The effect of post-tensioning force magnitude and eccentricity on the natural bending frequency of cracked post-tensioned concrete beams

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Abstract. The effect of prestress force magnitude on the dynamic properties of uncracked prestressed concrete structures is something that has been widely debated among researchers to date. The effect of pre- and post-tensioning force magnitude on the natural bending frequencies of cracked prestressed concrete structures is something that is more established, and widely agreed upon. This paper describes the results of dynamic impact testing on damaged post-tensioned concrete beams. The natural bending frequency of the cracked beams were determined through experimental modal analysis. Dynamic impact response signals were obtained at different levels of post-tensioning force for the cracked beams. The Fast Fourier Transform was implemented and a peak picking algorithm was subsequently used to determine the natural bending frequencies of the beams. The relationship between prestressing force and natural frequency for both the cracked and uncracked beam sections was determined. The results for the cracked beams were compared to the results for the same uncracked beam sections. A marked difference in vibration behaviour was observed for the cracked beams between the non-fully prestressed and the fully prestressed case. Conclusions from the study are drawn and have profound implications in the fields of system identification and structural health monitoring in pre- and post-tensioned concrete structures.

1. Introduction

The effect of prestress force magnitude on the modal properties (frequency, damping and mode-shape) of uncracked prestressed concrete structures is something that has been widely debated among researchers to date [1]. Some researchers suggest that *"compression softening"* theory governs for uncracked pre- and post-tensioned concrete structures, therefore the natural frequency decreases with increasing prestress force magnitude [2, 3]. Other researchers [4] have produced non-linear mathematical models showing there is no relationship between natural bending frequency and prestress force magnitude, while others [5, 6] primarily through experimental approaches, have suggested that there is actually an increase in natural bending frequency with increasing pre- and post-tensioning force magnitude.

The effect of pre- and post-tensioning force magnitude on the natural bending frequencies of cracked prestressed concrete structures is something that is more established, and widely agreed upon. Saiidi et al. [5] report an increase in natural frequency with increasing post-tensioning force. As pointed out by Bruggi et al. [7] the tests carried out by Saiidi et al. [5] were conducted

on cracked beam sections only. Uncracked sections were not tested. Williams & Falati [8] present a formula to calculate the average effective second moment of area of a cracked concrete cross section. The effect of crack closure in accordance to this method is that it increases the effective second moment of area of the cross section, and subsequently the natural bending frequencies. Hop [9] agrees, reporting a decrease in natural bending frequencies with increased cracking, and states that increasing the prestressing force acts as to close the cracks, stiffen the section and increase the natural bending frequencies of the beam sections. Grace & Ross [10] also report a decrease in girder stiffness leading to a decrease in natural frequency also attributing it to cracking in the cross section. Unger et al. [11] state that a loss in post-tensioning increases the appearance of cracks which reduces the bending stiffness and subsequent natural frequencies of the system. DeRoeck concurs [12] that prestress loss results only in measurable changes in frequency if accompanied by originating cracks. Hamed & Frostig also report that large cracking damage yields drastic reduction in the natural frequencies of cracked prestressed concrete beams [13].

Pavic et al. [14] agree that prestressing is used to overcome excessive cracking and static deflection but argue that prestressing does not significantly improve dynamic behaviour, as that is governed by stiffness, mass and damping, on which prestress force has little influence. Dall'Asta & Dezi [15] consider it is possible to determine the prestressing force by measuring the natural frequency of a PSC structure in its cracked state only. Rodriguez et al. [16] acknowledges this fact in relation to post-tensioned concrete wind turbine towers, stating that uncracked towers maintain their original stiffness and frequency, but once the towers are cracked and the cracks have been decompressed, any vibrations in the tower will mobilise smaller stiffness, which will be shown by the vibration frequencies.

This paper will describe the outcome of dynamic impact testing conducted on nine cracked post-tensioned concrete beam sections in the laboratory. The results for one beam will be discussed in detail. The relationship between the natural bending frequency and the post-tensioning force for cracked concrete sections will be determined through experimental modal analysis of the cracked beams. Comparisons will be made between the behaviour of the uncracked beams and the cracked beams. The conclusions have profound implications in the fields of system identification, damage detection and structural health monitoring.

