (kg)	m_{total} (kg)
Steel beam	42.7	-	42.7	0.785	-	0.785
Composite beam	33.8	10.4	33.9	0.471	0.052	0.523

From Table 1 results that the stiffness of composite structures is comparable to that of steel, but the mass decreases significantly when PVC layers are introduced. It can be also noted that the central layer brings an insignificant contribution to the structure rigidity, thus it is preferable to use lightweight materials in the centre of the beam. However, in structural design, it is important to take attention of the natural frequency increase associated to the weight decrease, which can be a disadvantage, in some cases.

3. Numerical model description

The model analysed by numerical simulations was a cantilever beam with the geometry described in the previous section, for which the modal analysis has been performed using the *SolidWorks* software. For the 3D model meshing, tetrahedral finite elements with an average element size of 1 mm were used. The materials available in the software library have identical properties to that earlier described. Additionally, the structural steel **AISI 1045** has the Yield strength $5.3 \cdot 10^8 \text{ N/mm}^2$, Tensile strength $6.25 \cdot 10^8 \text{ N/mm}^2$ and Poisson's ratio 0.29, while the **PVC rigid** has the Yield strength $4.5 \cdot 10^7 \text{ N/mm}^2$,

Tensile strength $4.07 \cdot 10^7 \text{ N/mm}^2$ and Poisson's ratio 0.3825. The ambient temperature has been set to 22°C . By using the *Modal analysis* module the first 20 natural frequencies were determined; from them the first weak-axis bending vibration modes are selected and used in this study.

To get a reference, first simulation has been made for the intact beam. Afterwards, a damage placed at $x = 0.274 \text{ m}$ from the fixed end was achieved. It has the width of 1 mm , the depth being gradually increased from 0.2 mm to 4.8 mm by a 0.2 mm step.

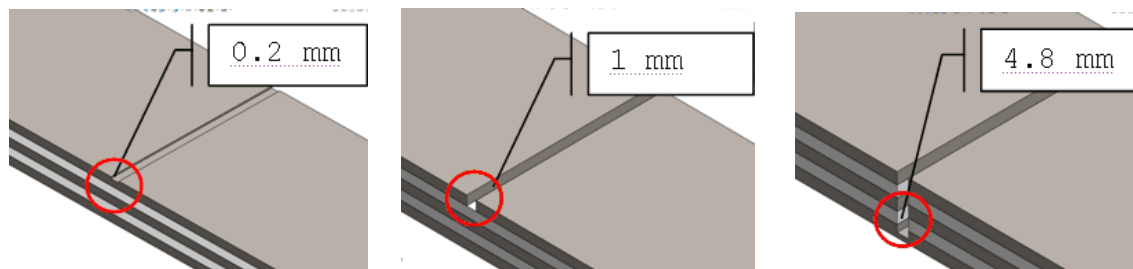


Figure 2. Beam subjected to different damage scenarios.

The beam section affected by the damage considered in the incipient stage, with dimensions $1 \times 0.2 \text{ mm}$, is illustrated in Figure 2. This Figure shows also the damage when the outer steel sheet is completely disrupted, i.e. damage dimensions $1 \times 1 \text{ mm}$, and the last analysed damage scenario, when damage has the dimensions $1 \times 4.8 \text{ mm}$.

4. Results and discussions

Simulation results revealed the natural frequency values for the healthy beam and the beam with a series of damages. The first conclusion regards the availability of equation (8) to provide correctly the natural frequency values. Table 2 compare the frequencies attained by FEM simulations and analytically; one can observe that the values fit well, especially for the first five modes. This leads to the conclusion that this configuration of the composite beam is at the limit of applicability of the equation (7). For more rigid beams, i.e. increased moment of inertia or decreased length, the Timoshenko or Share model are more adequate.

