

## Probability distribution functions for cover used in 3-D model simulating concrete deterioration in port assets.

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**Abstract.** In previous studies, a 1-D numerical predictive tool to simulate the salt induced corrosion of port assets in Australia has been developed into a 2-D and 3-D model based on current predictive probabilistic models. These studies use a probability distribution function based on the mean and standard deviation of the parameters for a structure incorporating surface chloride concentration, diffusion coefficient and cover. In this paper, this previous work is extended through an investigation of the distribution of actual cover by specified cover, element type and method of construction. Significant differences are found for the measured cover within structures, by method of construction, element type and specified cover. The data are not normally distributed and extreme values, usually low, are found in a number of locations. Elements cast insitu are less likely to meet the specified cover and the measured cover is more dispersed than those in elements which are precast. Individual probability distribution functions are available and are tested against the original function. Methods of combining results so that one distribution is available for a structure are formulated and evaluated. The ability to utilise the model for structures where no measurement have been taken is achieved by transposing results based on the specified cover.

### 1. Introduction

The corrosion of reinforcing steel in concrete structures is one of the primary causes of concern for owners worldwide. The reinforcing bars in concrete are normally in a passive condition due to the alkaline nature of the cementitious matrix. However corrosion can be initiated when this passive layer is destroyed, either due to a drop in pH caused by carbonation, or by the build-up of chloride ions to a level sufficient to break down the passive layer. The chloride ions can come from three principal sources: inclusion in the initial mix, ingress from de-icing salts and from exposure in a marine environment. In Europe and North America, de-icing salts are the most common cause, while in Australasia it is from marine exposure. The most common models in the literature are 2-stage models based on Tuutti [1-4].

The service life in the 2-stage approach comprises initiation followed by propagation stages [2, 5-7]. The initiation stage refers to the time required for the chloride concentration at the steel surface to reach a critical threshold concentration to initiate corrosion [2]. The propagation stage is the time between initiation of active corrosion until the structural integrity is compromised, where major repair or in the worst case structural failure occurs [1, 8].

The most commonly used solution to model the initiation phase of the service life model is based on Fick's 2<sup>nd</sup> Law of diffusion, and was developed in the early 1970's[9]. The solution is given in Eqn 1:



$$C_{(x,t)} = C_s \left[ 1 - \operatorname{erf} \left( \frac{x}{2\sqrt{D_c t}} \right) \right] \quad (1)$$

where  $x$  is the distance from concrete surface (m);  $t$  is time (s);  $D_c$  is the diffusion coefficient ( $\text{m}^2/\text{s}$ );  $C_s$  is the chloride concentration at the concrete surface;  $C_{(x,t)}$  is chloride concentration at the depth of  $x$  from the surface at time  $t$ ; and  $\operatorname{erf}$  is the error function.

This model has been modified as a greater understanding of the process has been developed [10-13] including the DuraCrete model which involves a limit state formulation for chloride induced corrosion initiation [14], Eqn 2.

$$\frac{C}{C_s} = 1 - \operatorname{erf} \left( \frac{x}{\sqrt{k_c \cdot k_e \cdot D_{RCM} \cdot \left( \frac{t_0}{t + t_{ex}} \right)^n \cdot t}} \right) \quad (2)$$

where  $C$  is chloride content at  $x$  depth;  $C_s$  is surface chloride concentration  $k_c$  is the curing factor;  $k_e$  is the environmental factor;  $D_{RCM}$  is the diffusion coefficient determined by the rapid chloride migration test;  $t_0$  is age  $D_{RCM}$  is measured at;  $t$  is duration of exposure;  $t_{ex}$  is age of concrete;  $n$  is age factor.

Further developments have been undertaken and incorporated in the current fip model[15], Eqn 3;

$$C_{(x,t)} = \left( C_0 + (C_{s,\Delta x} - C_0) \left[ 1 - \operatorname{erf} \left( \frac{x - \Delta x}{2\sqrt{D_{app,c} t}} \right) \right] \right) \quad (3)$$

where  $x$  is the distance from concrete surface (m);  $t$  is time (s);  $D_{app,c}$  is the apparent diffusion coefficient ( $\text{m}^2/\text{s}$ );  $C_{s,\Delta x}$  is the chloride concentration at the depth of  $x$  from the surface;  $C_0$  is the initial chloride concentration;  $\Delta x$  depth of the convection zone (m); and  $\operatorname{erf}$  is the error function.

In order to be able to solve these equations a number of variables must be known, eg the depth of the rebar, the chloride diffusion coefficient of the material, the age factor and the depth of the convection zone. Each of these variables has a level of uncertainty, ie while the specified cover may be given the actual cover will be a distribution function, which will affect the reliability of the solution to Fick's equation.