2. Experimental Set-Up

Following dynamic impact tests in the lab on nine uncracked post-tensioned concrete beam sections, which have been described previously [17], 4-point bending was applied to each beam as outlined in Figure 1. The individual beams differed in that they had different straight profiled post-tensioning strand eccentricities. The transverse load was applied incrementally and the crack patterns were observed at different values of load application. The loading increments and yield moment for Beam 1 (e=0mm) are outlined in Table 1. First, second and third cracks were identified by visual inspection only, and were not determined scientifically.

Table 1: The incremental damage states of Beam 1 (e=0) until yielding.

Beam	1st **crack (kN)**	2nd **crack (kN)**	3rd **crack (kN)**	Yield (kN)	M_{yield} (kN-m)
C1	24.6	41.3	49.8	70.4	21.12

Yielding was identified as the point where the beams could no longer hold any additional load and deflections increased significantly. Visible structural cracks formed in the section. Vertical

9No. 150 wide x 250 deep PS beams with following PS
bar/strand eccentricities:
e = -52, -39, -26, -13, 0, +13, +26, +39, +52

11No. PS Load testing levels:
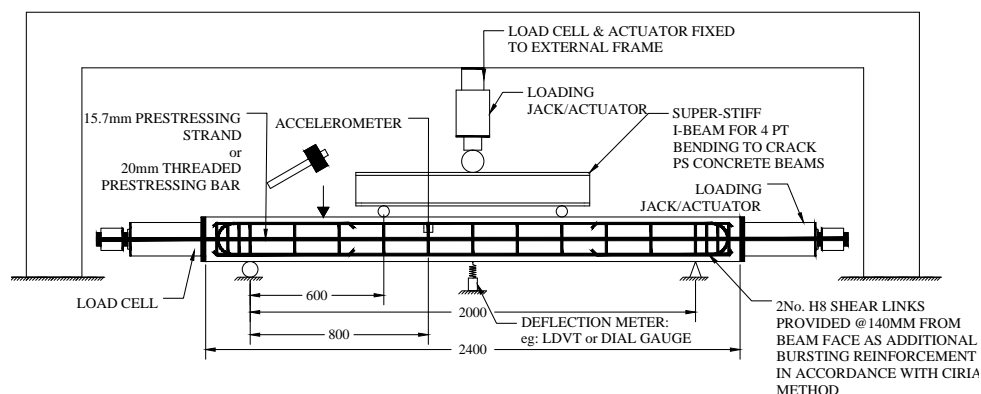P = 0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200 kN

Figure 1: Experimental set-up.

flexural cracks formed in the area of high moment, between the span of the spreader beam, whereas diagonal shear cracks formed in the areas between the end of the spreader beam and the supports. Following cracking, dynamic impact testing was conducted on the beams. Figure 1 shows the experimental set-up. An accelerometer was placed 800mm from one of the supports as shown, enabling the first three modes of vibration to be captured. Three strain gauges were placed equi-distant between the support and mid-span, in order to capture the mode shapes of vibration. The beam was struck ten times at each strain gauge location using an impact rig assembled in the laboratory. This ensured repeatability of the experiment and enabled the error in the estimation of the frequencies to be calculated. This experimental procedure was then repeated at incremental values of post-tensioning load, as outlined in Figure 1. The post-tensioning load was applied using a 15.7mm Freyssinet post-tensioning strand, secured either side of the 300 ton loading jacks as shown. The different straight profiled strand eccentricities are also outlined in Figure 1. A similar testing procedure was applied to the uncracked beam sections and has been outlined in previous works by the same authors [17].