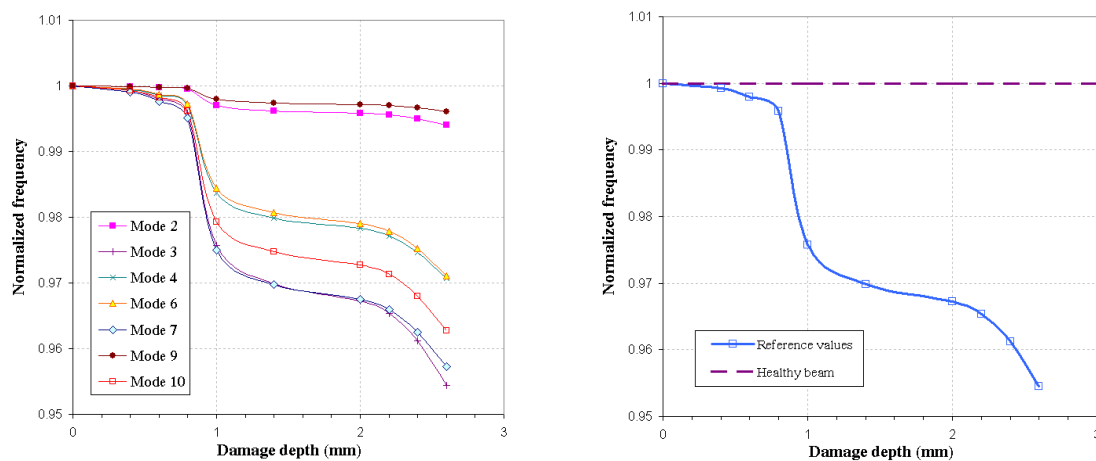
Table 2. Natural frequencies of first ten bending vibration modes of the healthy beam

Mode i	Frequency - analytical f_{i-U}^a (Hz)	Frequency - FEM f_{i-U}^{FEM} (Hz)	Error ε (%)
1	4.48443	4.5076	-0.51668
2	28.10534	28.158	-0.18736
3	78.69577	78.439	0.326278
4	154.21230	152.578	1.05977
5	254.92397	249.857	1.987641
6	380.81227	369.034	3.092935
7	531.87831	508.711	4.355753
8	708.12200	667.375	5.754235
9	909.54337	843.447	7.266984
10	1136.14241	1035.34	8.87234

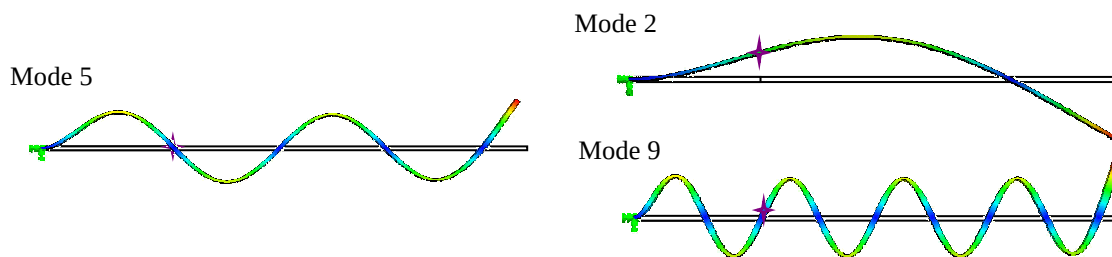
The frequency values for the beam with the damage located at $x = 0.274 \text{ m}$ from the fixed end, and having different levels of depth, are presented in Table 3. The damage depth is indicated as part of the subscript, the value 0.4 in the $f_{i-0.4}^{FEM}$ meaning the depth of 0.4 mm . Figure 3 shows the changes occurring for the frequency values with respect to the damage depth. Note that normalized frequencies are indicated, thus for all vibration modes the frequency for the undamaged beam is 1.

Table 3. Natural frequencies of the first ten bending vibration modes of the healthy beam and the beam with a damage at distance $x = 0.247\text{ m}$ from the fixed end (Hz)

