

# A theoretical model for fault diagnosis of localized bearing defects under non-weight-dominant conditions

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**Abstract.** Fault diagnosis of localized bearing defects under non-weight-dominant conditions is studied in this paper. A theoretical model with eight degrees of freedom is established, considering two transverse vibrations of the rotor and bearing raceway and one high-frequency resonant degree of freedom. Both the Hertzian contact between rolling elements and raceways, bearing clearance, unbalance force and self-weight of rotor are taken into account in the model. The localized defects in both inner and outer raceways are modeled as half sinusoidal waves. Then, the theoretical model is solved numerically and the vibrational responses are obtained. Through envelope analysis, the fault characteristic frequencies of inner/outer raceway defects for various conditions, including the weight-dominant condition and non-weight-dominant condition, are presented and compared with each other.

## 1. Introduction

Rolling element bearings are important components of rotating machinery. Localized defects are frequently found in inner/outer raceways. The growth of localized defects can cause the fault of rolling bearings and even severe accidents to the rotating machinery if undetected early. For years, extensive efforts have been devoted to the modelling of localized defects and then predict their effects upon the vibrational response of the bearing system [1-3].

Most current studies focused on the bearing systems under the weight-dominant condition [4-6]. The load zone due to the self-weight of the rotor is time-invariant. The rolling elements periodically pass through the load zone and impact with the inner/outer raceway defects. The fault characteristic frequencies for the defects could be predicted accordingly.

However, the self-weight might not be the dominant load in many rotating machinery, such as the vertical rotors, lightly weight rotors, and et.al. In these rotors, the unbalance force is the dominant load, and cause the load zone of the rolling bearing varies with time. In this case, the impacts between the rolling element and raceway defects become more complicate. Obviously, the fault characteristic frequencies would differ distinctly with that of the weight-dominant case.

Therefore, fault diagnosis of localized bearing defects under non-weight-dominant conditions is studied in this paper. A theoretical model with eight degrees of freedom is established, considering two transverse vibrations of the rotor and bearing raceway and one high-frequency resonant degree of freedom. Both the Hertzian contact between rolling elements and raceways, bearing clearance, unbalance force and self-weight of rotor are taken into account in the model. The localized defects in both inner and outer raceways are modelled as half sinusoidal waves. Then, the theoretical model is

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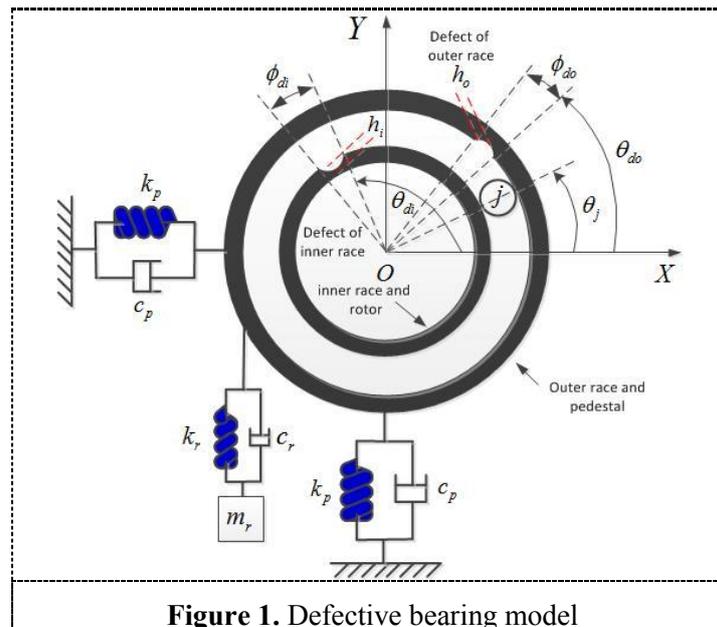


solved numerically and the vibrational responses are obtained. Through envelope analysis, the fault characteristic frequencies of inner/outer raceway defects for various conditions, including the weight-dominant condition and non-weight-dominant condition, are presented and compared with each other. Finally, some conclusions are summarized.