3. Experimental Analysis

Each beam was struck ten times at 3 different impact locations, for 11 different post-tensioning load levels. The response was measured by one accelerometer and three strain gauges. As such, this method of dynamic testing is known as the multiple input, multiple output (MIMO) method. The Fast Fourier Transform (FFT) was then applied to the response signals to convert into the frequency domain. No data was collected on the impact signal, and as such, the frequency response functions (FRFs) of the beams could not be determined. The FRFs are defined as the transfer function, or the ratio between the input and output signals in the frequency domain. Since the input signal was not quantified, the FRFs were not obtained. In this way, the study is an output-only study. However, FFTs of the output signal were sufficient to determine the natural vibration frequencies of the structural system.

As outlined in previous work [17, 18], having calculated the FFTs of the signals, the fundamental vibration mode was intelligible, however, due to the presence of significant noise in the data, higher modes were not. In order to eliminate this noise and identify higher modes, a signal processing technique was established by the authors and applied to the raw data.

Figure 2 and Figure 3 show the results of two processed signals. The higher modes are now intelligible. A peak picking algorithm is invoked to determine the natural frequencies of the system. Figure 2 shows an accelerometer response signal of a cracked beam for zero post-tensioning load, indicating a structural dynamic response of the cracked beam for a non-fully prestressed condition, in which the cracks are open. It is evident that the structural response of the non-fully prestressed condition is complex. The fundamental mode, in this case contributes most to the response, but its contribution is not dominant. The contribution of the other modes is relatively similar.

Figure 3 shows an accelerometer response signal of the same cracked beam for a post-tensioning load level of 180kN, indicating a structural dynamic response of the cracked beam for a fully prestressed condition. Here, the dominance of the first mode of vibration is evident. The beam, when fully prestressed, behaves monolithically and vibrates as one entity rather than a series of individual cracked entities, and hence the dominance of the first vibration mode returns. This response is in line with the response of the uncracked 'virgin' beams.
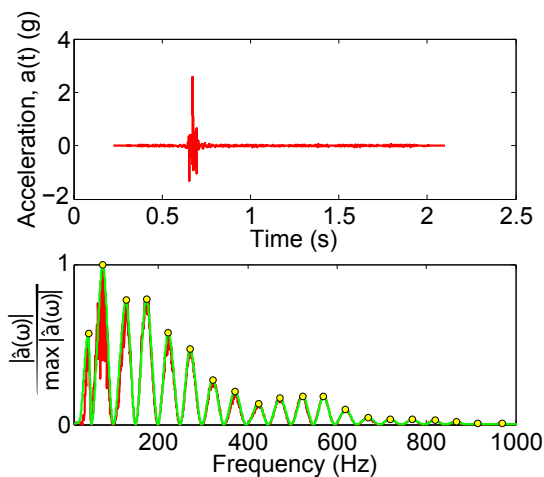

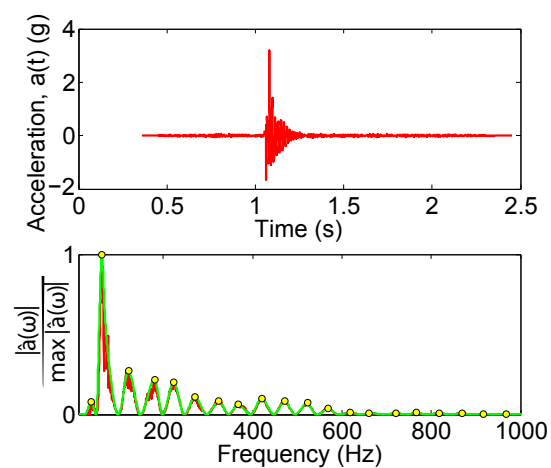Figure 2: Structural dynamic response of cracked beam for non-fully prestressed case, Beam 1, e=0mm

Figure 3: Structural dynamic response of cracked beam for fully prestressed case, Beam 1, e=0mm

4. Experimental Results

Figure 4 shows the structural dynamic response of Beam 1 (e=0) in its uncracked condition. The dominance of the first mode of vibration is evident in this case, since the relative modal amplitude of the fundamental bending mode accounts for a large proportion of the modal mass of the structure. Figure 5 shows the structural dynamic response of Beam 1 in its cracked condition. In this case, the dominance of the first mode of vibration is not clear. The structural dynamic response is much more complex and the contribution of the higher modes of vibration are clearly far more significant in the cracked case. This is evident up to a certain threshold value of post-tensioning force. The dominance of the first vibration mode again becomes apparent in the cracked case when the post-tensioning force causes the structure to behave as one monolithic structure rather than a series of cracked entities. This is the fully prestressed condition. In this case, for Beam 1, e=0mm, it was identified to be at a value of 160kN.

