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Load identification approach based on basis pursuit denoising algorithm

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Abstract. The information of the external loads is of great interest in many fields of structural analysis, such as structural health monitoring (SHM) systems or assessment of damage after extreme events. However, in most cases it is not possible to measure the external forces directly, so they need to be reconstructed. Load reconstruction refers to the problem of estimating an input to a dynamic system when the system output and the impulse response functions are usually the knowns. Generally, this leads to a so called ill-posed inverse problem, which involves solving an underdetermined linear system of equations. For most practical applications it can be assumed that the applied loads are not arbitrarily distributed in time and space, at least some specific characteristics about the external excitation are known a priori. In this contribution this knowledge was used to develop a more suitable force reconstruction method, which allows identifying the time history and the force location simultaneously by employing significantly fewer sensors compared to other reconstruction approaches. The properties of the external force are used to transform the ill-posed problem into a sparse recovery task. The sparse solution is acquired by solving a minimization problem known as basis pursuit denoising (BPDN). The possibility of reconstructing loads based on noisy structural measurement signals will be demonstrated by considering two frequently occurring loading conditions: harmonic excitation and impact events, separately and combined. First a simulation study of a simple plate structure is carried out and thereafter an experimental investigation of a real beam is performed.

1. Introduction

Online monitoring of external loads is a very important technique in the field of structural health monitoring (SHM). For SHM-Systems, the information of the time history of acting forces in combination with the location where these forces are applied, is useful to make a statement about the occurrence of damage and its extent after adverse loading conditions. In reality, external loads are reconstructed using measured structural responses, such as acceleration, velocity, position or strain. A direct measurement is mostly difficult or sometimes even not possible, e.g. when the force application position is unknown. The reconstruction of external forces from dynamic measurements results in an inverse problem, which can be considered as mathematically ill-posed in most cases, which means that either the existence, the uniqueness or the stability of the solution is violated [1][2][3]. If this inversion can be done, the structure itself becomes its own force sensor [4]. In order to overcome the problem of ill-posedness several studies have been carried out and a considerable amount of literature has been published on this topic, e.g. [3][4][5][6][7].

In this contribution the inverse problem is tackled by the so called basis pursuit denoising algorithm. Some characteristics of the applied force are employed to transform the force reconstruction problem in a sparse recovery task. This paper is structured as follows: Section 2 briefly discusses the general problem of force reconstruction and explains how it can be transformed into a sparse identification problem. In Section 3, the applied approach for solving the sparse reconstruction problem is introduced. A simulation study and an experimental investigation in Section 4 and 5 show the capability of the proposed identification method. Finally, concluding remarks are presented in Section 6.

2. Problem

Dynamic external forces applied on a mechanical structure will stimulate the system to vibrate. In general these structural responses can easily be measured by appropriate sensors. For linear systems the structural responses $\mathbf{y}(t) \in \mathbb{R}^r$ are linked to the external forces input $\mathbf{u}(t) \in \mathbb{R}^f$ in the time continuous case by the linear convolution integral

$$\mathbf{y}(t) = \int_0^t \mathbf{h}(\tau)\,\mathbf{u}(t-\tau)\,\mathrm{d}\tau \tag{1}$$

where $\mathbf{h}(t) \in \mathbb{R}^{r\times f}$ denotes a matrix containing the impulse response functions (IRF) $h_{i,j}$. It is assumed that $\mathbf{u}(t) = \mathbf{h}(t) = \mathbf{y}(t) = \mathbf{0}$ for $t < 0$. The IRF $h_{i,j}$ describes the transmission of an external force applied at a discrete input position $j = 1, 2, \ldots, f$, to the structural responses at output position $i = 1, 2, \ldots, r$. In a discrete time domain ($t = n\Delta t,\ n \in \mathbb{N}$), the convolution integral becomes an algebraic equation:

$$\mathbf{y}_k = \sum_{i=0}^{k} \mathbf{h}_i \mathbf{u}_{k-i}\ , \quad (k = 0, 1, 2, \ldots, n) \tag{2}$$

Here, $\mathbf{h}_i \in \mathbb{R}^{r\times f}$ are the so-called Markov parameters. Rewriting Equation (2) in vector matrix form leads to

$$\mathbf{Y} = \mathbf{H}\mathbf{U}\ , \tag{3}$$

with the transfer matrix $\mathbf{H} \in \mathbb{R}^{r\cdot n\times f\cdot n}$, the measurement vector $\mathbf{Y} \in \mathbb{R}^{r\cdot n}$ and the input vector $\mathbf{U} \in \mathbb{R}^{f\cdot n}$:

