Evaluation of damage indexes for structural health monitoring in nonlinear mechanical systems based on Volterra series

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Abstract. Nonlinear behavior can be important for the monitoring of the structural state of mechanical systems since it can be mistakenly classified as a damage in the systems. However, nonlinear tools are still not consolidated and need further research for applications in mechanical system. Discrete-time Volterra series is an interesting mathematical framework to deal with nonlinear dynamics. It is a clear generalization of the linear convolution for weakly nonlinear systems. In the present work two different damage indicators are proposed based on Volterra models by considering linear and nonlinear contributions of the total response. The goal is to evaluate these indicators using an experimental test rig where the damages are simulated and considering the nonlinear regime of motion. The main advantages and drawbacks of the proposed methodology are highlighted in the final remarks.

1. Introduction

There is an increasing importance on considering nonlinear phenomena in structural dynamics due to the need of better performance in terms of comfort, weight, noise, durability, among others features [1]. Nonlinearities can bring complex effects as jumps, limit cycle oscillations, harmonic distortion, chaos and others [2]. These effects can be strong in structures which are subjected to clearance, impacts, dry friction and bolted connections [3]. All these features shows that the study of the effect of nonlinearities in the monitoring of mechanical systems are highly important for the development of structural health monitoring (SHM) methodologies since these effects can mask or be mistakenly viewed as structural changes [4]. Many techniques as harmonic balance [5], nonlinear autoregressive models [6], nonlinear normal modes [7], among others, have been already presented in the literature. However, there is still no general technique to deal with this kind of system.

In this sense, the Volterra series expansion can be an interesting framework since it is a generalization of the linear convolution and the impulse response function [8]. Because of this, methodologies that are useful in the linear domain can be extended, until certain point, to nonlinear structures. Another advantage is that this model can separate the linear and nonlinear part of the response enabling to have an idea of the degree of nonlinearity of the system.

A few papers have already applied similar formulations in the damage detection problem. Chatterjee [9] identified a Volterra model representing a clamped beam with a breathing crack modeled by a bilinear oscillator. The model was identified using the harmonic probing method

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and the variations in the harmonics in the response signal were related to the opening of the crack. Tang *et al* [10] applied the Volterra expansion for the diagnosis of a rotor-bearing system. In this paper the Volterra kernels were monitored and compared to a baseline model in order to detect unbalance and rubbing conditions. Rébillat *et al* [11] used a Hammerstein model to calculate damage indicators based on the degree of nonlinearity of the response of the system. The technique was tested in initially linear systems subjected to breathing cracks that allows the system to behave in nonlinear way.

The goal of the present paper is to apply the Volterra series to identify the system using the input and output data. Numerical issues of this model are avoided using orthonormal Kautz functions. The identified model of the structure in the healthy condition is used to monitor the structural state of the system. In previous papers of the authors of the present work the Volterra model was already applied in the problem of damage detection in simulated [12] and experimental examples [13]. The current paper tries to improve the previous results in a benchmark structure with a better control on the application of the damage and also with a more detailed characterization of the behavior of the proposed damage index. A few analysis of the quality of the damage indicators are presented with the results. Finally, the main advantages and drawbacks of the technique are highlighted.

2. Application of discrete-time Volterra series for damage detection

The total response of the system in discrete-time, $y(k)$, can be separated in a linear component, $y_1(k)$, and a sum of nonlinear components $y_2(k) + y_3(k) + \ldots$ using the Volterra series by:

$$y(k) = \sum_{\eta=1}^{+\infty} y_\eta(k) = y_1(k) + y_2(k) + y_3(k) + \ldots \tag{1}$$

Despite being a infinite expansion [14], this expression can usually be truncated until a low-order component (i.e. order 2 or 3) for the case of smooth nonlinearities [15, 16] .