## 2. Theoretical model for rotor-bearing system with localized bearing defects

### 2.1. Nonlinear restoring force of rolling bearing with localized defects

The schematic diagram for a deep groove ball bearing is shown in Fig. 1. Generally, the outer race is fixed on the pedestal. The inner race, connected with rotor, rotates under constant angular speed  $\omega_s$ . The coordinate for dynamic analysis is  $O-X-Y$ . There are two degrees of freedom for the inner race and rotor, i.e.  $x_s, y_s$ . The mass and damping are  $m_s, c_s$ , and the stiffness is provided by the nonlinear Hertz deformation between the ball and raceways. The outer race and pedestal also have two degrees of freedom, i.e.  $x_p, y_p$ . The corresponding mass, stiffness and damping are  $m_p, k_p, c_p$ . Besides, in order to simulate the high frequency resonant response of the rolling bearing, a spring-mass-damping system ( $m_r, k_r, c_r$ ) with high resonant frequency is added to the model [3], as shown in Fig. 1.



**Figure 1.** Defective bearing model

The location angle of the  $j$ th ball is  $\theta_j(t) = \omega_{bc}t + 2\pi(j-1)/N_b$ , where  $N_b$  the number of balls,  $\omega_{bc}$  the orbital speed of the ball. Under pure rolling conditions, the orbital speed of deep groove rolling bearing is expressed by  $\omega_{bc} = \omega_s(1 - d_b/D_b)/2$ , in which  $d_b$  the ball diameter and  $D_b$  the pitch diameter of the bearing. According to the geometric relations shown in Fig. 1, the contact deformation for the the  $j$ th ball at time  $t$  is

$$\delta_j = (x_s - x_p)\cos\theta_j + (y_s - y_p)\sin\theta_j - c_i \quad (1)$$

Where  $c_i$  denotes the radial clearance.

A localized defect in the outer race is considered and shown in Fig. 1. The defect is modelled by a half sinusoidal wave, and its location, size and depth are, respectively, expressed by  $\theta_{do}, \phi_{do}$  and  $h_o$ . Similarly, the location, size and depth for the inner race defect are denoted by  $\theta_{di}, \phi_{di}$  and  $h_i$ . When

the ball is contact with the defect, the contact deformation should be modified to take into account the additional clearance induced by the defect. For the rolling bearing with outer race defect, the contact deformation could be rewritten as

$$\delta_j = \begin{cases} \left( (x_s - x_p) \cos \theta_j + (y_s - y_p) \sin \theta_j - \left( c_l + h_o \sin \left( \frac{\pi}{\phi_{do}} (\theta_j - \theta_{do}) \right) \right) \right) & \theta_{do} \leq \theta_j < \theta_{do} + \phi_{do} \\ (x_s - x_p) \cos \theta_j + (y_s - y_p) \sin \theta_j - c_l & \text{else} \end{cases} \quad (2)$$

As the outer race is fixed, so the location angle of the defect  $\theta_{do}$  is also constant. However, as the inner race is rotating, so the inner race defect also rotates with time. Its location angle should be  $\theta_{di} = \omega_s t + \varphi_i$ , in which  $\varphi_i$  the initial angle with respect to the  $X$  axis at time  $t = 0$ . Thus, when the inner race defect is considered, the contact deformation could be expressed similar with Eq. (2), while the parameters  $\theta_{do}$ ,  $\phi_{do}$  and  $h_o$  should be substituted by  $\theta_{di}$ ,  $\phi_{di}$  and  $h_i$ .

According to the Hertz contact theory, the contact force between the inner race and ball is given by  $Q_j = \lambda_j K_c \delta_j^{3/2}$ , in which  $K_c$  the contact stiffness,  $\lambda_j$  the force coefficient. For  $\delta_j > 0$ ,  $\lambda_j = 1$ ; otherwise,  $\lambda_j = 0$ . The contact stiffness  $K_c$  is related to the geometry and material property of the contact pairs, and could be solved from Harris' monograph. Considering the number of balls in contact, the total restoring forces of the rolling bearing in the two axes are expressed as follows

$$\begin{aligned} F_{bx} &= \sum_{j=1}^{N_b} \lambda_j K_c \delta_j^{3/2} \cos \theta_j \\ F_{by} &= \sum_{j=1}^{N_b} \lambda_j K_c \delta_j^{3/2} \sin \theta_j \end{aligned} \quad (3)$$

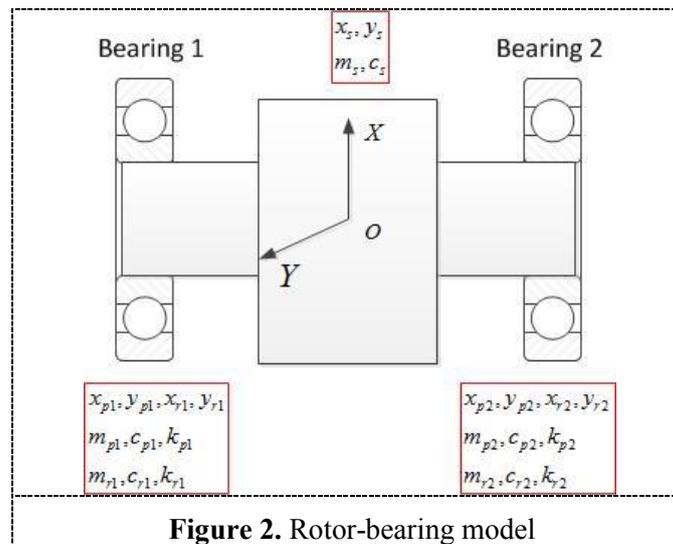
## 2.2. Equations of motion for the rotor-bearing system

