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Improved Stochastic Subspace System Identification for Structural Health Monitoring

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Abstract. Structural health monitoring acquires structural information through numerous sensor measurements. Vibrational measurement data render the dynamic characteristics of structures to be extracted, in particular of the modal properties such as natural frequencies, damping, and mode shapes. The stochastic subspace system identification has been recognized as a power tool which can present a structure in the modal coordinates. To obtain qualitative identified data, this tool needs to spend computational expense on a large set of measurements. In study, a stochastic system identification framework is proposed to improve the efficiency and quality of the conventional stochastic subspace system identification. This framework includes 1) measured signal processing, 2) efficient space projection, 3) system order selection, and 4) modal property derivation. The measured signal processing employs the singular spectrum analysis algorithm to lower the noise components as well as to present a data set in a reduced dimension. The subspace is subsequently derived from the data set presented in a delayed coordinate. With the proposed order selection criteria, the number of structural modes is determined, resulting in the modal properties. This system identification framework is applied to a real-world bridge for exploring the feasibility in real-time applications. The results show that this improved system identification method significantly decreases computational time, while qualitative modal parameters are still attained.

1. Introduction

Performance of civil infrastructure directly affects public safety and society cost. Civil infrastructure refers to the integration of various systems such as buildings, bridges, transportation networks, lifeline, etc. Components in such systems need to be functional; otherwise, a huge amount of economic loss and dead lives would occur and impact the entire society. For example, the I-35W Mississippi River Bridge collapsed on August 1, 2007, resulting in 13 people killed and 145 people injured. A replacement bridge, the I-35W Saint Anthony Falls Bridge, was then constructed and opened on September 18, 2008. The estimated users' economic loss was US$71,000 to US$220,000 a day, while more than 50,000 users needed to reroute [1]. This example demonstrates that deficient and aging infrastructure systems require diagnosis of present conditions to prevent catastrophic failures. Thus, structural health monitoring (SHM) is of need to early identify these problems in structures [2, 3]. Operational modal analysis is to extract the dynamic characteristics of structures based on the structural vibration responses. These dynamic characteristics are composed of natural frequencies,

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damping ratios, and mode shapes. Deviations in these dynamic characteristics reflect the changed properties of structures. Detailed inspection may be required for the structures. A number of researchers applied system identification to real-world structures for operational modal analysis such as Farrar and James [4], Brownjohn [5], Lynch *et al*. [6], Siringoringo and Fujino [7], Weng *et al*. [8] and Jang *et al.* [9]. With growth of sensing technology, structural health monitoring has drawn a lot of attentions to researchers in order to assure structures of their serviceability and safety. Structural integrity can be then studied through operational modal analysis to the sensor measurements of structures.

Due to unavailability of input excitation, the operational modal analysis is directed to the stochastic system identification. This type of system identification methods is emphasized on performance assessment of structures using measured outputs. In terms of time-domain approaches, one of the popular methods is stochastic subspace system identification (SSI) proposed by Van Overschee and De Moor in 1991 [10]. This method utilizes the extended observability to derive modal parameters Moreover, Peeters and De Roeck in 1999 further extended the SSI method to improve computational efficiency [11]. Peeters and De Roeck in 2001 also proposed the stabilization diagram for SSI to enhance the quality of identification results [12]. Even though such nice methods are developed for operational modal analysis, both identification efficiency and quality are still a concern in practice.

One of common seen problems in SSI is the noise modes obtained in results. This noise modes may be reduced or removed by means of time series analysis prior to system identification such as the Singular Spectrum Analysis (SSA). Numerous fields has employed SSA to investigate the trend and periodic components to various components [13-16]. Moreover, SSA can be extended to multi-channel time series. For structural health monitoring applications, a large amount of data are measured from structures. When applying SVD to the Hankel matrix, the loading in computation becomes a problem. Further reductions on the dimension of the Hankel matrix is a solution to expedite the computational rate as well as to be suitable for structural health monitoring problems.