Each $\eta-$th order component of the expansion, $y_\eta(k)$, can be represented as a multidimensional convolution between the $\eta-$th Volterra kernel, $\mathcal{H}_\eta(n_1, \ldots, n_\eta)$, and the input signal $u(k)$:

$$y_\eta(k) = \sum_{n_1=0}^{N_1} \ldots \sum_{n_m=0}^{N_\eta} \mathcal{H}_\eta(n_1, \ldots, n_\eta) \prod_{i=1}^{\eta} u(k - n_i) \tag{2}$$

where $N_1, ..., N_\eta$ is the number of terms considered to represent the $\eta-$th Volterra kernel. The main problem of this approach is that this number of terms can be high, specially in higher-order kernels. This can make it difficult to calculate the kernels since it is an ill-posed numerical problem with severe problems of convergence [17]. This issue can be minimized by using orthonormal functions $\psi_{i_j}(n_j)$ to represent the $\eta-$th Volterra kernel $\mathcal{H}_\eta(n_1, \ldots, n_\eta)$:

$$\mathcal{H}_\eta(n_1, \ldots, n_\eta) \approx \sum_{i_1=1}^{J_1} \ldots \sum_{i_\eta=1}^{J_\eta} \mathcal{B}_\eta\left(i_1, \ldots, i_\eta\right) \prod_{j=1}^{\eta} \psi_{i_j}(n_j) \tag{3}$$

where $J_1, ..., J_\eta$ is the number of samples in each $\eta-$th orthonormal projection of the kernel $\mathcal{B}_\eta\left(i_1, \ldots, i_\eta\right)$. With this reduced representation of the Volterra kernels, it is possible to write the Volterra model in a matrix form by grouping the input signal in a matrix $\mathbf{\Gamma}$ and the output signal in the vector $\mathbf{y}$. In this way the vector with the orthonormal projections of the kernels, $\mathbf{\Phi}$, can be calculated by a least-squares estimation:

$$\mathbf{\Phi} = (\mathbf{\Gamma}^T \mathbf{\Gamma})^{-1} \mathbf{\Gamma}^T \mathbf{y} \tag{4}$$

With the identified Volterra kernels it is possible to make the multidimensional convolution as depicted in the eq. 2 to calculate each $\eta-$th component of the response. In this work, this model is considered to represent the studied system in its reference undamaged condition. Therefore, this model can be used to monitor the state of an initially nonlinear system.

The damage index was based on the prediction error of the reference Volterra model [12]. The main idea of this metric is that the response of the model should deviate from the measured response if a change in the structure happens. The prediction error, e_η, is defined considering η components in the Volterra series expansion:

$$e_\eta = y_{exp} - \sum_{m=1}^{\eta} y_m \tag{5}$$

where y_{exp} is the measured response, and y_m is the $m-$th component of the series expansion. An $\eta-$th order metric, named λ_η, was defined as the ratio between the standard deviation of the prediction error in the unknown condition, $e_{\eta,unk}$, and in the reference condition, $e_{\eta,ref}$:

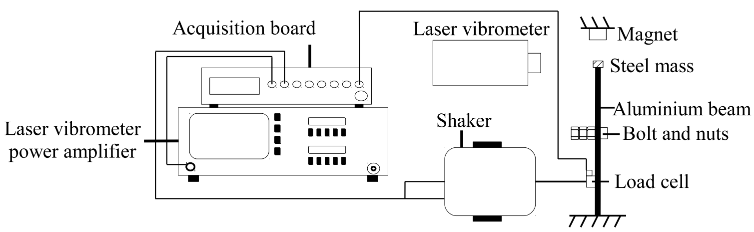
$$\lambda_\eta = \frac{\sigma\left(e_{\eta,unk}\right)}{\sigma\left(e_{\eta,ref}\right)} \tag{6}$$

In the cases where the structure is undamaged, it is expected that $\lambda_\eta \approx 1$ since the prediction errors in the reference and unknown conditions should have similar statistical distributions. Otherwise, any other effect modified the structure making the index to deviate from 1.

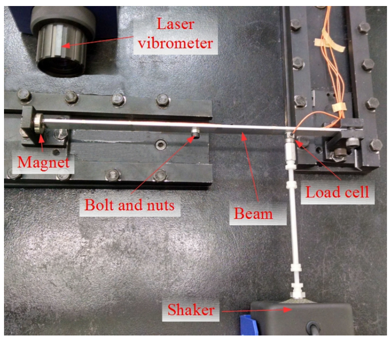
The effects of structural variations and input levels will be showed in the next section.