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{h}_1 & \mathbf{h}_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{h}_n & \mathbf{h}_{n-1} & \cdots & \mathbf{h}_0 \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_n \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_n \end{bmatrix} \tag{4}$$

In case of a forward problem the IRFs or the Markov parameters and the external forces are known, so that the desired system response can be calculated by Equation (1), (2) or (3). However, generally the external force respectively the input to the system is unknown and the structural responses are determined by (noisy) measurements. Thus force identification becomes a deconvolution problem. But this also requires the determination of the system dynamics in advance. If the input position of a single force is known a priori, the number of inputs f is reduced to 1 and the force reconstruction problem is a bit more simplified. Since in general the

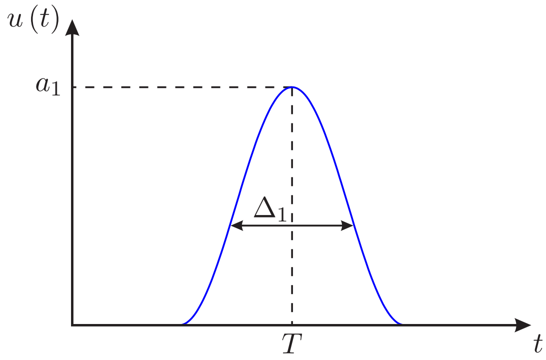

Figure 1. General representation of an impact force history

input position is unknown, every location on the surface becomes a potential input position. Depending on the type of structural model, these positions are usually coinciding with the degrees of Freedom (DOF). For real applications, the number of measurement sensors r is normally smaller than the number of DOFs, so that $r < f$ and the deconvolution problem becomes even more challenging. Nevertheless, in most practical applications at least some characteristics about the acting forces are known and can be taken into account in the estimation process. Two frequently occurring loading conditions are harmonic forces and impact events. These loading types will be further investigated in this paper.

Impact forces are characterized by a short time duration and a spatial concentration. The profile of the force history looks approximately like the pulse function shown in Figure 1. This pulse-shape is commonly modeled as half or quarter cycle-sine pulses function (see e.g. [8] and [9]). The force history can be described by three parameters: the pulse width, indicated by the parameter $\Delta \in [\Delta_{\min}, \Delta_{\max}]$, the time $T \in [0, T_{\max}]$ at which the magnitude is reached and the magnitude value of this pulse $a_{T,\Delta} \in \mathbb{R}$ itself. However, if the impact is intended to be reconstructed using measured structural responses these parameters are unknown and need to be identified. Additionally the location where the load is applied is unknown as well.

But, employing these parameters an impact dictionary $\tilde{\mathbf{U}}_{\mathrm{I}}$ can be created which contains the force time history of all potential parameter combinations at all potential force input locations. To this end, the interval $[0, T_{\max}]$ needs to be discretized according to the sampling time (see Equation (2)) with $T = l\Delta t \;\; (l = 0, 1, 2, \ldots, m)$ and $T_{\max} = m\Delta t$ and also the interval $[\Delta_{\min}, \Delta_{\max}]$ describing the range of potential impact durations, has to be discretized appropriate, with $\Delta = \Delta_{\min} + (p-1)\,\Delta_p$ and $(p = 1, 2, \ldots, q)$. By multiplying this dictionary $\tilde{\mathbf{U}}_{\mathrm{I}} \in \mathbb{R}^{f \cdot n \times f \cdot m \cdot q}$ by a vector $\mathbf{A}_{\mathrm{I}} \in \mathbb{R}^{f \cdot m \cdot q}$

$$\hat{\mathbf{U}}_{\mathrm{I}} = \tilde{\mathbf{U}}_{\mathrm{I}} \cdot \mathbf{A}_{\mathrm{I}} \;, \tag{5}$$

$\hat{\mathbf{U}}_{\mathrm{I}}$ equals now the actual force input of a single impact, if all entries in $\mathbf{A}_{\mathrm{I}}$ are zeros, except the one which belongs to the actual impact. Here $\mathbf{A}_{\mathrm{I}}$ can be considered as magnitude vector, if the force histories in $\tilde{\mathbf{U}}_{\mathrm{I}}$ are normalized to unit.