Mode i	f_{i-U}^{FEM}	$f_{i-0.4}^{FEM}$	$f_{i-0.6}^{FEM}$	$f_{i-0.8}^{FEM}$	f_{i-1}^{FEM}	$f_{i-1.4}^{FEM}$	f_{i-2}^{FEM}	$f_{i-2.2}^{FEM}$	$f_{i-2.4}^{FEM}$	$f_{i-2.6}^{FEM}$
1	4.5076	4.5035	4.4975	4.487	4.3908	4.3611	4.3475	4.3374	4.3155	4.2794
2	28.158	28.1562	28.152	28.145	28.073	28.050	28.040	28.033	28.018	27.992
3	78.439	78.3794	78.282	78.108	76.537	76.074	75.868	75.719	75.394	74.8667
4	152.578	152.493	152.35	152.12	150.09	149.51	149.27	149.09	148.71	148.106
5	249.857	249.857	249.85	249.86	249.85	249.84	249.83	249.83	249.83	249.831
6	369.034	368.838	368.53	368	363.27	361.89	361.28	360.84	359.88	358.351
7	508.711	508.234	507.49	506.22	495.98	493.31	492.18	491.37	489.64	486.988
8	667.375	657.463	666.90	666.41	662.63	661.67	661.26	660.97	660.37	659.444
9	843.447	843.387	843.29	843.12	841.72	841.28	841.05	840.92	840.64	840.18
10	1035.34	1034.58	1033.4	1031.3	1013.9	1009.2	1007.1	1005.5	1002.1	996.663

**Figure 3.** Normalized frequencies for the beam subjected to damages with different depths.

It is obvious that the curves depicted in left image of Figure 3 have the same allure, just the amplitude is different. Recent research demonstrated that the amplitude is totally dependent on the energy stored in the slice where the damage is located [10], i.e. propositional to the squared mode shape curvature or, similar, to the bending moment. The image on the right side of Figure 3 represents the reference for the frequency changes for a cantilever having a given damage. To obtain the normalized frequency values for the affected beam, for any vibration mode, the curve is scaled by a factor which is the value of the dimensionless squared of the mode shape curvature. Each location provides different sequences of values for the squared of curvatures, which individualize the sequence and make it suitable for the location recognition.

**Figure 4.** Modes for which the damage produces low influence upon the natural frequencies.

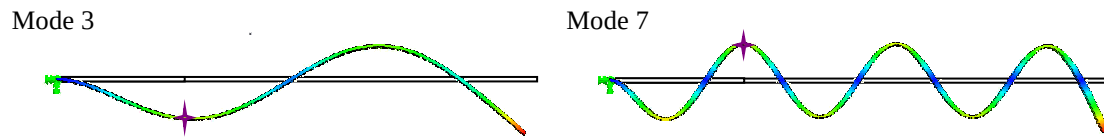


Figure 5. Modes for which the natural frequencies are mostly affected by damage.

The modes shapes for which the damage located at $x = 0.274 \text{ m}$ produce small or inexistent frequency changes, namely mode 5 and modes 2 respectively 9, are presented in Figure 4. One can observe that, for mode 5, the slice on which the damage is placed is not subjected of curving, so that the stored energy is null. In the modes 2 and 9, the damaged slices undergo low curving, thus the slice store a reduced amount of modal strain energy. Consequently, the lost energy is low as well as the frequency decreases.

Opposite, Figure 5 indicates the modes where the frequencies have most important changes, namely modes 3 and 7. It is visible that the damage is located on the slice that achieves the highs local value of curvature in these modes, storing important amount of modal strain energy. In this way, the damage causes important modal strain energy loss and relevant frequency decreasing.

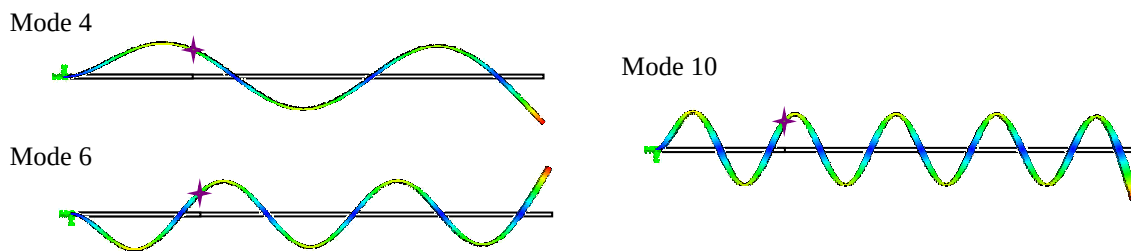


Figure 6. Modes for which the natural frequencies are subject of significant changes.

Figure 6 presents the cases in which an intermediate frequency drop occurs. This happens, for instance, at modes 4 and 6 in a moderate way, and in mode 10 more accentuated. The curves representing the frequency changes due to damages of various depths, plotted in Figure 3, confirm this finding; the stronger the curvature, the bigger the frequency drop.

