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Damage Detection in Bridge Structure Using Vibration Data under Random Travelling Vehicle Loads

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Abstract. Due to the random nature of the road excitation and the inherent uncertainties in bridge-vehicle system, damage identification of bridge structure through continuous monitoring under operating situation become a challenge problem. Methods for system identification and damage detection of a continuous two-span concrete bridge structure in time domain is presented using interaction forces from random moving vehicles as excitation. The signals recorded in different locations of the instrumented bridge are mixed with signals from different internal and external (road roughness) vibration sources. The damage structure is also modelled as the stiffness reduction in one of the beam element. For the purpose of system identification and damage detection three different output-only modal analysis techniques are proposed: The covariance-driven stochastic subspace identification (SSI-COV), the blind source separation algorithms (called Second Order Blind Identification) and the multivariate AR model. The advantages and disadvantages of the three algorithms are discussed. Finally, the null-space damage index, subspace damage indices and mode shape slope change are used to detect and locate the damage. The proposed approaches has been tested in simulation and proved to be effective for structural health monitoring.

1. Introduction

One of questions in research attention is the use of structural response from operational dynamic loads in the damage detection procedures. In the long service life and continuous operation process, bridge may suffer resistance and damage accumulation under the comprehensive effect of random vehicle loads and environmental effects such as wind load and rain load. By placing sensors on bridges, the bridge health status can be monitored online through health monitoring system in order to determine the degree of damage. For bridge monitoring the vehicle-bridge structure interaction effect needs to be considered in the modal parameter identification. Generally, the modal parameters of the vehicle-bridge system are changing when the random vehicles are moving on top of the bridge (include the road roughness). For such a system, system identification tools must be developed and tested using numerically generated time histories so as to verify the methodologies. For identification, using time-frequency tools, which are non-parametric methods for processing responses of non-linear or non-stationary system, had been studied [1, 2]. A linear approach derived from the stochastic subspace identification (SSI) method, together with the moving window SSI to examine the time-varying system, has also been adopted for vibration-based system identification [3].

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In this study, comparison of three different time-domain identification and damage detection methods are proposed to identify the bridge model properties by using the simulation data of a vehicle-bridge structural interaction under random travelling loads and with consideration of surface roughness. First, the automatic covariance-drive stochastic subspace identification (SSI-COV) is proposed to extract the system poles through stability diagram. In cooperated with the wavelet packet transform (WTP) sifting process for modal phase consistency, spurious modes can be removed from using SSI. For well-separated modes, the blind source separation (called Second-order Blind Identification, SOBI) algorithm can also be used for identification and damage detection. Advantages and disadvantages of the two methods are discussed. Finally, a parametric identification method, using multivariate AR-model, is adopted to enhance the damage detection and compare the result with other methods.

2. Simulation of bridge-vehicle interaction system

A two span bridge model is proposed in this study, as shown in Fig.1. It shows a bridge carrying a series of random moving vehicles which are modelled as a sprung-mass system. The bridge is modelled as a Bernoulli-Euler beam by finite element method. The information for the bridge and vehicles are provided in Table 1. The traffic loads are assumed as random and to be moving as a group from both sides (two ways) of the bridge with random velocity. The prescribed velocity of each car is random generated and shown in the travel time across the bridge with span length of total 200 m, as shown in Fig.2. A total of 16 cars are on the bridge during the measurement. From Fig.2 one can see that during the recorded time window (i.e. 30. sec) some cars are still on the bridge and some cars may just enter/leave the bridge. In this paper, the simulation of road roughness in vehicle–bridge interaction is also considered [4, 5]. The random road surface roughness of the bridge can be described by a kind of zero-mean, real-valued, stationary Gaussian process defined by ISO 8608 [6]. The moving vehicle is also modelled as a moving mass with spring-mass system. Based on the finite element method a total of 10 beam elements (Bernoulli-Euler beam) are used and the nodal points are considered as measurements. First, the system equation of motion is formed, then the equations of motion of the bridge-vehicle interaction system can be solved by Newmark-β method. Acceleration responses from each nodal point (similar to measurement point) are collected with duration of 30. sec. Both intact and damaged situation of the bridge structure are simulated. For damaged case a 50% of stiffness reduction at element 7 (between measurement point 5 and 6) is assumed.