This paper reports the analysis of data taken for a number of elements from a number of structures, both precast and cast insitu, to determine their statistical distributions and any statistically significant differences. The intention is to utilise this knowledge to create a set of more tailored probability distribution functions (pdfs) of cover which may be used in the solution of Fick's equation. Systematic sampling was used to select the 30 data points which form the pdf.

## 2. Site data

The depth of cover has been measured on various elements of four port structures and a bridge in Australia. The port structures and the bridge have concrete elements some of which were manufactured insitu and some of which were precast. The date of construction of the structures ranges from 1968 to 2008, their age at date of measurement has been used to report the analysis. The depth of cover was measured using a covermeter. The sampling strategy was to systematically sample within the accessible parts of the structure. The original plans were consulted to establish the specified cover.

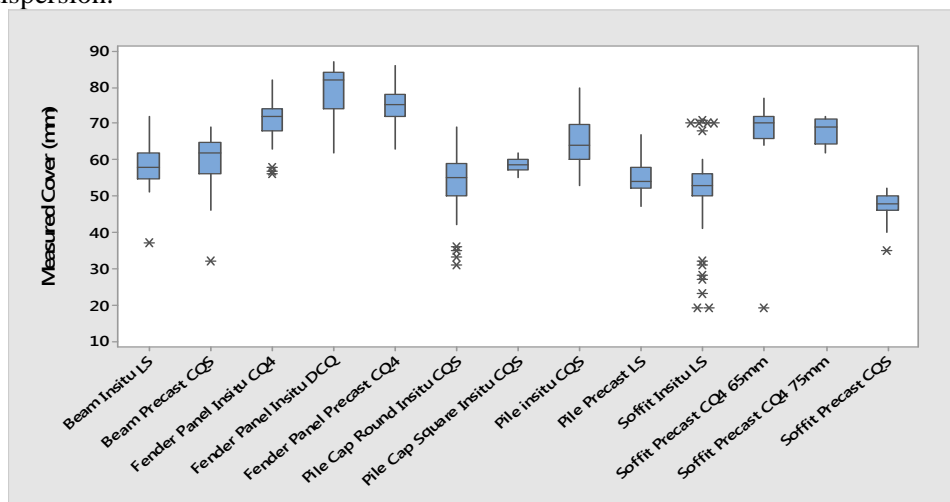
The two oldest structures had specified cover of 44.45mm and 50.8mm (1.75 and 2 inches), with the later structures having specified covers of 65mm and 75mm. CQS (48 years) and LS (44 years) are referred to as "old" structures and DQC (21 years) and CQ4 (2 years) as "new" structures. These variations in cover reflect the changes in design practices and material innovations over the last 50 years.

Due to access restriction only the piers of the bridge were accessible and measurements were taken of the vertical and horizontal elements from 5 of the 18 piers. The specified cover is believed to be 65mm.

### 3. Analysis of the data

#### 3.1 The port structures.

The port structures had 12 distinct data sets of measured cover from different elements. This included 5 elements, using both methods of construction. The 12 data sets all had a negative skew and did not follow a normal distribution. Figure 1 illustrates the disparity in homogeneity for different element types. Extremely low values were observed in several data sets. Square pile caps manufactured insitu have the smallest dispersion.