A confirmation of the enunciated theory is also obtained by analysing the normalized frequency shifts, derived from the mathematical relation

$$\Delta f_i^* = \frac{f_{i-U} - f_{i-D}}{f_{i-U}} \cdot 100 \text{ (\%)} \quad (8)$$

Values of this feature, for a damage located at $x = 0.274 \text{ m}$ from the fixed end, and three damage levels (1, 2 and 2.6 mm), are presented in Figure 7 and Table 4. The input values for the calculus are introduced by Table 3. For the modes in which the damaged slice attains a comparable curvature, modes 4 and 6 for instance (see Figure 6), the normalized frequency shifts are similar (see Figure 7). Remarkable in Figure 7 is also the fact that the frequency shift ratio is identical for all modes of any pair of damage scenarios/depths. Additionally, this ratio is similar, for the pairs of damage depths, with that identifiable in Figure 3. This validates the existence of a unique curve describing the damage severity, as depicted in Figure 3, scalable in respect to the damage depth. This curve is, for the multi-layer structure, analogous to the curves contrived from the stress intensity factor for isotropic materials [15-17].

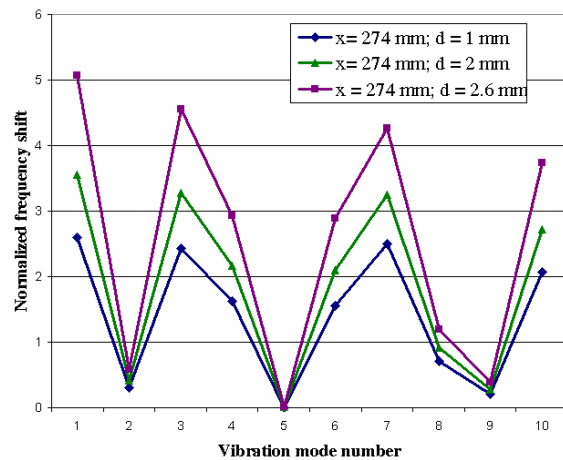


Figure 7. Normalized frequency shifts for the damage located at $x = 274 \text{ mm}$ and three severity scenarios: depth 1 mm , 2 mm and 2.6 mm .

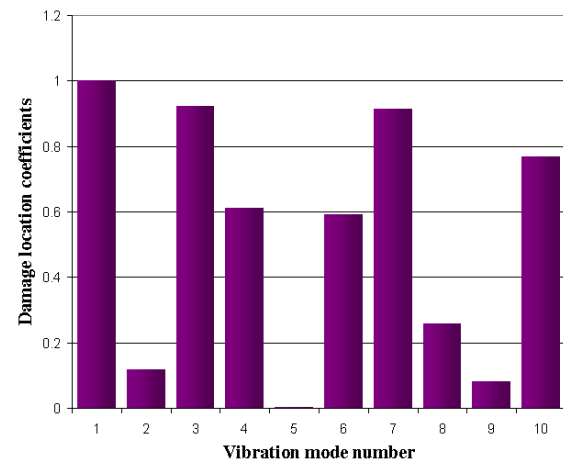


Figure 8. Damage location coefficients for location $x = 274 \text{ mm}$.

Table 4. Normalized frequency shifts of the first ten bending vibration modes of the beam with the damage at distance $x = 0.247 \text{ m}$ from the fixed end

Mode i	Δf_{i-1}^*	Δf_{i-2}^*	$\Delta f_{i-2.6}^*$
1	4.3908	3.550226	5.062561
2	28.073	0.417643	0.589531
3	76.537	3.277069	4.55424
4	150.09	2.168043	2.932232
5	249.85	0.009605	0.011606
6	363.27	2.098745	2.893803
7	495.98	3.248216	4.270016
8	662.63	0.915234	1.187647
9	841.72	0.284664	0.387693
10	1013.9	2.722882	3.731962

The study revealed that, if the same damage is located on different slices of the beam it will produce different sets of values regarding normalized frequency shifts. For the cantilever, as an asymmetric beam, these sets are unique for every location and represent, in fact, the damage signatures. After a new normalization, by dividing the normalized frequency shift values for a location to the highest value of the sequence, the damage location coefficients are resulted, as a new set of sub-unitary values. These values are the same for a given damage location irrespective to the damage severity, thus proper to be used as a benchmark in non-destructive vibration-based damage location.