A rigid rotor system supported by two deep groove rolling bearings is shown in Fig. 2. Thus, the equations of motion for the system could be derived as follows

$$\begin{aligned} m_s \ddot{x}_s + c_s \dot{x}_s + F_{bx1} + F_{bx2} &= m_u e_u \omega_s^2 \cos \omega_s t \\ m_s \ddot{y}_s + c_s \dot{y}_s + F_{by1} + F_{by2} &= m_u e_u \omega_s^2 \sin \omega_s t \\ m_{p1} \ddot{x}_{p1} + c_{p1} \dot{x}_{p1} + k_{p1} x_{p1} - F_{bx1} &= 0 \\ m_{p1} \ddot{y}_{p1} + (c_{p1} + c_{r1}) \dot{y}_{p1} - c_{r1} \dot{y}_{r1} + (k_{p1} + k_{r1}) y_{p1} - k_{r1} y_{r1} - F_{by1} &= 0 \\ m_{r1} \ddot{y}_{r1} + c_{r1} (\dot{y}_{r1} - \dot{y}_{p1}) + k_{r1} (y_{r1} - y_{p1}) &= 0 \\ m_{p2} \ddot{x}_{p2} + c_{p2} \dot{x}_{p2} + k_{p2} x_{p2} - F_{bx2} &= 0 \\ m_{p2} \ddot{y}_{p2} + (c_{p2} + c_{r2}) \dot{y}_{p2} - c_{r2} \dot{y}_{r2} + (k_{p2} + k_{r2}) y_{p2} - k_{r2} y_{r2} - F_{by2} &= 0 \\ m_{r2} \ddot{y}_{r2} + c_{r2} (\dot{y}_{r2} - \dot{y}_{p2}) + k_{r2} (y_{r2} - y_{p2}) &= 0 \end{aligned} \quad (4)$$

In which  $x_{pi}, y_{pi}, x_{ri}$  ( $i = 1, 2$ ) represents the degrees of freedom for the bearing 1 and 2,  $m_{pi}, m_{ri}$ ,  $c_{pi}, c_{ri}$  and  $k_{pi}, k_{ri}$  ( $i = 1, 2$ ), respectively, denote the mass, damping and stiffness of bearing 1 and 2.

Obviously, considering the degrees of freedom of the rotor, pedestal and high-frequency spring-mass system, there are eight degrees of freedom for the rotor-bearing system. Due to the nonlinear restoring forces induced by the rolling bearing, Eq. (4) could only be solved by the numerical integration method. In the following section, we will show the fault features of the dynamic response for the system under various load conditions, including the non-weight-dominant condition.



### 3. Simulations and discussions

In the simulation, the parameters for the rotor-bearing system are shown in Table 1. Two types of load conditions are considered: (1) only weight condition (just the weight-dominant condition); (2) only unbalanced force condition (just the non-weight-dominant condition). In the following analysis, the envelope spectra for the rolling bearing without defect, with inner race defect and with outer race defect are obtained to show the values and distributions of fault frequencies.