Another possible reason to result in noise modes in identification results is the orthogonal projection. Yang and Nagarajaiah in 2014 found that the outliers in measure signals can contaminate identification results and introduce noise modes to be obtained [17]. Moreover, the low-amplitude signals in measurements can also induce the difficulty in the calculation of a subspace through the orthogonal projection. To effectively eliminate the noise modes in identification results, these unfavourable components need to be removed in advance.

As introduced in [12], a stabilization diagram is helpful for determining true modes in identification. However, establishing a stabilization diagram may take a large amount of time, especially state-space realization. Many studies developed a series of methods that allow determining the number of states [18-20]. The resulting number of states from these methods are tended to be overestimated. Thus, these methods can be employed in a stabilization diagram to limit the numbers of states used. To accelerate the determination of true modes, modifications should be made to a stabilization diagram.

In this study, an improved stochastic subspace system identification method is developed. This method consists of 1) signal preprocessing using SSA, 2) forming a modified Hankel matrix with a reduced dimension, 3) effectively orthogonal projection to obtain multiple subspaces, and 4) extraction of modal parameters from a given range of numbers of states. The signal preprocessing using SSA is employed to reduce the noise effect in system identification as well as to retain the principle components in measured signals. A modified Hankel matrix is developed to eliminate the low-energy components in this matrix. Then, multiple subspaces are calculated using the proposed effective projection method. Each subspace can render a set of modal parameters. By analysing these sets of modal parameters, the dynamic characteristics of a structure can be obtained. Moreover, the proposed method is evaluated by field measurements from a cable-stayed bridge. The identification results illustrate that the efficiency and quality of the proposed stochastic subspace system identification method suppress those of the conventional SSI.

2. Framework of Proposed System Identification Method

Figure 1 illustrates the improved stochastic subspace system identification method. This improved method contains four sequential components: 1) preprocessing, 2) forming Hankel matrix, 3) obtaining subspaces through projection, and 4) determining stable modes. The signal preprocessing eliminates signal trends and offsets and prepares denoised signals. The Hankel matrix is then formed, and only those columns with higher norms are retained. In comparison to the convectional stochastic subspace system identification, the improved method calculates several subspaces instead of one. The state-space system and measurement matrices are derived based on the extended observability matrix that is calculated from a subspace. A range of orders (or a range of numbers of states) are used to examine different sets of identified modes, and the range is determined by proposed criteria. Finally, a number of modes are obtained with respect to subspaces and orders. The stable modes are determined using a similar approach to the stabilization diagram.

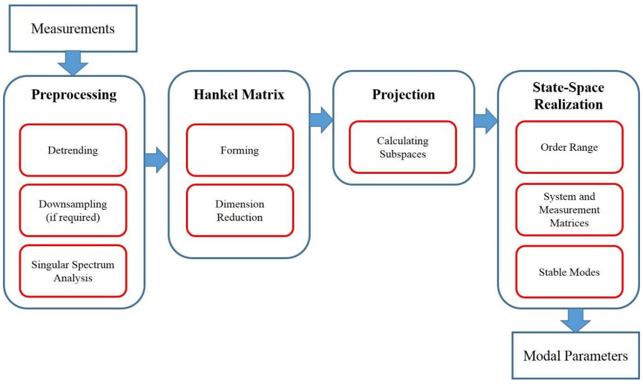

Figure 1. Framework of improved stochastic subspace system identification.