Figure 6 and Figure 7 show regression analysis of fundamental frequency versus post-tensioning load for the same beam in its uncracked and cracked states. From the regression analysis of the uncracked case in Figure 6, no statistically significant change in fundamental
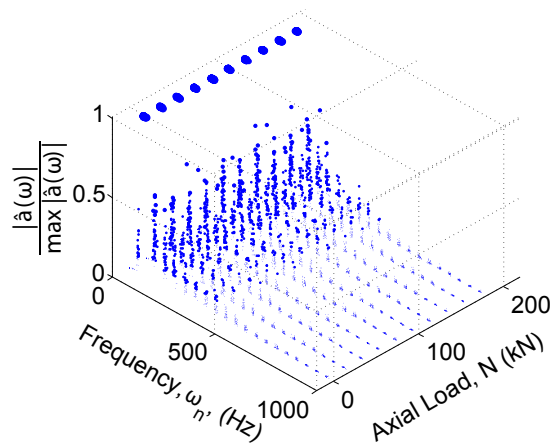
Figure 4: Rel. Modal Amp. Uncracked Beam 1, e=0mm
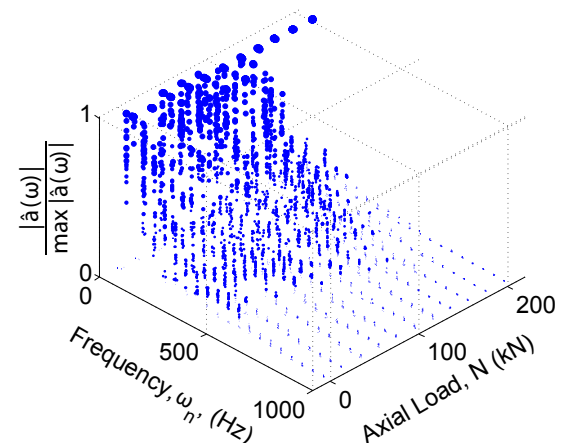

Figure 5: Rel. Modal Amp. Cracked Beam 1, e=0mm

frequency with increasing post-tensioning force can be observed, as previously reported [17]. From first glance at the simple linear regression analysis of the cracked beam case, an overall decreasing trend in fundamental frequency versus post-tensioning load is observed. However, when analysed in conjunction with Figure 5, it becomes apparent that there are two distinct trends in the data. For the non-fully post-tensioned case (0-160kN), the structural dynamic response is complex, the error in the estimation of the natural frequency is high, the structure behaves dynamically as a series of independent vibrating entities and a decreasing trend in fundamental frequency is observed. However, at a threshold post-tensioning load level and upward (160-200kN), the dominance of the first vibration mode returns, the error in the estimation of the fundamental frequency decreases, the structure begins to behave monolithically again, and an increasing trend can be observed for the final 3 data points. This increasing trend, has been widely reported for cracked beam sections [5, 8, 10, 11, 12, 13, 14, 15], however, is not observed for uncracked beam sections [4, 15, 17]. The stiffness of the section is increased by increasing the prestressing force due to crack closure, and the second moment of area moves from the cracked to the uncracked value.
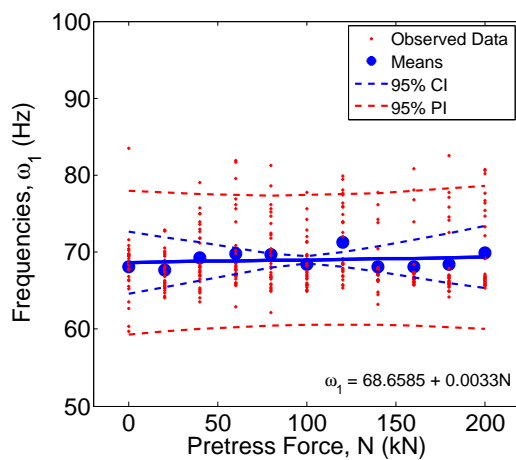