3. Experimental study of damage detection using Volterra-based metrics

The experimental setup is composed by a clamped-free aluminum beam with dimensions of $300 \times 19 \times 3.2$ mm. The beam have a small steel mass attached to the tip of the free end of the beam and a neodymium magnet positioned 2 mm distant to the mass as seen in figure 1. A shaker is attached 50 mm distant from the clamped end instrumented with a load cell. A laser vibrometer is used to measure the velocity of the oscillation in the free end.



(a) Schematic diagram.



(b) General view.

Figure 1. Experimental setup of the magneto-elastic system.

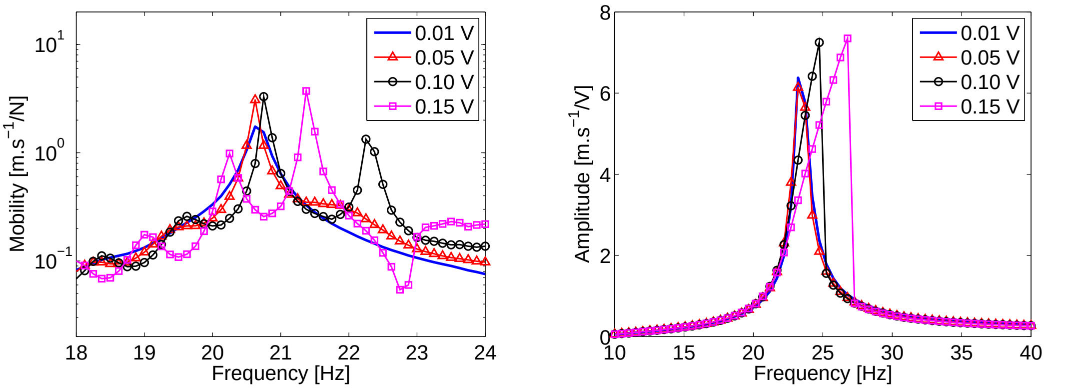
The structure was excited by a sine sweep signal from 10 to 50 Hz in 4 amplitude levels applied in the shaker: 0.01, 0.05, 0.10 and 0.15 V. For each level 40 blocks were collected in a sample rate of 1024 Hz with 8192 samples for each block. A bolt with 4 nuts was placed in a hole in the center of the beam. The reference condition was considered to be the structure with 4 nuts (state 1) and the damage was simulated by removing these nuts one-by-one until only 1 nut remains. After this the same kind of tests were performed again by putting back the nuts in the bolt one-by-one in order to simulate a gradual repair in the structure. This was done in order to investigate if the damage index shows the structure returning to the reference condition. A summary of the tests is illustrated in the table 1.

Table 1. Structural states simulated in the nonlinear system.

State	Condition	State	Condition
1	4 nuts (reference)	5	1 nut (repair)
2	3 nuts (damaged)	6	2 nuts (repair)
3	2 nuts (damaged)	7	3 nuts (repair)
4	1 nut (damaged)	8	4 nuts (repair)

A stepped sine test was also performed in the beam mapping the frequency range from 10 to 40 Hz in 4 different input amplitudes. The frequency response function (FRF) calculated with the sweep sine input and the stepped sine response are showed in the figure 2.

In the sweep sine FRF it is possible to observe the hardening behavior of the system and many distortions in the FRF with the increase of the excitation. The hardening effect can be more clearly seen in the stepped sine response where it is also possible to observe the jump phenomenon characterized by the sudden fall in the response level [3]. The main issue with these effects is that they can be mistakenly viewed as a damage if a proper nonlinear model is not used. Another possibility is that the nonlinearity can also hide effects of structural damage. These facts make it important to identify a model that take in account this kind of phenomenon.



(a) FRF with 4 nuts in the beam.

(b) Stepped sine response for the beam with 4 nuts.