In this study, the proposed system identification method is developed in accordance to the stochastic state-space model for a structure. The stochastic discrete-time state-space model is defined by

$$\begin{aligned} &\mathbf{x}[\mathrm{k}+1]=\mathbf{A}_d x[\mathrm{k}]+\mathbf{w}[\mathrm{k}] \\ &\mathbf{y}[\mathrm{k}]=\mathbf{C}_d[\mathrm{k}]+\mathbf{v}[\mathrm{k}] \\ &\mathbf{x}\in\mathfrak{R}^{n_S\times 1}, \mathbf{y}\in\mathfrak{R}^{n\times 1}, \mathbf{w}\in\mathfrak{R}^{n_S\times 1}, \mathbf{v}\in\mathfrak{R}^{n\times 1} \end{aligned} \tag{1}$$

where $\mathbf{x}$ and $\mathbf{y}$ are the state and output measurement vectors at time step, k; $\mathbf{A}_d$ and $\mathbf{C}_d$ are the system and measurement matrices in the stochastic state-space representation; and $\mathbf{w}$ and $\mathbf{v}$ are the input and output noise. The time span is defined as $k\in[1,N]$ where N is the total number of samples.

2.1 Pre-processing

Measured signals from structures are sometimes distorted or contaminated by drift, offset, and noise. A drift and offset can be corrected by a simple function such as low-order polynomial function and piecewise linear functions. Noise can be reduced or eliminated by filtering or time series analysis. Downsampling signals to the frequency range of interest is an approach to limit the frequency content in low frequencies, and then noise in high frequencies can be reduced. Another method is to utilize the Multi-channel Singular Spectrum Analysis (MSSA). MSSA consists of three steps: embedding, singular value decomposition, and reconstruction [16]. The basic concept of MSSA is to find a transformation matrix that satisfies

$$\mathbf{Y}=\begin{bmatrix} \mathbf{y}[1] & \mathbf{y}[1+h] & \cdots & \mathbf{y}[1+(n_c-1)h] \\ \mathbf{y}[2] & \mathbf{y}[2+h] & \cdots & \mathbf{y}[2+(n_c-1)h] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}[l] & \mathbf{y}[l+h] & \cdots & \mathbf{y}[l+(n_c-1)h] \end{bmatrix}=\mathbf{TY}+(\mathbf{I}-\mathbf{T})\mathbf{Y} \tag{2}$$

where **Y** is the extended Hankel matrix (or the delay coordinates) with a specific delay, h; l is the window length each column vector; n_c is the number of delays in the row; **T** is the transformation matrix; and (**I-T**) indicates the noise or less significant components in the **Y** space. The transformation matrix represents the mapping of most significant components in the space of the measurements, $\mathbf{y}[k]$.

Embedding in SSA is to form an extended Hankel matrix such as **Y** in Eq. (2). In the SSA or MSSA theory, h in Eq. (2) is typically equal to 1; l should be in a range of 2~N/2; and n_c is equal to N-l+1. To reduce the dimension in embedding, an additional parameter, h, is introduced. A smaller size of the extended Hankel matrix can expedite the rest of the SSA process (e.g., singular value decomposition and reconstruction). When using h, some rules should be satisfied such as

$$\begin{gathered} 1 \le h \le l \\ \left(1+\left(n_c-1\right)h\right) > nl \\ 2 < l < N/2 \end{gathered} \tag{3}$$

Singular value decomposition in SSA is to obtain a transformation matrix. When **Y** in Eq. (2) is decomposed by SVD, the transformation matrix can be represented by

$$\begin{aligned} \mathbf{Y} &= \mathbf{U\Lambda V}^T = \mathbf{U}_m\mathbf{U}_m^T\mathbf{Y} + \left(\mathbf{I}-\mathbf{U}_m\mathbf{U}_m^T\right)\mathbf{Y} \\ \mathbf{T} &= \mathbf{U}_m\mathbf{U}_m^T \end{aligned} \tag{4}$$

where $\mathbf{\Lambda}$ is a matrix which contains nonnegative singular values of **Y** in diagonal terms; **U** and **V** are unitary matrices. The singular values in $\mathbf{\Lambda}$ are in a decreasing order so that the first few column vectors in $\mathbf{U}_m$ indicate the most significant components in the space of **Y**. Assume that the number of modes in noise-free structural measurements is known, and the number of significant components are equal to two times of number of modes; the rest of singular values are very close to zero. Consider structural measurements with noise, and the rest of singular values will be nonzero. For a single-channel time series (e.g., n = 1), the variance of noise can be estimated through SVD to the covariance matrix of **Y**, given by