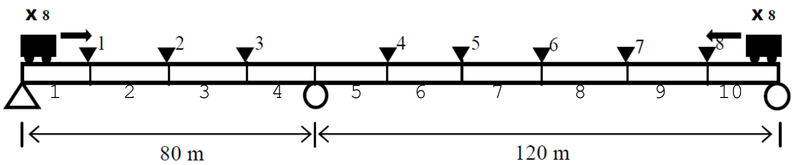

Figure 1. Bridge model with two-way travelling vehicles and the location of measurement points.

Table 1. Natural frequencies of bridge model and vehicle information.

Bridge Data	Bridge Freq. (Hz)	1st mode	2nd mode	3rd mode	4th mode	5th mode	6th mode	7th mode	8th mode
		2.12	5.29	8.37	16.42	20.50	30.31	41.55	48.65
Vehicle Data	Vehicle system natural frequency: 1.96 Hz ~ 6.5 Hz								
	Vehicle Weight (ton) : 1.5 ~ 9.0								

3. Identification of bridge-vehicle interaction system

To identify the dynamic characteristics of the bridge directly from the vibration measurement during random travelling vehicles, two different approaches are employed: the covariance-drive stochastic subspace identification (SSI-COV) and the second-order blind identification (SOBI). Damage detection will also be studied from the extracted vibration features and system modal properties.

Discussion on the identification of damage location using parametric Multivariate-AR model is also presented for comparison.

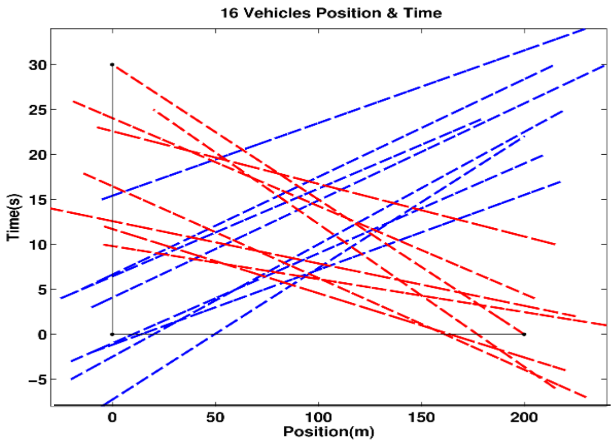

Figure 2. Plot the relationship between the recorded time window (30. sec) and the location of vehicles (8 cars from right and another 8 cars from left) travelling with different speed.

3.1. Automatic covariance-drive stochastic subspace identification (SSI-COV)

A fully automatic identification scheme of reference-base SSI-COV algorithm is developed to extract the dynamic characteristics of the monitoring structure. Technique to remove the spurious modes is included in the automatic identification scheme and the calculation of uncertainty on the identified modal parameters is to confirm the accuracy of the identified modes. The following 4 stages are developed to provide a good quality of the stability diagram from using SSI-COV [13]:

a. Conduct down-sampling on the recorded data (from 200. Hz to 50 Hz), and then adopt singular spectrum analysis (SSA) on the data Toeplitz matrix so as to construct the stability diagram (fix number of rows in data Hankel matrix and change number of order in Singular Value Decomposition).
b. Select output modal amplitude criteria (OMAC) and mode phase co-linearity (MPC) to remove the spurious modes from stability diagram. The hard criteria, such as threshold values on mode assurance criteria, system natural frequency and damping ratio are also employed.
c. Use the 4-means clustering algorithm to minimize the sum of the squared Euclidean distance between the extracted poles, and determine the suitable poles satisfy the criteria.
d. Grouping of poles with each individual mode and identify the physical modal parameters of each mode by using uncertainty bounds of modal frequency and damping ratio as weighting factor for each pole in each specific group of poles.