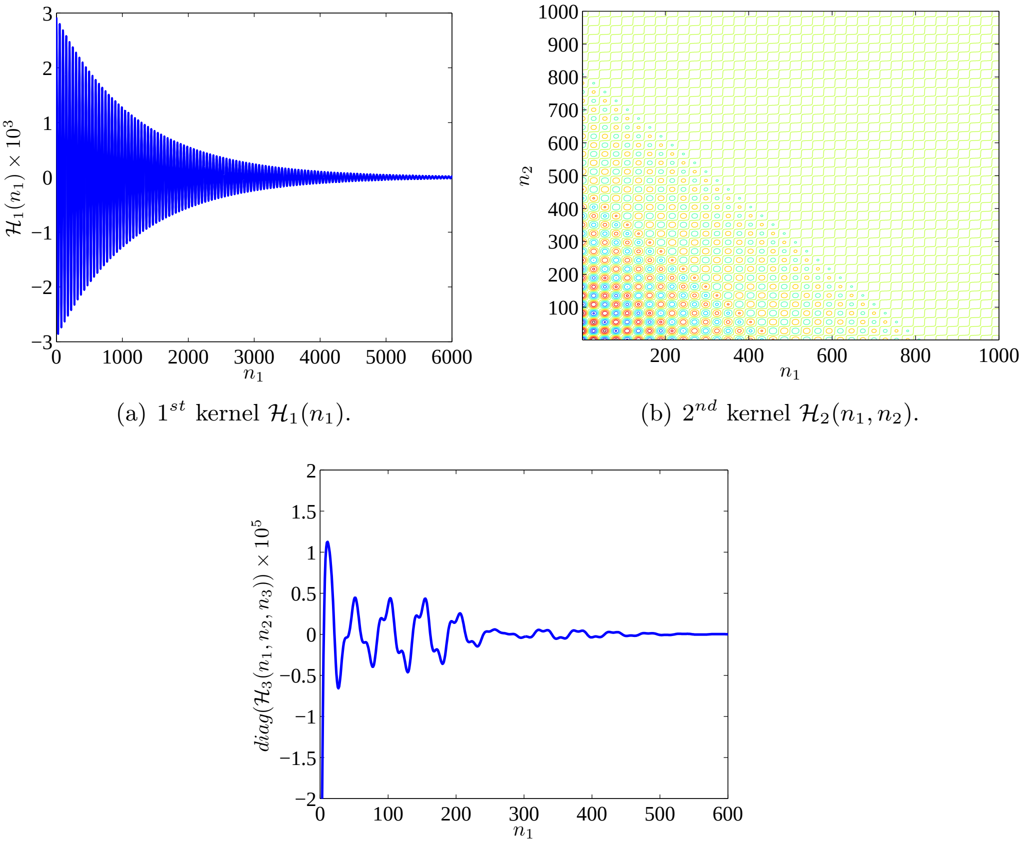
Figure 2. Nonlinearity detection of the magneto-elastic system for different input amplitudes.

In order to model the nonlinearity of the system a 3^{rd} order Volterra model was employed to identify the benchmark structure. The Kautz orthonormal functions were defined through an optimization procedure aiming to minimize the prediction error of the model. The frequencies and damping ratios representing the orthonormal functions for each kernel are presented in table 2. The high-level input (0.15 V) was used to estimate the first three kernels that are illustrated in figure 3. The 1^{st} kernel is analogous to the impulse response function while the other kernels have additional dimensions in the representation. The 3^{rd} kernel can not be fully represented and only the main diagonal is showed for illustration.

With the identified model it is possible to estimate the linear and nonlinear components of the response of the system using the multidimensional convolutions showed in the eq. 2 . The performance of the model in the high-level input is illustrated in figure 4 where it is showed a direct comparison between the responses of the experimental system and of the model, and also the linear and nonlinear components of this response. The response of the model to a sine excitation with 21 Hz and 0.15 V of amplitude is illustrated by the power spectral densities (PSD) in a direct comparison with the experimental response and the components of the response in figure 5. The harmonics are clearly showed in the figure 5 (b) where it is seen that the 1^{st}, 2^{nd} and 3^{rd} kernels reproduces the first three harmonics of the input frequency.

Table 2. Information for the identification of the Kautz functions.

f_1 [Hz]	ζ_1[%]	f_2 [Hz]	ζ_3[%]	f_3 [Hz]	ζ_3[%]
20.65	0.65	19.03	0.27	19.91	1.38



(a) 1^{st} kernel $\mathcal{H}_1(n_1)$.