A similar dictionary matrix can be created for harmonic loads. It is well known that harmonic signals can also be described by a small number of parameters: the frequency of the force and its cosine and sine amplitude. If the applied load is in a known frequency bandwidth $\omega \in [\omega_{\min}, \omega_{\max}]$ the harmonic force dictionary $\tilde{\mathbf{U}}_{\mathrm{H}} \in \mathbb{R}^{f \cdot n \times f \cdot 2 \cdot s}$ reads as:

$$\tilde{\mathbf{U}}_{\mathrm{H}} = \begin{bmatrix} \tilde{\mathbf{U}}_{\mathrm{HC}} & \tilde{\mathbf{U}}_{\mathrm{HS}} \end{bmatrix} \tag{6}$$

It is combined from a dictionary $\tilde{\mathbf{U}}_{\mathrm{HC}}$ containing normalized cosine force histories of different frequencies in the interval $[\omega_{\min}, \omega_{\max}]$ and a corresponding sine dictionary $\tilde{\mathbf{U}}_{\mathrm{HS}}$. The discretization of the frequency interval depends on the length of the measured response time

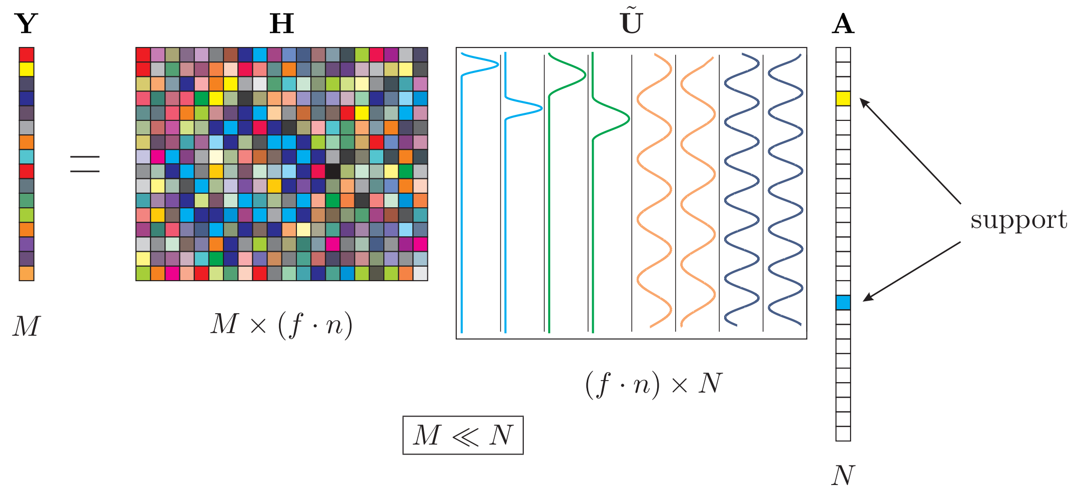

Figure 2. By pre-multiplying a force dictionary, identification becomes a sparse recovery problem

history, $\omega = \omega_{\min} + (v-1)\,\Delta\omega$ with $(v = 1, 2, \ldots, s)$. Again, if the dictionary $\tilde{\mathbf{U}}_{\mathrm{H}}$ is multiplyed by a magnitude vector $\mathbf{A}_{\mathrm{H}} = \left[\mathbf{A}_{\mathrm{HC}}^T \quad \mathbf{A}_{\mathrm{HS}}^T\right]^T$

$$\hat{\mathbf{U}}_{\mathrm{H}} = \tilde{\mathbf{U}}_{\mathrm{H}} \cdot \mathbf{A}_{\mathrm{H}} \tag{7}$$

$\hat{\mathbf{U}}_{\mathrm{H}}$ equals the actual harmonic load (with the same frequency, phase and amplitude), if all elements of $\mathbf{A}_{\mathrm{H}}$ are zero, except cosine and sine magnitudes corresponding to the actual frequency of the harmonic force. The impact and the harmonic force dictionary are united to entire input dictionary $\tilde{\mathbf{U}}$ with an associated entire magnitude vector $\mathbf{A}$:

$$\tilde{\mathbf{U}} = \left[\tilde{\mathbf{U}}_{\mathrm{H}} \quad \tilde{\mathbf{U}}_{\mathrm{I}}\right] \in \mathbb{R}^{f \cdot n \times f(m \cdot q + 2 \cdot s)} \text{ and } \mathbf{A} = \left[\mathbf{A}_{\mathrm{H}}^T \quad \mathbf{A}_{\mathrm{I}}^T\right]^T \in \mathbb{R}^{f(m \cdot q + 2 \cdot s)} \tag{8}$$