5. Conclusions

This paper introduces a new view of interpreting frequency changes occurring due to damage in composite beams. The results highlighted that a unique curve representing the frequency change of transverse modes in respect to damage depth exists as presented in Section 4. This curve is similar for all modes, if a scaling factor is applied. This factor depends on the locally stored energy in the damaged slice, i.e. the local value of the squared of the mode shape curvature. If the slice is on an inflection point the change is null - see mode five and its normalized frequency shift. Opposite, if damaged slices achieve important curving, as mode three and seven are, the scaling factor and consequently the normalized frequency shifts are important. The ratios between the normalized

frequency shifts due to damages of similar position but different depths are identical for any transverse modes. Normalizing again the set of certain normalized frequency shifts, the effect of damage depth is annulated, and a Damage Location Indicator in form of a one row vector is obtained. Thus, for location of damage in multi-layered structures the method is also applicable.

Acknowledgement

The authors gratefully acknowledge the support of the Romanian Government by co-financing the project “Doctoral and postdoctoral grants to sustain innovation and competitiveness in research” (InnoRESEARCH), ID 132395.

References

- [1] Plantema F J 1966 *Sandwich construction: The bending and buckling of sandwich beams, plates and shells* (New York: Wiley)
- [2] Magnucka-Blandzi E and Magnucki K 2007 Effective design of a sandwich beam with a metal foam core *J. Sound Vib.* **301** 253–77
- [3] Shahdin A, Morlier J, Mezeix L, Bouvet C and Gourinat Y 2011 Evaluation of the impact resistance of various composite sandwich beams by vibration tests *Shock Vib.* **18**(6) 789-805
- [4] Malekzadeh K, Khalili M R and Mittal R K 2007 Response of composite sandwich panels with transversely flexible core to low-velocity transverse impact: A new dynamic model *Int. J. Impact Eng.* **34** 522–43
- [5] Carrera E 2004 Assessment of theories for free vibration analysis of homogeneous and multilayered plates *Shock Vib.* **11**(3-4) 261-70
- [6] Hause T and Librescu L 2006 Flexural free vibration of sandwich flat panels with laminated anisotropic face sheets *J. Sound Vib.* **297** 823–41
- [7] Gillich G R, Praisach Z I, Wahab M A and Vasile O 2014 Localization of transversal cracks in sandwich beams and evaluation of their severity *Shock Vib.* **9**(4-5) 607125
- [8] Moreira R A S and Dias Rodrigues 2010 J Static and dynamic analysis of soft core sandwich panels with through-thickness deformation *Compos. Struct.* **92** 201–15
- [9] Schwartz-Givli H, Rabinovitch O and Frostig Y 2007 Free vibrations of delaminated unidirectional sandwich panels with a transversely flexible core - a modified Galerkin approach *J. Sound Vib.* **301** 253–77
- [10] Deshpande V S and Fleck N A 2001 Collapse of truss core sandwich beams in 3-point bending *Int. J. Solids Struct.* **38**(36–37) 6275–305
- [11] Gillich G R and Praisach Z I 2014 Modal identification and damage detection in beam-like structures using the power spectrum and time–frequency analysis *Signal Process.* **96**(Part A) 29–44
- [12] Praisach Z I, Gillich G R and Birdeanu D E 2010 Considerations on natural frequency changes in damaged cantilever beams using FEM *Latest trends on engineering mechanics, structures, engineering geology* 214-9
- [13] Gillich G R and Praisach Z I 2013 Damage-patterns based method to locate discontinuities in beams *Proceedings of SPIE (San Diego)* vol 8695 article number 869532
- [14] Gillich G R, Praisach Z I and Negru I 2012 Damages influence on dynamic behaviour of composite structures reinforced with continuous fibers *Mater. Plast.* **49**(3) 186-191
- [15] Liebowitz H and Claus Jr. W D S 1968 Failure of notched columns *Eng. Fract. Mech.* **1**(1) 379–383
- [16] Ostachowicz W M and Krawczuk C 1991 Analysis of the effect of cracks on the natural frequencies of a cantilever beam *J. Sound Vib.* **150**(2) 191–201
- [17] Chondros T J, Dimarogonas A D and Yao J 1998 A continuous cracked beam vibration theory *J. Sound Vib.* **215**(1)17–34