**Figure 1.** Box and whisker plot of the individual data sets for the port structures

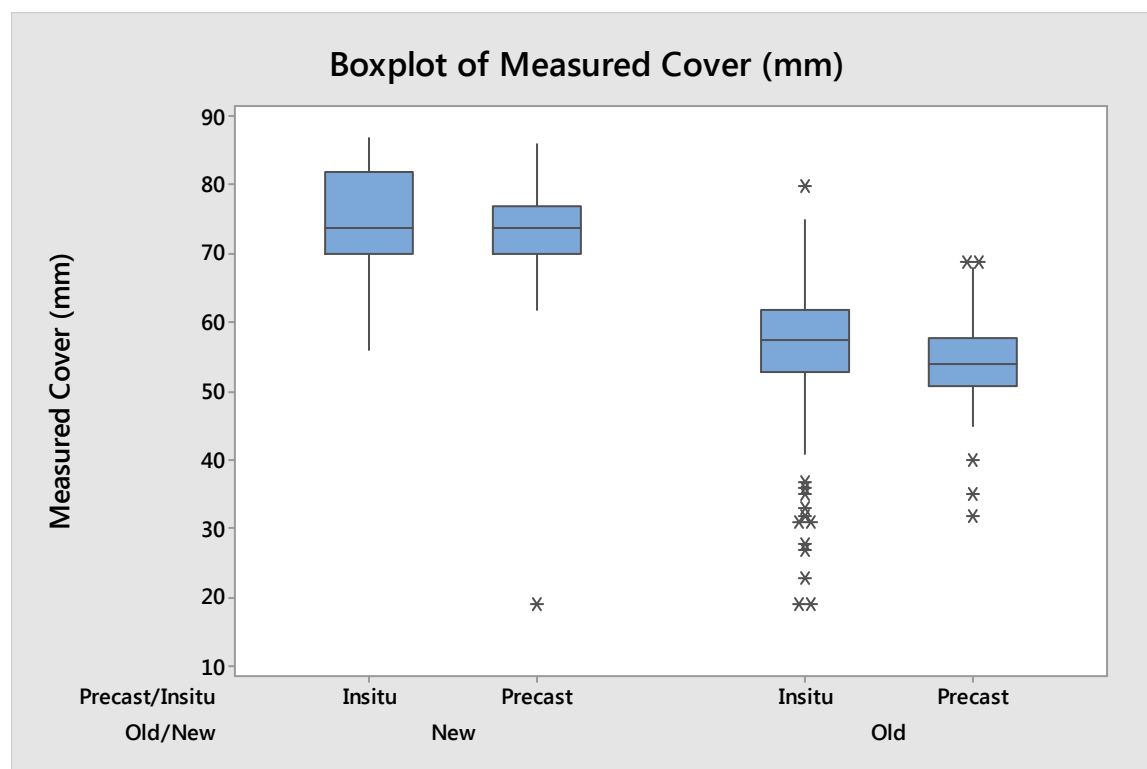
In addition to the presence of outliers, usually small values, there was variation in the proportion of measurements that exceeded specification. Table 1 summarises the size of the spread of the data and the proportion of measurements that exceed specification.

**Table 1.** Summary of the element types, with specified cover and % exceeding specified cover

	Structure	Sample size (n)	Specified Cover (mm)	Median Measured Cover (mm)	Spread (mm)	Proportion above specified cover
Beam insitu	LS	30	50.8	58	35	97%
Beam Precast	CQS	52	44.45	62	37	98%
Fender Panel Insitu	DCQ	70	75	82	25	100%
Fender Panel Insitu	CQ4	81	75	72	26	23%
Fender Panel Precast	CQ4	104	65	75	25	92%
Pile Insitu	CQS	96	50.8	64	27	100%
Pile Precast	LS	158	50.8	54	20	91%
Round pile cap insitu	CQS	47	50.8	55	38	70%
Square Pile cap insitu	CQS	32	50.8	58.5	7	100%
Soffit Insitu	LS	121	44.45	53	54	91%
Soffit Precast	CQS	47	44.45	48	17	96%
Soffit Precast	CQ4	27	65	70	58	81%
Soffit Precast	CQ4	6	75	69	10	0%

### 3.1.2 The construction method

The change in specification of cover requires the analysis of elements constructed insitu and those precast, for the older and younger structures. There was a statistically significant difference between the measured cover for all the elements constructed insitu or precast for both old and new structures, illustrated in Figure 2 (( $W=23301$ ,  $p=0.0358$ ) for new structures and ( $W=106799$ ,  $p<0.001$ )) for old structures. However, in each case the difference in the median between precast and insitu was less than 3mm. The cover for elements constructed insitu for old structures was greater than for precast elements, however in CQ4 cover for elements cast insitu was lower than for those precast.



**Figure 2.** Box and whisker plot comparing the construction method within new and old port structures

In addition to the actual dispersion decreasing between the old and new structures, the prevalence of outliers has decreased with only one outlier present in the data for the new structure. This indicates an improvement in the consistency of construction with fewer instances of insufficient cover achieved.

### 3.1.3 Port structures

The nature of the sampling mechanism used to take the measurements and the change in cover specification results in no opportunity to compare like for like results. However, some comparisons are possible. Table 2 summarises the results of individual elements manufactured insitu or precast. With the exception of beams there was a statistically significant difference between elements constructed using different methods. Table 3 contains the findings from comparisons where one or more characteristics differs, for example the comparison of the distribution of cover of fender panel was made for the same specified cover, and method of construction, but different locations (and hence different construction teams). Mann-Whitney (Non-parametric) tests were used to test for differences between distributions from different sites or using different methods.