If this entire dictionary is used as an input to the system, it follows from Equation (3):

$$\mathbf{Y} = \mathbf{H} \cdot \tilde{\mathbf{U}} \cdot \mathbf{A} = \tilde{\mathbf{H}} \cdot \mathbf{A} \tag{9}$$

Thus, a new transition matrix $\tilde{\mathbf{H}} = \mathbf{H} \cdot \tilde{\mathbf{U}} \in \mathbb{R}^{r \cdot n \times f(m \cdot q + 2 \cdot s)}$ can be obtained (see Figure 2).

Now force reconstruction becomes a problem of solving Equation (9) for $\mathbf{A}$. However, it can be easily seen that $\tilde{\mathbf{H}}$ tends to have a lot more columns $(N := f\,(m \cdot q + 2 \cdot s))$ than rows $(M := r \cdot n)$. Hence the linear system of equations in Equation (9) has a lot more unknowns than knowns and thus solving for $\mathbf{A}$ is in general not possible. But it is known that $\mathbf{A}$ has just a very few nonzero entries and can be considered as sparse vector. The property of sparsity of the desired magnitude vector $\mathbf{A}$ is used in the following to obtain an optimal estimate.

3. Sparse solution

The sparse solution of a high-dimensional underdetermined problem is an application of Occam's Razor: in face of many possibilities, all of which are plausible, favor the simplest candidate solutions [10]. Here, it can also been regarded as a problem of finding the columns in $\tilde{\mathbf{H}}$ which correlate the most with the measurement vector $\mathbf{Y}$.

A possibility to obtain a sparse solution for the linear system in Equation (9) is to minimize the l_0-norm of the solution vector $\mathbf{A}$:

$$\hat{\mathbf{A}} = \underset{\mathbf{A} \in \mathbb{R}^N}{\arg\min} \|\mathbf{A}\|_0 \text{ subject to } \mathbf{Y} = \tilde{\mathbf{H}}\mathbf{A} \tag{10}$$

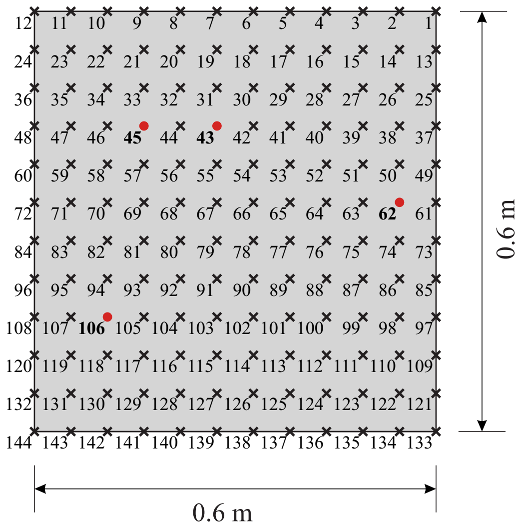

Figure 3. Node numbering of the finite element plate model; node 43, 45, 62 and 106 are acceleration measurement positions

The l_0-norm specifies the number of nonzero entries of $\mathbf{A}$. Equation (10) leads to the sparsest solution which agrees with the measurement vector $\mathbf{Y}$. Unfortunately, solving Equation (10) requires a combinatorial search, which makes it nearly impossible to solve computationally. Under the assumption of a sparse solution, e.g. [11] and [12] have shown that by replacing the l_0-norm with the l_1-norm almost the same solution can be obtained as solving Equation (10):

$$\hat{\mathbf{A}} = \underset{\mathbf{A}\in\mathbb{R}^N}{\arg\min} \|\mathbf{A}\|_1 \text{ subject to } \mathbf{Y} = \tilde{\mathbf{H}}\mathbf{A} \tag{11}$$

Now, the solution to equation (11) can be found by applying linear programming techniques, since l_1-regularized optimization is a convex problem.