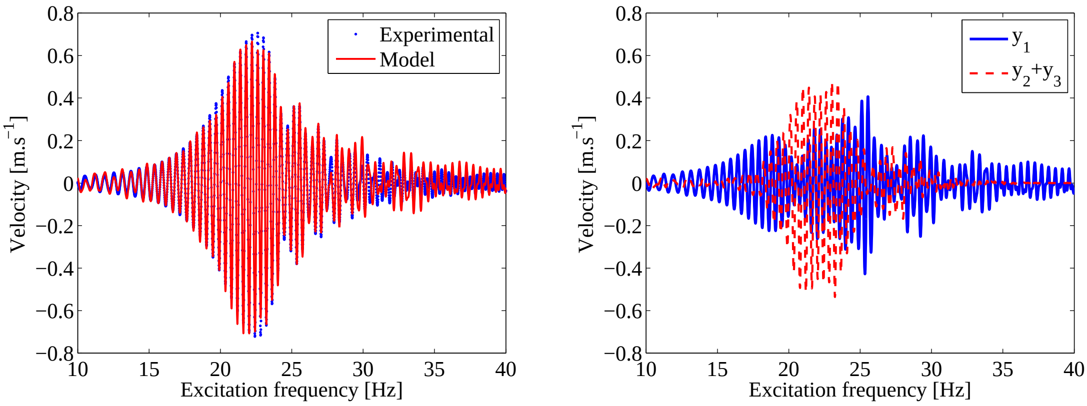
(b) 2^{nd} kernel $\mathcal{H}_2(n_1, n_2)$.

(c) Main diagonal of the 3^{rd} kernel $\mathcal{H}_3(n_1, n_2, n_3)$

Figure 3. Volterra kernels of the magneto-elastic system in the physical basis.

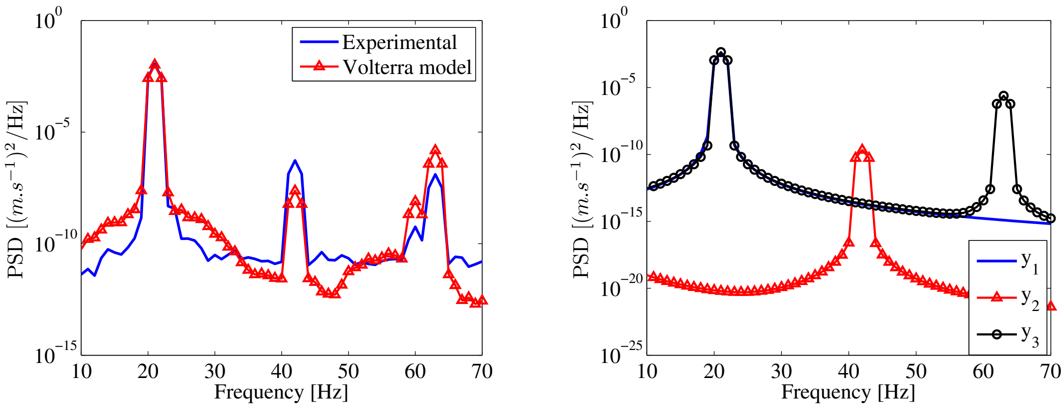
The damage index based on the prediction error of the reference Volterra model showed in eq. 6 was applied in the acquired dataset for each one of the structural states showed in the table 1. The indexes were calculated with the last 39 blocks in each combination of amplitude levels and structural states. The first block was rejected due to the presence of transients related to the start in the testing. The results for the 39 realizations considering the error bars with $2 \times \sigma$, where σ is the standard deviation of the samples, are depicted in the figure 6 for the low-level input (0.01 V) and in the figure 7 for the high-level input (0.15 V). These figures show a linear

(λ_1) and a nonlinear metric (λ_3) while the 2^{nd}-order index λ_2 was not taken in account since the contributions of y_2 does not showed to be relevant to the response. In the figure 6, for the low-input level (0.01 V), both indexes are sensitive to structural variations since it is possible to observe the increase in the index from the states 1 to 4, where the damage is applied, and the decrease in λ_η from the states 5 to 8, where the repair of the bolts was done.



(a) Direct comparison. (b) Components of the response.

Figure 4. Output estimated by Volterra model in healthy condition.



(a) Direct comparison between the PSDs. (b) PSDs of the components of the response.

Figure 5. Response of the Volterra model to a sine excitation.