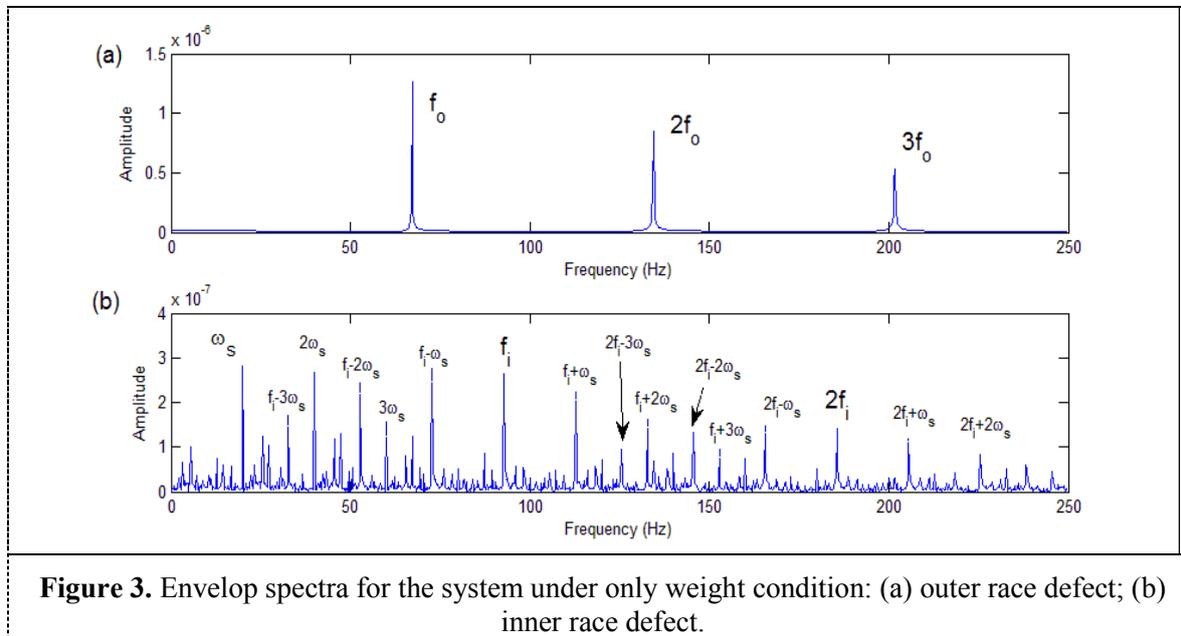
**Table 1.** System parameters

Parameters	Values	Parameters	Values
Rotor mass (Kg)	3	Pedestal mass (Kg)	12.638
Rotor damping (N/s)	1200	Pedestal stiffness (N/m)	15e6
Rotor stiffness (N/m)	1e6	Pedestal damping (N/s)	2210
Unbalanced mass (Kg)	0.01	Spring's mass (Kg)	1
Mass eccentricity (m)	5e-4	Spring's damping (N/s)	9500
Bearing pitch diameter (m)	0.0449	Spring's stiffness (N/m)	8e9
Ball diameter (m)	0.0072		
Contact stiffness (N/m <sup>3/2</sup> )	8e9		
Number of balls	8		
Radial clearance (m)	1e-5		

#### 3.1. Only rotor self-weight condition (weight-dominant condition)

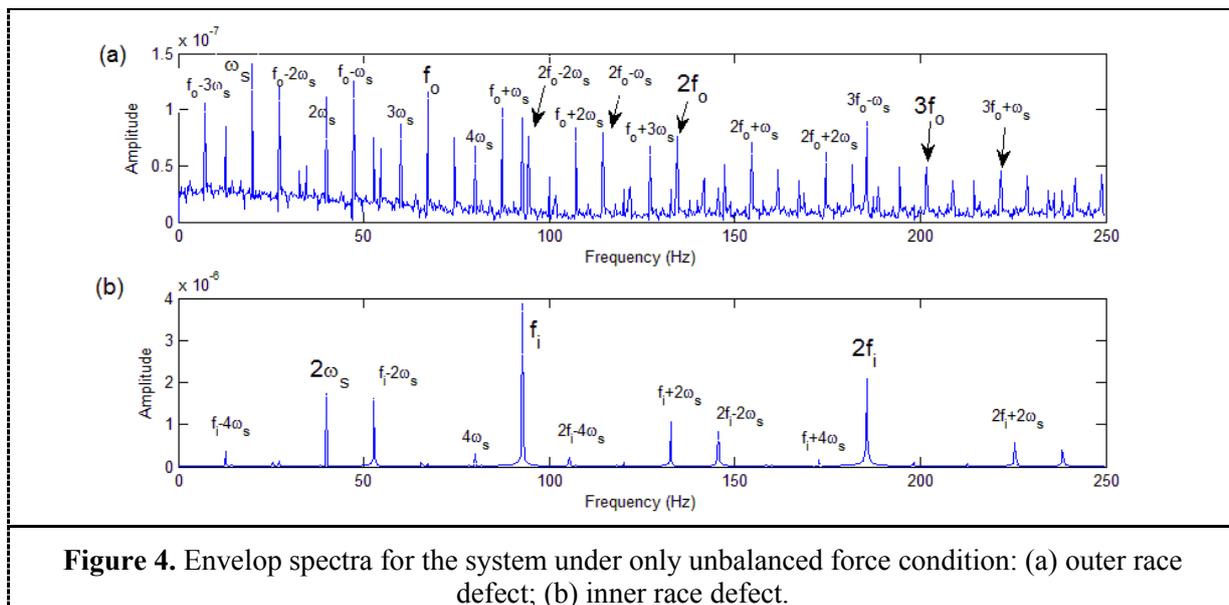
The unbalanced force excitation is ignored in this section in order for comparisons with the following analysis. Rotor speed is 20 rad/s, and the localized defect parameters are assumed to be  $\theta_{do} = 0.1rad$ ,  $\phi_{do} = 1e-3rad$  and  $h_o = 1e-3m$  for the outer race defect;  $\theta_{di} = 0.1rad$ ,  $\phi_{di} = 1e-3rad$  and  $h_i = 1e-3m$  for the inner race defect. The envelope spectra for the system with outer race defect and inner race defect are, respectively, shown in Fig. 3. It is shown that the fault characteristic frequencies are consistent with current results. For the outer race defect, the characteristic frequencies could be expressed by  $nf_o$  ( $n=1,2,3,\dots$ ), where  $f_o = N_b\omega_{bc}$  denotes the ball passing frequency. When the inner race has a localized defect, one can see from Fig. 3(b) that the characteristic frequencies are the