For a measured output vector $\hat{\mathbf{Y}}$, the measurement data are usually polluted by some measurement noise $\boldsymbol{\omega}$, so Equation (9) becomes:

$$\hat{\mathbf{Y}} = \tilde{\mathbf{H}}\mathbf{A} + \boldsymbol{\omega} \tag{12}$$

To obtain a sparse solution for Equation (12) the minimization problem (11) needs to be modified, since it is not capable to deliver a proper estimate of $\mathbf{A}$ in the presence of noise:

$$\hat{\mathbf{A}} = \underset{\mathbf{A}\in\mathbb{R}^N}{\arg\min} \left\|\hat{\mathbf{Y}} - \tilde{\mathbf{H}}\mathbf{A}\right\|_2^2 + \lambda \|\mathbf{A}\|_1 \tag{13}$$

This problem is known as basis pursuit denoising (BPDN) [13]. In this case, a least-squares minimization is combined with a l_1-norm which penalizes solutions with many nonzero elements. The parameter λ regularizes the trade-off between sparsity of the solution and congruence of $\hat{\mathbf{Y}}$ and $\tilde{\mathbf{H}}\mathbf{A}$. λ can be adjusted according to the noise level $\boldsymbol{\omega}$ [14].

In recent years, the possibility of finding a sparse solution via l_1-minimization has been attracted a lot of attention, particularly in the field of Compressive Sensing (CS). Hence, a number of algorithms have been developed to solve Equation (11) and Equation (13) (e.g. [15] and [16]). In this contribution the so called *In-Crowd Algorithm* developed by Gill et al. in 2011 [10] is employed. This algorithm is one of the fastest solvers for very large sparse problems. A detailed description of the *In-Crowd* Algorithm, including a precise prescription of each calculation step, can be found in [10].

The so obtained magnitude vector $\hat{\mathbf{A}}$ has just a few nonzero elements, called the support of $\hat{\mathbf{A}}$. By considering this values and its corresponding columns in the dictionary $\tilde{\mathbf{U}}$ the

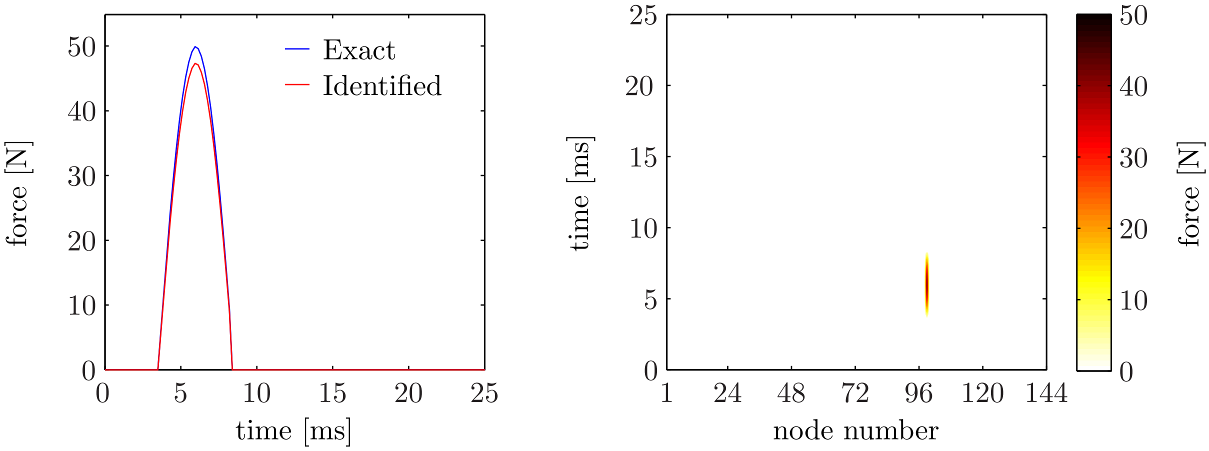

Figure 4. Impact reconstruction result; force history at node 99 (left), which was correctly identified as impact location (right)

force application points and the associated parameters of impact force and harmonic load are identified. The estimated force history at each potential input location $\hat{\mathbf{U}} \in \mathbb{R}^{f \cdot n}$ can be calculated by:

$$\hat{\mathbf{U}} = \tilde{\mathbf{U}}\hat{\mathbf{A}} \tag{14}$$

4. Simulation study

The capability to identify the force history and the force input location of impacts and harmonic loads from noisy measurements by solving the BPDN problem will be shown first by means of simulation studies. For illustration purposes, a completely free uniform 0.6 m by 0.6 m square steel plate of 5 mm thickness is investigated. The structural dynamics of the plate due to external forces are described by a finite element model. The plate is modelled by quadratic shell elements and 144 nodes (each with 6 degrees of freedom), see figure 3. The employed structural responses used in the simulation studies as well as in the experimental investigations are acceleration measurements perpendicular to the plate plane. White Gaussian noise, with a standard deviation of three percent of the maximum measurement value, is added to the simulated outputs to imitate real measurement data. It should be mentioned that the shown identification strategy is also applicable for the use of other structural models, e.g. analytical models and other types of measurement data, e.g. strain [17].