Through the proposed 4 stages in SSI-COV analysis, one can significantly increase the autonomous ability to remove the spurious modes and create a clear and identifiable stability diagram. The system dynamic characteristics can also be identified.

3.2. Second-order blind identification (SOBI)

In applying SSI-COV some parameters need to be determined, such as the number of row in data Hankel matrix and the number of order, which depends on the dynamic characteristics of the system. Different from the SSI-COV algorithm, blind source separation (BSS) is also a signal processing technique, which attempts to recover individual unknown but statistically independent components with only knowledge of measurements [7]. There is no need to provide parameters in using BSS algorithm. The simple model for BSS assumes the existence of *n* independent signals $\mathbf{S}^{\mathbf{T}} = [s_1(t), \cdots, s_n(t)]$ and *m* measurements $\mathbf{X}^{\mathbf{T}} = [x_1(t), \cdots, x_m(t)]$. This is completely represented by the mixing equation

$$\mathbf{X}(t) = \mathbf{AS}(t) \tag{1}$$

where $A \in R^{mxn}$. If the number of measurements and the excitation sources are remain the same, then the source vector **S** can be obtained once the mixing matrix **A** is estimated. But the physical meaning of the mixing equation needs to be established. Now consider a linear time-invariant system with n degree-of-freedom (DOF). The system response can be formulated as the product of mode shapes $[\mathbf{\Phi}] = [\phi_1, \phi_2, \cdots, \phi_n]$ and modal coordinates **U**(t)**:**

$$\mathbf{X}(\mathrm{t}) = [\mathbf{\Phi}]\mathbf{U}(\mathrm{t}) \tag{2}$$

Compare Eqs.(1) and (2), under the conditions where the modal coordinates are mutually uncorrelated with non-similar spectra for dynamic system which is similar to the assumption of BSS that provided a one-to-one mapping between the mixing matrices (sources) and the vibration modes (modal responses) of the system, therefore, the BSS can be interpreted as the modal expansion.

Since most BSS techniques are based on a model in which the sources are mutually independent, however, the second order blind identification (SOBI) belongs to a BSS technique that makes use of the temporal structure. It means that the sources have different autocorrelation function (or power spectra) and are mutually uncorrelated [8, 9]. To perform SOBI two computation steps are required: whiting process and determining the matrix **U** that approximately diagonal the whitened covariance matrix. For whiting process, first, conduct the eigenvalue decomposition of the signal covariance matrix:

$$[\mathbf{R_X}(0)] = \mathbf{EDE^T} \tag{3}$$

From which the weighting matrix is defined:

$$\mathbf{W} = \mathbf{D}^{-1/2}\mathbf{E}^T \tag{4}$$

Additionally, the whitening observed data **Z** can be expressed as:

$$\mathbf{Z} = \mathbf{W}\mathbf{X} \tag{5}$$

The covariance matrix of the whitening data becomes:

$$E[\mathbf{ZZ}^{\mathrm{T}}] = \mathbf{WA}E[\mathbf{SS}^{\mathrm{T}}]\mathbf{A}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}} = \mathbf{WAA}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}} = \mathbf{UU}^{\mathrm{T}} = \mathbf{I} \tag{6}$$

where $E[\mathbf{SS}^{\mathbf{T}}] = I$ and $\mathbf{U} = \mathbf{WA}$ **.** Because sources are stationary and uncorrelated, therefore, the time-delayed covariance matrix of sources is diagonal. From this one can easily determine the **U** matrix through an eigenvalue decomposition of the time-delayed covariance matrix of whitened data. For different time-delayed τ_i, the whitened covariance matrix is also different. SOBI algorithm employs the joint approximation diagonalization (JAD) technique to determine the matrix **U** that approximately diagonalized the whitened covariance matrix with different time-delayed value τ_i [10]. Procedures of using SOBI for modal identification are listed as follows:

1. Pre-Processing： Zero-mean can remove the offset and slope of the measured signals. Whitening makes the measured signals uncorrelated and have unit variance.
2. JAD Analysis： After applying joint approximation diagonalization, the **U** matrix which diagonalized the whitened covariance matrices with different time-delayed value can be obtained.
3. Modal Analysis： When mixing matrix **A** is obtained; the source vector **S** can be determined and apply Fourier transform to the source vector to determine the system natural frequencies.