$$\mathbf{M} = \frac{1}{\left(n_c-1\right)h}\mathbf{YY}^T = \mathbf{U}\bar{\mathbf{\Lambda}}\mathbf{U}^T \tag{5}$$

where

$$\begin{gathered} diag\left(\mathbf{\Lambda}\right) = \left[\lambda_1 \quad \cdots \quad \lambda_{no} \quad \lambda_{no+1} \quad \cdots \quad \lambda_l\right] \\ \lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_{no} > \lambda_{no+1} \ge \cdots \ge \lambda_l \end{gathered} \tag{6}$$

Because **Y** has a maximum rank equal to the row dimension, the left unitary matrix, **U**, in both Eqs. (4-5) is the same. no is assumed to be two times of number of modes, and the measurement noise is assumed to be Gaussian white noise. The variance of measurement noise, σ, in this single-channel time series is estimated by

$$\sigma \cong \frac{1}{\left(l-no\right)}\sum_{i=no+1}^{l}\lambda_i \tag{7}$$

To extend the same idea in multi-channel structural measurements, Eq. (7) is modified by

$$\frac{1}{n}\sum_{j=1}^{n}\sigma_j \cong \frac{1}{\left(nl-no\right)}\sum_{i=no+1}^{nl}\lambda_i \tag{8}$$

where σ_j is the variance of measurement noise with respect to channel. When the number of modes or no is unknown, Eq. (8) can be treated as a criterion to examine no. Because no ranges from 2 to l, only a few trial-and-error attempts are required to approach no using Eq. (8). Consequently, the transformation matrix is determined.

The final step in MSSA is reconstruction that averages out the processed measurements (e.g., **TY**). When single-channel measurements are considered and h equals 1, the reconstruction cam be completed by the diagonal averaging of **TY** [16]. For MSSA, a block-diagonal averaging method can be applied if h is equal to 1. When h is greater than 1, the reconstruction can be completed by a similar approach, counting the number of occurrences per step in **Y** and then averaging the signals at a step by this number. The reconstructed signals contains the most dominate components in a structure.

2.2 Forming Hankel Matrix

As described in [10], a Hankel matrix is formed in the beginning of the SSI method. Likewise to Eq. (2), the Hankel matrix, **H**, is composed of the processed measurements, given by

$$\mathbf{H}=\begin{bmatrix}\mathbf{Y}_{\mathrm{p}}\\ \hline \mathbf{Y}_{\mathrm{f}}\end{bmatrix}=\left[\begin{array}{cccc}\bar{\mathbf{y}}[1] & \bar{\mathbf{y}}[2] & \cdots & \bar{\mathbf{y}}[n_{\mathrm{h}}]\\ \bar{\mathbf{y}}[2] & \bar{\mathbf{y}}[3] & \cdots & \bar{\mathbf{y}}[n_{\mathrm{h}}+1]\\ \vdots & \vdots & \ddots & \vdots\\ \bar{\mathbf{y}}[l] & \bar{\mathbf{y}}[l+1] & \cdots & \bar{\mathbf{y}}[n_{\mathrm{h}}+l]\\ \hline \bar{\mathbf{y}}[l+1] & \bar{\mathbf{y}}[l+2] & \cdots & \bar{\mathbf{y}}[n_{\mathrm{h}}+l+1]\\ \bar{\mathbf{y}}[l+2] & \bar{\mathbf{y}}[l+3] & \cdots & \bar{\mathbf{y}}[n_{\mathrm{h}}+l+2]\\ \vdots & \vdots & \ddots & \vdots\\ \bar{\mathbf{y}}[2l] & \bar{\mathbf{y}}[2l] & \cdots & \bar{\mathbf{y}}[n_{\mathrm{h}}+2l]\end{array}\right] \tag{9}$$