In a first study an impact load is applied perpendicular to the plate at Node 99. The acceleration responses due to this impact are recorded only at four output positions (Node: 43, 45, 62 and 106). The impact dictionary is composed of 13 different impact durations ($q = 13$). In order to investigate the effect of deviations in the employed reconstruction dictionary, the actual applied impact duration Δ_{true} is not included. The relative deviation to the best fitting impact Δ_{dic} amounts $(\Delta_{\text{dic}} - \Delta_{\text{true}})/\Delta_{\text{true}} = 3\%$. The reconstructed force history at Node 99 is shown in the left part of figure 4, the estimated force history at other nodes is zero (or nearly zero), thus the impact location is identified as well (figure 4 right). The magnitude estimation error amounts 5%.

In the next simulation a harmonic load with a frequency of 158 Hz is applied at node 79. The harmonic force dictionary is composed of frequencies from 38.5 Hz up to 275.5 Hz, the step size is (according to the recorded time history) 3 Hz. Again, the harmonic force dictionary does not contain the true frequency. Figure 5 displays the identification results. In the left plot the estimated and the true force frequency spectra are compared. The right plot illustrates once

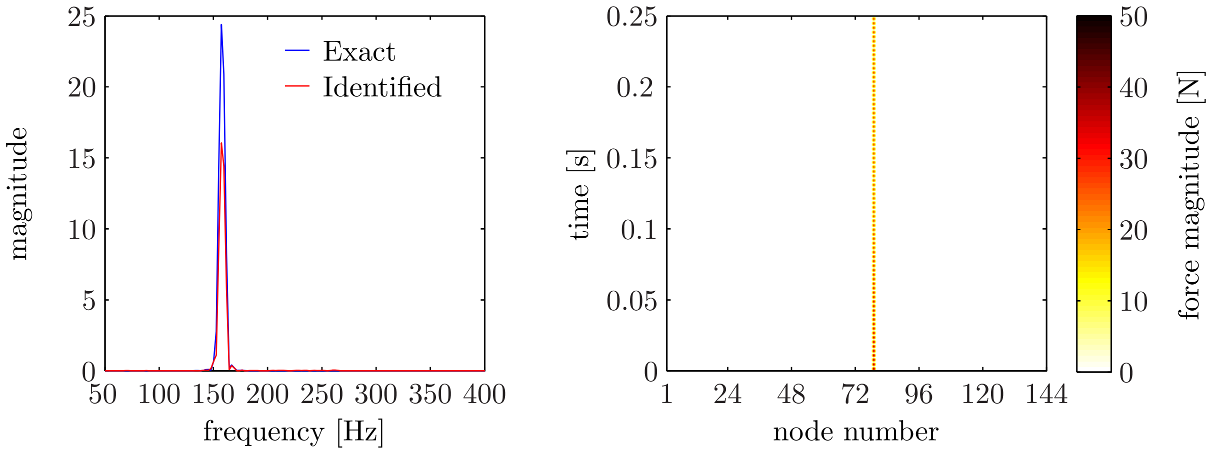

Figure 5. Reconstructed harmonic load; left: comparison of force spectra, right: identified force application at node 79

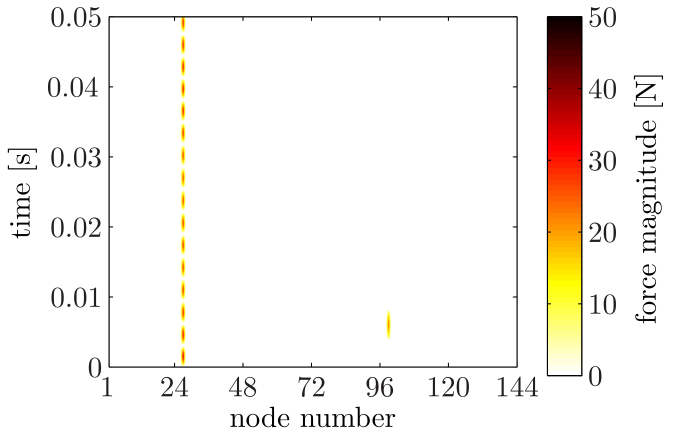

Figure 6. Reconstruction of superimposed impact force (node 99) and harmonic load (node 27); both application points are identified correctly

more the amount of force history for all nodes. It can be seen that the estimated force is assigned to the correct input node. However, the amplitude of the signal is slightly attenuated.