**Table 2.** Results for tests for difference between the distributions of elements from different structures, different construction methods or different specifications

Element type	Construction method	Specified cover	Location	Results of Mann Whitney test. W=Test statistic, N=sample size	
Pile	Insitu Precast	50.8mm	CQS LS	Significant difference (p<0.05,) W=18814, (N96,158)	Insitu (64mm) >Precast (54mm) Difference 10mm
Fender panel	Insitu	75mm	DQC CQ4	Significant difference (p<0.05), W=6963.5, (N70,81)	Age 21 (82mm) >Age 2(72mm) Difference 10mm
Fender panel	Insitu Precast	75mm 65mm	CQ4	Significant difference in distribution. (p<0.05) W=1113.5 (N=81,104)	Insitu (72) <Precast (75mm) Difference 3mm
Soffits	Insitu Precast	44.45mm	LS CQS	Significant difference in distribution. (p<0.05) W=2132.5 (N=121,47)	Insitu (53) >Precast (48mm) Difference 5mm
Beam	Insitu Precast	50.8mm 44.45mm	LS CQS	No significant difference p=0.13, W=1087 (N=52,30)	

The second set of tests was performed to establish if differences exist between different elements with the same specification in the same structure. The results in Table 3 show the significant differences; however these are small. This validates the decision to combine the measurements of different elements to produce a probability distribution function (pdf) to use in the model based on specified cover.

**Table 3.** Results for tests for difference between the distributions of elements in CQS with different construction methods or different specifications

Element type	Construction method	Specified cover	Location	Results of Mann Whitney or Kruskal Wallis tests	
Round pile caps Square pile caps	Insitu	50.8mm	CQS	Significant difference in distribution. (p<0.05) W=1605 (N47,32)	Round cap (55mm) <Square cap (58.5mm) Difference 3.5mm
Beam Soffit	Precast	44.45mm	CQS	Significant difference in distribution. (p<0.05) W=1261 (N52,47)	Beam (62mm) > Soffit (48mm) Difference 14mm
Pile Round pile cap	Insitu	50.8mm	CQS	Significant difference in distribution. (p<0.05) W=8537 (N=96,47)	Pile (64mm) > Round cap (55mm) Difference 9mm
Pile Square pile cap	Insitu	50.8mm	CQS	Significant difference in distribution. (p<0.05) W=7205 (N=96,32)	Pile (64mm) > Square cap (58.5mm) Difference 5.5mm

### 3.2 Bridge

A total of 162 measurements were made, 44 of horizontal and 118 of vertical elements. The data was normally distributed, with one low value outlier measured. This data is normally distributed, but does contain one very low value. The distribution of the data is much less dispersed than for the port structures. Horizontal and vertical elements were measured and a significant difference in the distribution found ( $p=0.008$ ,  $T=2.71$ ,  $N=44,118$ ), with the mean measured cover for horizontal elements less than that for vertical elements. The mean for horizontal measurements (66.0mm) was only 2.2mm smaller than the mean for vertical measurements (68.2mm).

## 4. Creating probability distribution functions (pdf)

### 4.1 Port structures

The development of the model from 1D to 3D involved the inclusion of surface chloride concentration, diffusion coefficient and cover. The input for the cover used the mean and the standard deviation to produce a distribution function based on a Gaussian distribution. The use of such a distribution is more representative of reality than the use of a single value. The hypothesis under test is that creating a distribution function based on actual measurements will produce output that will produce a more realistic model.

The statistical analysis revealed that the data has a negative skew. The mean cover is smaller than the median cover and the implication of using a pdf based on the mean rather than median is that the predicted time to onset will be shorter than the actual time to onset. The distributions of the individual elements were compared to a number of known distributions, including Weibull, Gamma and Poisson and none of the elements could be represented by one of these distributions. Additionally the cover data was transformed using log, exponential and Box-Cox transformation to establish if this would result in a normal distribution. None of the transformations was successful.

Some exploratory work was undertaken looking at transforming the data with different specified cover. Precast soffits are present in two structures, with three different specified covers. The data for the cover was transformed to produce a set with a notional specified cover of 44.45mm. There is no significant difference between the distributions of the two normalised precast situations, but there is a significant difference between the distribution of the soffits manufactured insitu and those precast. This result confirms that transformation of data is an appropriate technique to employ. Hence pdfs created using transformed data will be evaluated.

A series of bespoke pdfs was created. These were tested by using them as the input for the model, with the value for Sodium Chloride concentration ( $C_s$ ) held constant at 2.9% and the Diffusion Coefficient ( $D_c$ ) at  $1.0 \times 10^{-12} \text{ ms}^{-2}$  in Eqn 1. It is important to evaluate the differences in the pdfs created using different sampling strategies. For every pdf 30 values were drawn from either the entire data set for structures or for specific subsets. The data was ordered and a systematic sampling method (1 in  $n$ ) was used to select 30 values from the data set. Pdfs were created with different starting places for the selection process and also on data sets which excluded the lowest 5%, highest 5% or lowest and highest 5% of readings.