However, in the figure 7 with the high-level input (0.15 V) only the nonlinear index λ_3 points to the real structural condition while λ_1 detailed in the figure 7(b) does not have a trend consistent with the evolution of the structural state of the system. This happens due to the fact that at this level of displacement of the beam the nonlinearity is very relevant in the response of the structure and so the linear index is not able to distinguish between the nonlinear effects and the damage. This fact illustrates what would happen if one chooses to model the system by assuming only the linear behavior of this kind of structure.

Figure 8 shows the indexes λ_1 against λ_3 for all the realizations in each structural state. In the diagram for the input of 0.01 V both indexes separate well the states of the structure while for the 0.15 V input only λ_3 really characterize the damage. Also a lot of dispersion is observed mainly in the case with 4 nuts and input level of 0.15 V during the application of the damage (blue circles) which was probably caused by the loosening of the nuts during the test cycles. This can also explain why index do not really return to the reference value during the repair.

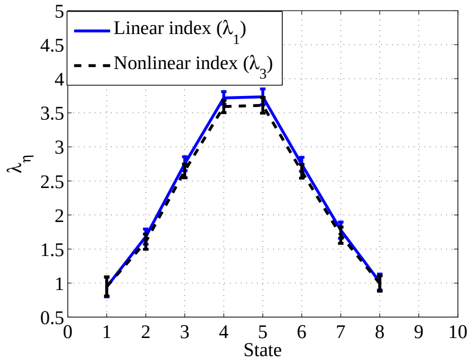
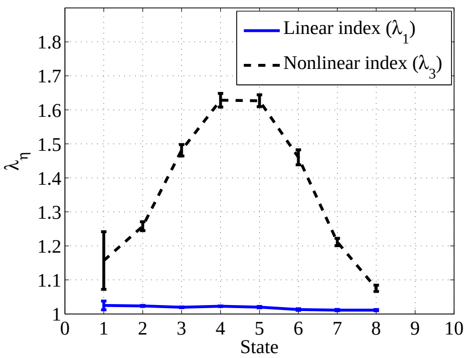
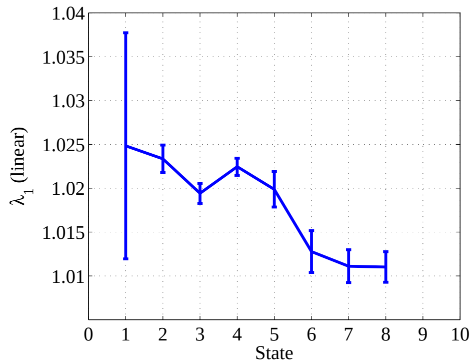


Figure 6. Damage index based on the prediction error (λ_η) for an input level of 0.01 V.

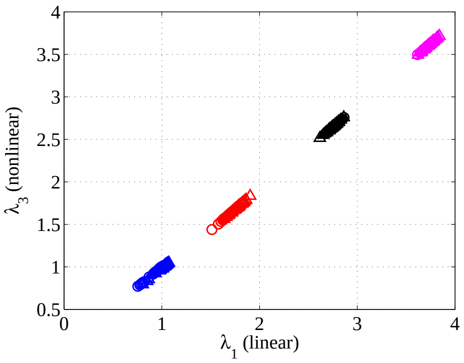


(a) Damage indexes λ_η for an input of 0.15 V.

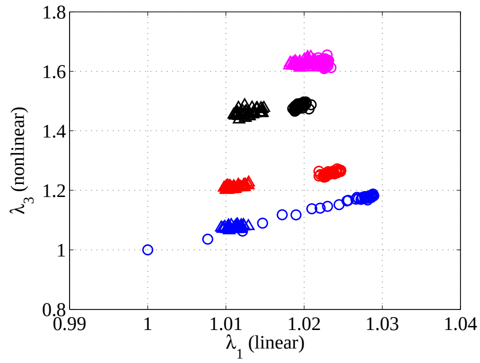


(b) Linear index λ_1 for an input of 0.15 V.

Figure 7. Damage index based on the prediction error (λ_η) for an input level of 0.15 V.



(a) λ_1 *versus* λ_3 for an input of 0.01 V.



(b) λ_1 *versus* λ_3 for an input of 0.15 V.