4. Vibration-based system identification of bridge under travelling vehicle loads

Acceleration response data (vertical vibration) of the vehicle-bridge structure interaction through numerical simulation was used for analysis. The original sampling rate was 500 Hz. Before adopting the SSI-COV algorithm to extract the system dynamic characteristics, the data was down-sampled to 125 Hz for the analysis (through Butterworth low-pass filter). In this simulation a total of 16 cars (both ways) travelled on the bridge with different speed simultaneously. Figs. 3a and 3b shows the stability diagram developed from SSI-COV. The size of the data Hankel matrix is (500x500). Fig. 3a used the OMAC criteria of 99.99% to remove the spurious mode while Fig. 3b used both OMAC (99.99%) and MPC (99.9%) criteria. It is clear that with adding extra MPC criteria some spurious modes near

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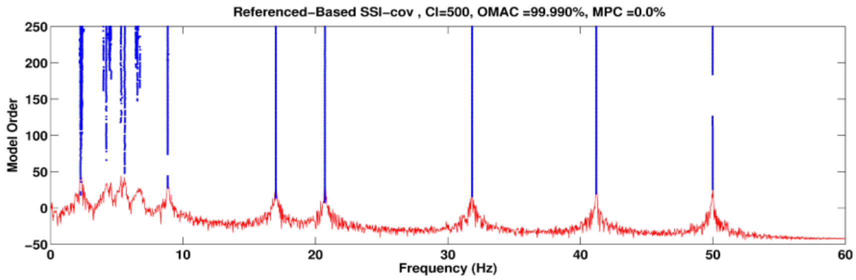
frequency 4.5 Hz and 6.4 Hz can be removed from the stability diagram, but still has some poles which still stay in the stability diagram. This is the phenomenon of complex modes (the effect of travelling waves). To check poles under such ambiguity, wavelet packet transform (WPT) is applied to extract signal near the frequency where ambiguity poles are located. As shown in Fig. 4, the response (from WPT) from each measurement is plotted at frequency of 4.28 Hz and 5.26 Hz (from the location of ambiguity poles) [14]. It is observed that the phase difference among the measurement at frequency of 4.28 is not consistent even though the identified poles are satisfy both the OMAC and MPC criteria. Through such an observation these ambiguity poles can be also be removed from the stability diagram.



. b

Figure 3a. Stability diagram with using OMPC criteria to remove the spurious modes.

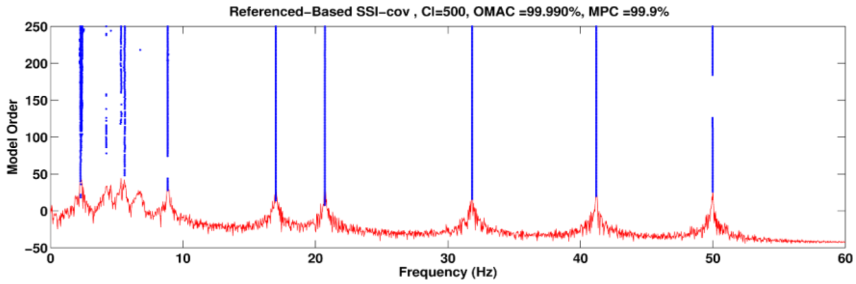


Figure 3b. Stability diagram with using both OMPC and MPC criteria to remove the spurious modes.