combinations of inner race fault frequency with rotating speed, i.e.  $|nf_i + m\omega_s|$  ( $n = 0, 1, 2, \dots, m = \dots, -2, -1, 0, 1, 2, \dots$ ). These frequencies are also identified and marked in Fig. 3.



### 3.2. Only unbalanced force excitation condition (non-weight-dominant condition)

For the lightly rotor, especially the vertical rotor, its weight might be small enough and even could be ignored in the analysis. In this section, only the unbalanced force is considered in the simulation. The envelop spectra for the system with outer race defect and inner race defect are present in Fig. 4. Compared with the weight-dominant condition (Fig. 3), one can see that the fault frequencies differ distinctly. For the outer race defect, it is shown from Fig. 4(a) that the fault frequencies are relatively complex. They could be summarized as:  $|nf_o + m\omega_s|$  ( $n = 0, 1, 2, \dots, m = \dots, -2, -1, 0, 1, 2, \dots$ ). When the inner race defect is consider, as shown in Fig. 4(b), it is shown that the fault frequencies are related to the inner race frequency and odd times of rotating speed, i.e.  $|nf_i + m\omega_s|$  ( $n = 0, 1, 2, \dots, m = \dots, -4, -2, 0, 2, 4, \dots$ ). Although the fault frequencies are still the combinations of inner race frequency and rotating speed, the difference is that only the odd times of rotating speed are found.



**Figure 4.** Envelop spectra for the system under only unbalanced force condition: (a) outer race defect; (b) inner race defect.

#### 4. Conclusions

The fault characteristics of a rolling bearing with localized defects under non-weight-dominant conditions are studied. Through envelope analysis, the fault characteristic frequencies of inner/outer raceway defects for various conditions, including the weight-dominant condition and non-weight-dominant condition, are presented and compared with each other. It is shown that the fault frequencies for the non-weight-dominant condition differ distinctly with that of the weight-dominant condition. Experiments on a vertical rotor with defective rolling bearing are being conducted to verify the theoretical results.

#### References

- [1] Sopianen J and Mikkola A 2003 Dynamic model of a deep-groove ball bearing including localized and distributed defects. Part 1: theory *Proc. Instn Mech. Engrs Part K: J. Multi-body Dynamics* **217** 201-211.
- [2] Zeki K and Hira K 2003 Simulation and analysis of vibration signals generated by rolling element bearing with defects *Tribology International* **36** 667-678.
- [3] Sawalhi N and Randall R B 2008 Simulating gear and bearing interactions in the presence of faults Part I. The combined gear bearing dynamic model and the simulation of localised bearing faults *Mechanical Systems and Signal Processing* **22** 1924-1951.
- [4] Rafsanjania A, Abbasiona S, Farshidianfarb A 2009 Nonlinear dynamic modeling of surface defects in rolling element bearing systems *Journal of Sound and Vibration* **319** 1150-1174.
- [5] Ashtekar A, Sadeghi F and Stacke L-E 2010 Surface defects effects on bearing dynamics *Proc. IMechE Vol. 224 Part J: J. Engineering Tribology* **224** 25-35.
- [6] Patel V. N., Tandon N and Pandey R K 2010 A Dynamic Model for Vibration Studies of Deep Groove Ball Bearings Considering Single and Multiple Defects in Races *Journal of Tribology* **132** 041101-1.