Figure 6: Regression analysis of fundamental frequency vs. post-tensioning load; uncracked case.
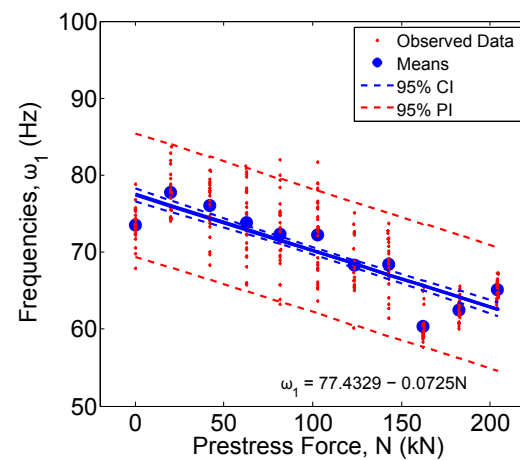

Figure 7: Regression analysis of fundamental frequency vs. post-tensioning load; cracked case.

Figure 8 shows the comparison between the simple linear regression models for the cracked and uncracked beam cases. As discussed previously, this is not strictly a reasonable comparison as two different trends can be observed in the cracked beam case. The initial fundamental frequency for the cracked beam is higher to begin with, which is counter-intuitive. If it is considered that the beam in its damaged state behaves as a series of different vibrating cracked entities rather than as one monolithic structure, then the span length of each individual vibration entity is significantly reduced. Since the span length is a denominator in the equation for the prediction of natural frequency for simply supported beams, this makes intuitive sense. This span length increases as the post-tensioning force increases and more and more structural cracks are closed, meaning the measured frequency decreases. On the other hand, when the beam begins to vibrate again monolithically, the structural dynamic response becomes less complex and the fundamental mode dominates, an increasing trend is observed in the frequency with increasing post-tensioning force. At the threshold value of post-tensioning force, the span length reaches its original value. From this point onwards, as shown in Figure 9 an increasing trend in fundamental frequency is observed, and this is attributed to the gain in geometric stiffness due to crack closure, in accordance with previous research works [5, 8, 10, 11, 12, 13, 14, 15].
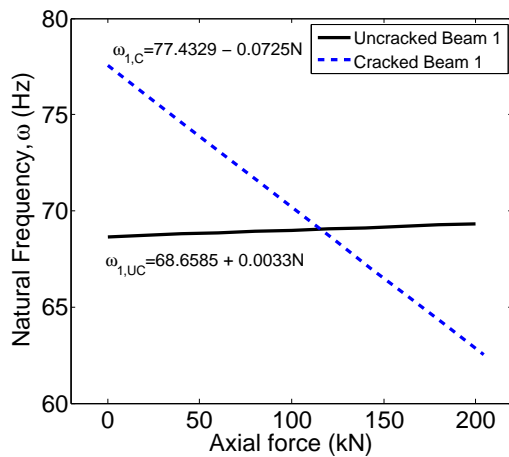

Figure 8: Regression comparison between cracked and uncracked beam cases.
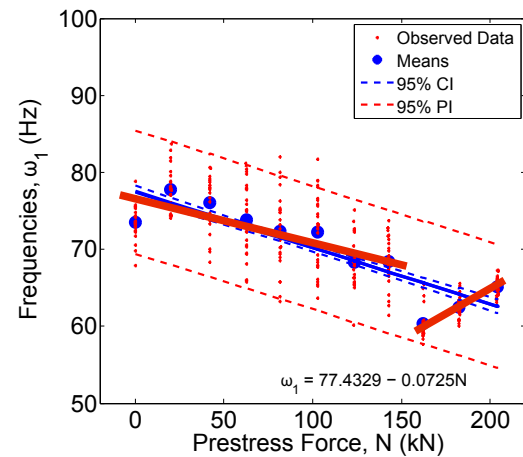
Figure 9: Evidence of the two trends present in the data.