(a) (b)

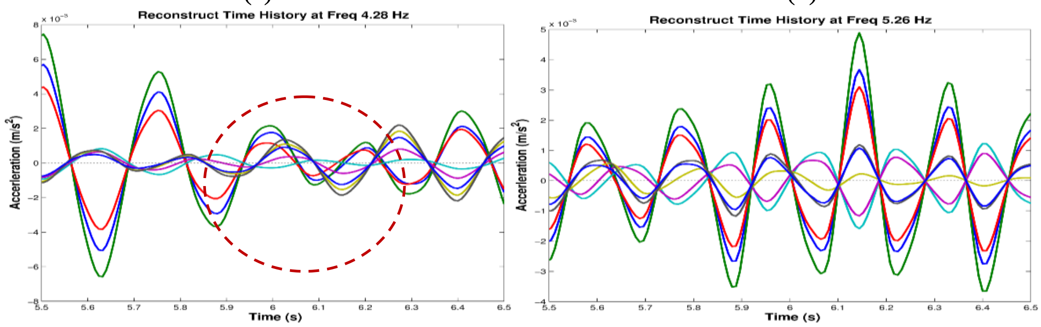


Figure 4: (a) Extract vibration signal at frequency of 4.28 Hz,
(b) Extract vibration signal at frequency of 5.26 Hz.

As discuss before, in using the SSI-COV to identify the system dynamic characteristics there is still have some hard criteria, such as the size of data Hankel matrix or threshold of damping ratio, that need to be determined (cannot automatically setup). On the contrary, in applying BSS less pre-determined parameters is required, therefore, the BBS technique is used to minimize of using model parametersfor system identification. Fig. 5 shows the Fourier amplitude spectrum of the identified source signals (a total of 8 modes which are equivalent to the system modal responses). There is no spurious mode needs to be removed. It is confirmed that the modal frequency from each identified source is consistent with the result from SSI-COV, as shown in Table 2. The result is very consistence. Results of identification from SSI-COV and BSS-SOBI algorithms can also be also applied to the damage case (a 50% reduction of flexible stiffness between measurment pont 5~6). Table 2 also lists the identified system natural frequencies for comparison. It is observed that the change of natural frequencies of the damage case is obvious and particulary focus on the higher modes. The identified mode shapes from the mixing matric **A** of BSS for both damaged and undamaged case is shown in Fig. 6. It is observed that there is no significant difference among them, so the identified mode shapes are not a good indicator for damage detection even though there have obvious chang on the identified modal frequencies. To perform damage detection further analysis is required.

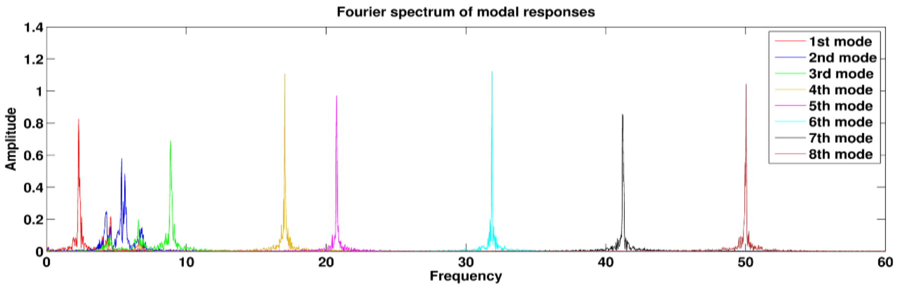


Figure 5. fourier amplitude spectrum of each identified source signal using SOBI

Table 2. Comparison on the identified natural frequencies between damaged and udamafged case.