An estimation result for a superposition of impact and harmonic loads is shown in figure 6. The proposed load reconstruction strategy identifies the input location for both loading types correctly (see figure 6). The estimated magnitude of the impact and the identified frequency is slightly attenuated as well in this case.

5. Experimental results

For experimental validation of the presented load identification strategy an experimental test bench is set up. A beam of $L = 3\,\mathrm{m}$ length is placed onto two bearing blocks. Two piezoelectric accelerometers are mounted on the beam bottom side. The sensors are placed at positions $x_{s1}/L = 0.5$ and $x_{s2}/L = 0.73$ of the normalized beam length. The beam structure is excitated by an impact of an impulse force hammer. The built-in force sensor is able to measure the force history applied by the hammer, which is used later to confirm the reconstructed load. Due to the simplicity of the simply supported beam structure an analytical model is used as structural model [17]. Figure 7 show the reconstruction results for an impact applied at the normalized beam position $x/L = 2/3$. The deviation of the calculated impact location amounts 1.94 cm. In assessing the localization accuracy, the hammer tip with a width of 2 cm needs to be considered. The relative magnitude estimation error amounts in this case 8.26%.

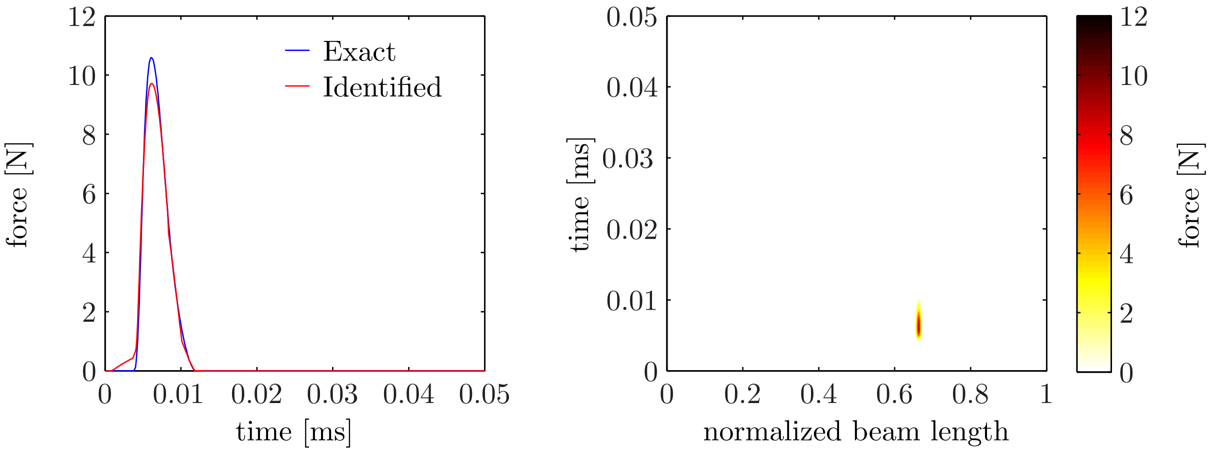

Figure 7. Experimental reconstruction results; left: comparison of force history, right: identified force over normalized beam length and time

6. Conclusion

In this paper the problem of reconstructing an impact from structural responses is addressed. It has been shown that by employing prior knowledge of the external force characteristics the proposed load identification method is able to reconstruct the force location and the force history, simultaneously. Generally it can be observed that this identification method estimates the input location very reliably. Also the force history deviations of the shown reconstruction studies are within reasonable limits.

Compared to other identification methods, significantly fewer sensors are required to estimate simultaneously the location and the force history. For the investigated beam structure two accelerometers are sufficient to obtain satisfying estimates. The proposed sparse identification method opens up a lot of new opportunity in the field of load reconstruction. The idea of employing prior knowledge of the force characteristics to create a sparse problem is transferable to other load reconstruction problem, e.g. spatial distributed loads.

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