Table 2 shows the statistical regression parameters - intercept parameter ($\alpha_{0,iuc/c}$), slope parameter ($\alpha_{1,iuc/c}$), standard error, t-values and 95% confidence intervals when regressing ω_1 on N for Beam 1, e=0 ($i = 1$) in its cracked (c) and uncracked (uc) case. The corresponding linear regression equations are obtained by substituting into Equation 1. Table 2 should be read in conjunction with Figure 8 for completeness.

$$\omega_1 = \alpha_{0,i} + \alpha_{1,i}N \tag{1}$$

5. Conclusions
Whereas there is significant disagreement among researchers as to the effect of prestress force magnitude on the natural vibration frequency of uncracked prestressed and post-tensioned concrete sections [1], the effect of prestressing force on the bending frequency of cracked pre- and post-tensioned sections is more established. Researchers agree that, for cracked pre- and post-tensioned concrete beams, increasing the post-tensioning force acts as to close the cracks, increase the second moment of area of the cross section, resulting in an increase in the natural bending frequencies [5, 8, 10, 11, 12, 13, 14, 15].

Table 2: Statistical analysis on regression parameters for ω_1 on N

Beam	e (mm)	Reg. P.	Value	SE	t-value	t-crit.	p	95% CI
B1	0	$\alpha_{0,1uc}$	68.6585	2.0515	33.4680	1.9672	0.0000	(64.6228,72.6942)
		$\alpha_{1,1uc}$	0.0033	0.0204	0.1635	1.9672	0.8702	(-0.0368,0.0434)
C1	0	$\alpha_{0,1c}$	77.4329	0.4465	173.4027	1.9680	0.0000	(76.5541,78.3117)
		$\alpha_{1,1c}$	-0.0725	0.0038	-18.8934	1.9680	0.0000	(-0.0801,-0.0650)

For the uncracked beam sections, the fundamental mode of vibration dominates the structural dynamic response and no statistically significant change in the fundamental vibration frequency can be observed. As such, for uncracked pre- and post-tensioned concrete beams, the prestressing force does not change the dynamic stiffness significantly. Pavic et al. [14] concluded similar, stating that the dynamic properties are dependent on mass, stiffness and damping properties, on which prestressing does not have a major influence. It should also be noted that the error in the estimation of the frequency is high. Changes in frequency are affected by temperature effects, lack of ideal support conditions and material variability, which is reported to be in the order of 6-18% [19, 20], therefore any potential effect of prestressing force on dynamic properties is lost within this error.

For the cracked beam sections, the implications of prestressing force on dynamic properties is more apparent. For the cracked beam sections, it was found that the structural dynamic response was far more complex than for the uncracked beams. There is no clearly dominant vibration mode and the response comprised of all modes of system. However, at threshold values of post-tensioning force there is evidence of dominance of first vibration mode. This threshold post-tensioning load value is the load value at which the structure begins to vibrate monolithically again. The trend from this point onwards is easily identified. There is an increase in fundamental frequency with increasing post-tensioning force. This is attributed to crack closure and the subsequently increase the flexural stiffness of the member, as reported by previous authors [5, 8, 10, 11, 12, 13, 14, 15].

The implications that this has for system identification, and structural health monitoring are profound. The above suggests that the identification of the existing prestressing force in uncracked prestressed concrete structures is not feasible through measurement of the natural frequency of the structure only, due to the confounding variables (temperature effects, support conditions, material properties). However, the existence of the relationship between the frequency and the level of cracking in the structure does allow for the calculation of the damage (cracking) state of the structure from the measurement of the natural frequency [12, 21, 22] and application of the formula presented in [8].

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