Bridge Model		Undamaged case		Damaged case	
Mode	Theory (Hz)	SSI-COV (Hz)	SSOBI (Hz)	SSI-COV (Hz)	SOBI (Hz)
1st	2.305	2.262	2.300	2.078 (-0,184)	2.133 (-0.167)
2nd	5.300	5.613	5.367	5.596 (-0.017)	5.367 (-0.0)
3rd	8.829	8.840	8.867	8.366 (-0,474)	8.367 (-0.500)
4th	17.071	17.002	17.033	16.377 (***-0.625***)	16.430 (***-0.603***)
5th	20.815	20.714	20.733	20.386 (-0.346)	20.430 (-0.303)
6th	32.251	31.823	31.867	29.941 (***-1.882***)	29.970 (***-1.897***)
7th	42.134	41.196	41.233	40.643 (***-0.553***)	40.070 (***-1.163***)
8th	51.692	49.981	50.033	47.193 (***-2.788***)	47.230 (***-2.803***)

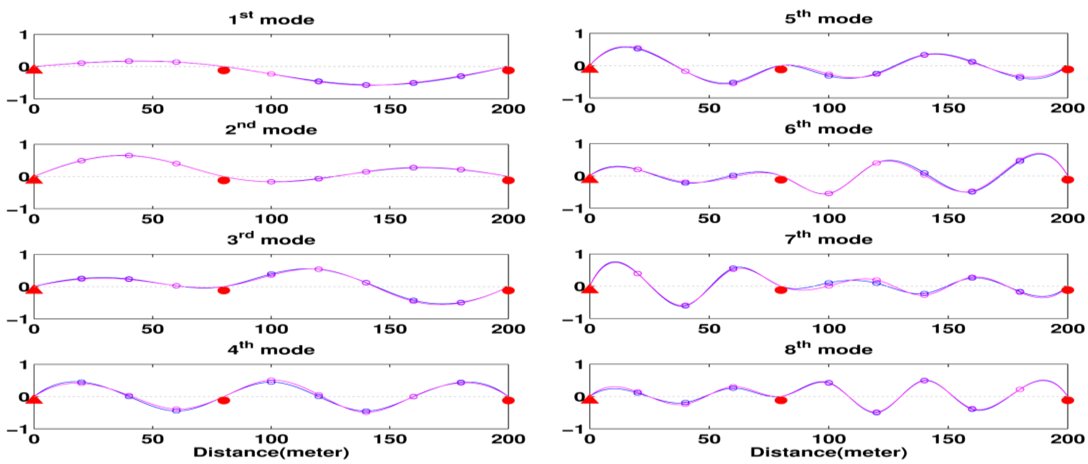


Figure 6. Comparison on the identified mode shapes between undamaged and damaged cases(SOBI method)

5. Damage detection using reponse measurement

To identify the damage location of the beam-like structure, three different approaches are used. First, taking the advantage of constructing the data Hankel matrix, a method based on null subspace and subspace damage detection is used [11]. It is defined the null-space damage index (DI_n) as:

$$DI_n = mean\left\{\left|\mathbf{U}_s \mathbf{U}_{n0}^{\mathbf{T}}\right|\right\} \tag{7}$$

where $\mathbf{U_s}$ is the subspace of the target analysis matrix $\mathbf{Y}$ (multiplication of the transpose of data Hankel matrix) and $\mathbf{U_{n0}}$ is the null-space of the reference analysis matrix $\mathbf{Y_0}$. Different from the null-space damage index, the subspace damage index, DIs, is also defined as [11]:

$$DI_S = \frac{\sum_{i=1}^{K} (\mathbf{y_i^T y_i} - \mathbf{y_i^T U_{s0} U_{s0}^T y_i})}{\sum_{i=1}^{K} \mathbf{y_i^T y_i}} \tag{8}$$

where $\mathbf{y_i}$ is the i[th] column of matrix **Y.** The term in the denominator is used to normalize the damage index DI_s so as to have DI_s ranges from 0 to 1. The reference data (undamaged data) is needed to calculate the damage indices. Larger DI_s or DI_n means the monitor structure is different from the reference structure.