### 4.2 Bridge

The data sets for vertical and horizontal measurements are normally distributed as is the data set of combined values. The creation of a pdf is less complex than for the port structures as there are only two variants of the element and the distributions have a low dispersion. The pdfs were created using data from the vertical, horizontal and combined data set. The inclusion of the low value was also investigated.

## 5. Analysis of the probability distribution functions

The bespoke pdfs created for port structures and bridge were analysed using the model. Constant values for the mean were set at 2.9% for the Sodium Chloride concentration ( $C_s$ ) and  $1.0 \times 10^{-12} \text{ m}^2/\text{s}$  for the diffusion coefficient ( $D_c$ ). The evaluation of the output will identify the impact of the created pdfs with the original input based on mean and standard deviation. The evaluation will also evaluate the method

of selecting the sample used to create the pdf, from the set of measured cover. The characteristics determining the different pdfs for port structures include:

- Different starting places when systematically sampling from the ordered data
- Data normalised on 44.45mm, 50.8mm, 96mm and 75mm
- Data excluding the lowest 5% of values, the highest 5% of values and both the lowest and highest 5% of values
- Data sets for old structures, in total and by method of construction
- Data sets for new structures, in total and by method of construction

Those for bridges include:

- Including and excluding the observed outlier
- Considering the vertical and horizontal measurements separately and combined

The impact of the pdf is evaluated using the 10% and 90% time to onset values. A 10% time to onset of 17.2 years means that there is a 10% chance that onset will have occurred by 17.2 years after construction. The associated 90% value of 31.9 years means that there is a 90% chance that onset will have occurred by 31.9 years after construction.

### 5.1 Findings

The pdfs using different starting values and the exclusion of the highest and lowest 5% were created to investigate the impact of including extremely low and high measured cover data points. The impact was very small, indicating that the model is robust and does not respond to extreme values. The non-parametric Kruskal-Wallis test was used to test for difference between the pdfs created with different starting positions. No significant differences were found at either the 10% or 90% years to impact values.

Pdfs were created excluding the lowest 5%, highest 5% and both the lowest and highest 5% of values. Kruskal-Wallis tests established that there were no significant differences for either the 10% or 90% years to impact values.

Significant differences were found comparing method of construction for the old structures (those with a specified cover of 44.45mm or 50.8mm) for the 90% to onset value. The time to onset for precast elements was 39.1 years compared to 47.2 years for insitu ( $W=15$ ). There was no significant difference at the 10% years to onset between precast and insitu.

There were no significant differences between time to onset (either 10% or 90%) between precast and insitu for the “new” structures (65mm and 75mm).

Pdfs were created to evaluate the scenario that the older structures had been built to the new specifications and the new structures had been built to the old specification. The outcome is shown in table 4.

**Table 4.** Results for 10% and 90% time to onset for the four port structures.

	<b>CQS</b>	<b>CQS built as new</b>	<b>LS</b>	<b>LS built as new</b>	<b>DCQ</b>	<b>DCQ built as old</b>	<b>CQ4</b>	<b>CQ4 built as old</b>
<b>10% years to impact</b>	21	46	24	40	40	18	42	19
<b>90% years to impact</b>	47	103	36	67	73	33	64	29

It can therefore be concluded that the impact of changing the specification has had a dramatic increase in the time to onset between the older and younger structures.

Small differences were found for vertical and horizontal elements of the bridge as shown in table 5.

**Table 5.** Results for 10% and 90% time to onset for vertical and horizontal elements of the bridge

	Vertical	Horizontal	Vertical and Horizontal
<b>10% years to onset</b>	35.8 years	33.2 years	36.8 years
<b>90% years to onset</b>	58.6 years	51.3 years	54.1 years

## 6. Conclusions and recommendations

The analysis of the data reveals substantial differences between the old structures and the new structures. The change in specified thickness has resulted in an increase of time to onset at 10% from 23 years to 42 years and at 90% from 42 years to 68 years. The impact of manufacturing method varies by element and hence is more complex. The use of the data, without weighting for the prevalence of the different element types, indicates that the manufacture method within new structures only impacts the 90% years to onset, with elements manufactured insitu lasting 6 years longer than precast elements.

Further analysis of pdfs created for individual elements is required to refine the method of creating a generic model to be used for structures where measurements have not been taken.

## 7. References

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