Figure 8. Comparison between the linear index λ_1 and the nonlinear index λ_3. Blue is the reference state with 4 nuts, Red is the case with 3 nuts, Black represents the case with 2 nuts and Purple is the case with 1 nut. Circles $\circ$ represent the case where the damage is being applied and triangles $\triangle$ represent the repaired cases.

4. Final remarks

This work proposed the application of the Volterra series in the problem of damage detection in an initially nonlinear structure. The structure showed to behave with a hardening type of nonlinearity. If linearity is assumed this kind of behavior can be mistakenly viewed as a damage in the system. Orthonormal Kautz functions were applied in order to minimize numerical problems of the identification of the kernels. An index based on the prediction error of the reference model was tested to detect structural variations. The metric showed to be sensitive to structural variations even under the nonlinear regime of motion. The index also illustrated a clear difference between considering or not the nonlinear behavior of the system since the linear version of the index can fail to represent the structural state.

The results used the interesting property of the Volterra expansion in separating the linear and nonlinear parts of the response of a nonlinear system. This was useful to show the possible drawbacks of the linearity assumption. However, one should keep in mind that the identified model has a limited range of operation since for higher levels of nonlinearity the model could fail since higher-order kernels may be necessary. This is still a problem in most of the nonlinear tools since there is still not a general tool to apply in this kind of system.

5. Acknowledgments

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References

[1] Kerschen G, Worden K, Vakakis A F 2006 Past, present and future of nonlinear system identification in structural dynamics *Mech. Syst. Signal Pr.* **20** 502-592.

[2] Virgin L N 2000 *Introduction to Experimental Nonlinear Dynamics: A Case Study in Mechanical Vibration* (Cambridge University Press)

[3] Worden K and Tomlinson G R 2001 *Nonlinearity in Structural Dynamics* (London : Institute of Physics Publishing)

[4] Bornn L, Farrar C R and Park G 2010 Damage detection in initially nonlinear systems *Int. J. Eng. Sci.* **48** 909-920

[5] Peng Z K, Lang ZQ, Billings S A and Tomlinson G R 2008 Comparisons between harmonic balance and nonlinear output frequency response function in nonlinear system analysis *J. Sound Vib.* **311** 56-73

[6] Fassois S D and Sakellariou J S 2007 Time-series methods for fault detection and identification in vibrating structures *Philos. T. R. Soc. A* **365** 411-448

[7] Peeters M, Kerschen G and Golinval J C 2011 Modal testing of nonlinear vibrating structures based on nonlinear normal modes: Experimental demonstration *Mech. Syst. Signal Pr.* **25** 1227-1247

[8] Schetzen M 1980 *The Volterra and Wiener theories of nonlinear systems* (New York: Wiley)

[9] Chatterjee A 2010 Structural damage assessment in a cantilever beam with a breathing crack using higher order frequency response functions *J. Sound Vib.* **329** 3325-3334

[10] Tang H, Liao Y H, Cao J Y and Hang X 2010 Fault diagnosis approach based on Volterra models *Mech. Syst. Signal Pr.* **24** 1099-1113

[11] Rébillat M, Hajrya R and Mehbal N 2014 Nonlinear structural damage detection based on cascade of Hammerstein models *Mech. Syst. Signal Pr.* **48** 247-259

[12] Shiki S B, Lopes Jr V and da Silva S 2013 Damage Detection in Nonlinear Structures using Discrete-Time Volterra Series *Key Eng. Mat.* **569** 876-883

[13] Shiki S B, Hansen C and da Silva S 2014 Nonlinear features identified by Volterra series for damage detection in a buckled beam *International Conference on Structural Nonlinear Dynamics and Diagnosis*

[14] Rugh W J 1991 *Nonlinear System Theory – The Volterra/Wiener Approach* (The Johns Hopkins University Press)

[15] da Silva S 2011 Non-linear model updating of a three-dimensional portal frame based on Wiener series *Int. J. Nonlin. Mech* **46** 312-320

[16] Shiki S B, Lopes Jr V and da Silva S 2013 Identification of nonlinear structures using discrete-time Volterra series *J. Braz. Soc. Mech. Sci.* **36** 523-532

[17] da Rosa A, Campello R J G B and Amaral W C 2007 Choice of free parameters in expansions of discrete-time Volterra models using Kautz functions *Automatica* **43** 1084-1091