It is also believed that data collected from near damage location may record significant response signals due to the occurrence of damage in the structural member. Damage index using the difference of Hilbert amplitude spectrum of the damaged data and the reference (undamaged) data can be used for locating the damage. It is now defined the Hilbert amplitude assurance criteria (HAAC) between the damage case and reference case of the Hilbert amplitude spectrum at sensor location *i* as $\mathbf{e^i_{t,f}}$:

$$\mathbf{e^j_{t,f}} = \frac{\sum_t \sum_f (\mathbf{H}^{d_j}(t,f) \times \mathbf{H}^{Ref}(t,f))}{(\sum_t \sum_f \mathbf{H}^{Ref}(t,f)^2 \cdot \sum_t \sum_f \mathbf{H}^{d_j}(t,f)^2)^{1/2}} \tag{9}$$

Large value of $\mathbf{e^i_{t,f}}$ indicated the similarity is higher at meaurement location *i*. As shown in Fig. 7a, at node 5 and node 6 the HAAC value is much lower which indicate the damage location.
Finally, to verify the accuracy of the proposed method in damage location identification, the the multivariate AR model with order of 3 was used to identify the model parameters [12]. The analysis of multivariate AR model is conducted using all the measurement data. By using the identified model parameter from undamaged data, and adopt the damaged data for one step prediction for all measurement node, predict error can determined. Fig. 8 shows the error norm increases significanrtly near the location where element stiffness reduced. Comparison with the estimated damage location using the change of mode shape curvature is als shown in the same plot. The result shows the identified damage location is almost the same from three different approaches.

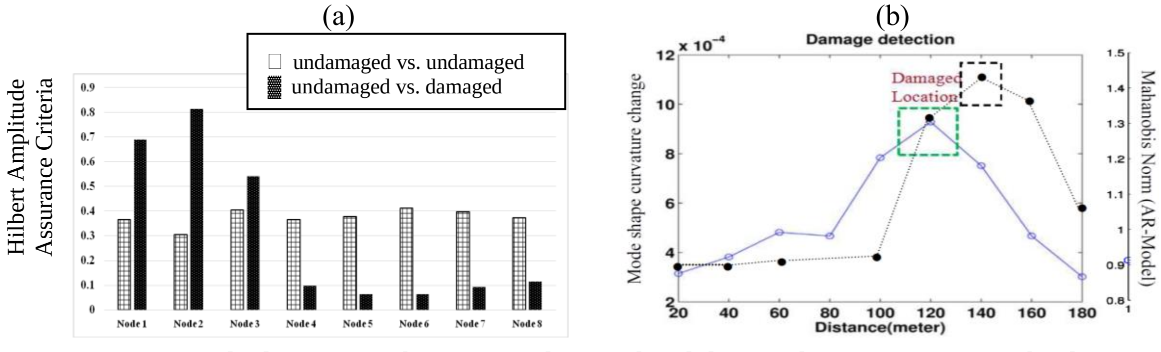


Figure 7. (a) Calculate HAAC between undamaged and damaged cases to estimate the damage location; (b) Plot the change of mode shape curvature change (open circle) and the prediction error norm (close circle) with respect to measurement location.

6. Conclusions

This study presented three different system identification methods to extract the dynamic characteristics and damage detection by using output-only measurement of the bridge-vehicle interaction system. Through numerical simulation the dynamic response of a 2-span bridge subject to random travelling vehicle loads and with the consideration of surface roughness was generated for this study. A new developed automatic covariance-drive stochastic subspace identification (SSI-COV) algorithm, the second-order blind identification (SOBI) and multivariate AR-model are used to extract the dynamic characteristics of the bridge system. Technique to remove the spurious modes in developing the stability diagram of using SSI-COV is discussed. It is demonstrated that both methods can exactly identify the system natual frequencies and mode shapes. With the implemeation of reduction in element stiffness to simulate the damage of the bridge, through the proposed identification merthods, one can also detect the damage and identify the damage location by considering the mode shape curvature change, norm of the prediction error and Hilbert amplitude assurance criteria. It demonstrated that each proposed identyification method has it goodness as well as the drawback in system identifiction. For system with well separate modes either one of the method can detect the damage location well.

Acknowlefgement
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