where the subscripts, "p" and "f", denote the "past" and "future" delay coordinates, and $\bar{\mathbf{y}}$ is the processed measurements using the preprocessing methods. The purpose of forming the Hankel matrix is to obtain a column subspace through projection. However, the subspace may be derived a low-resolution projection if the column vectors in **H** has some small numbers. Thus, the improved method introduces a criterion that eliminates "ill-conditioned" column vectors in **H**. These ill-conditioned column vectors can be viewed as those have lower norms. Define that the column vector has the maximum norm in **H** as h_{max}, and then only those column vectors having a norm greater than $\alpha_h h_{\max}$ are retained, given by

$$\begin{gathered}\bar{\mathbf{H}}=\begin{bmatrix}\bar{\mathbf{Y}}_{\mathrm{p}}\\ \hline \bar{\mathbf{Y}}_{\mathrm{f}}\end{bmatrix}=\begin{bmatrix}\mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_{m_{\mathrm{h}}}\end{bmatrix}\\ \|\mathbf{h}_i\|\geq\alpha_h h_{\max},\ i\in[1,m_{\mathrm{h}}]\end{gathered} \tag{10}$$

where m_{h} is the number of columns in $\bar{\mathbf{H}}$. Utilizing Eq. (10), $\bar{\mathbf{H}}$ is derived with a reduced dimension, as well as the column vectors which may induce the projection errors are deleted.

2.3 Projection

The second step in the proposed SSI method is projection that allows one or multiple subspace(s) to be obtained. In Eq. (9), the Hankel matrix is divided into the past and future portions. As given in [10], the projected subspace is calculated using the past and future matrices in Eq. (10) and defined as

$$\mathbf{O}=\bar{\mathbf{Y}}_{\mathrm{f}}/\bar{\mathbf{Y}}_{\mathrm{p}}=\bar{\mathbf{Y}}_{\mathrm{f}}\bar{\mathbf{Y}}_{\mathrm{p}}^{T}\left(\bar{\mathbf{Y}}_{\mathrm{p}}\bar{\mathbf{Y}}_{\mathrm{p}}^{T}\right)^{-1}\bar{\mathbf{Y}}_{\mathrm{p}} \tag{11}$$

where **O** is the projected subspace. Consider using SVD to represent $\mathbf{Y}_{\mathrm{p}}$ and assume that $\mathbf{Y}_{\mathrm{p}}$ is a low-rank matrix, and then Eq. (11) can be simplified as

$$\begin{gathered}\bar{\mathbf{Y}}_{\mathrm{f}}/\bar{\mathbf{Y}}_{\mathrm{p}}\cong\bar{\mathbf{Y}}_{\mathrm{f}}\mathbf{V}_{\mathrm{p,m}}\mathbf{V}_{\mathrm{p,m}}^{T}\rightarrow\bar{\mathbf{Y}}_{\mathrm{f}}\mathbf{V}_{\mathrm{p,m}}\\ \bar{\mathbf{Y}}_{\mathrm{p}}=\mathbf{U}_{\mathrm{p,m}}\mathbf{\Lambda}_{\mathrm{p,m}}\mathbf{V}_{\mathrm{p,m}}^{T}+\mathbf{U}_{\mathrm{p,0}}\mathbf{\Lambda}_{\mathrm{p,0}}\mathbf{V}_{\mathrm{p,0}}^{T}\end{gathered} \tag{12}$$

where the subscripts, "m" and "0", denote the main and null spaces. All singular values in $\mathbf{\Lambda}_{\mathrm{p,0}}$ are assumed to be close to zero. $\mathbf{V}_{\mathrm{p,m}}^{T}$ in Eq. (12) is neglected because this matrix can be a similarity matrix to both extended observability and controllability matrices in the SSI theory. Therefore, a projected subspace is derived from $\overline{\mathbf{Y}}_{\mathrm{f}}\mathbf{V}_{\mathrm{p,m}}$.

In the SSI theory, a Hankel matrix in Eq. (9) only supports to calculate one subspace. The separation line in this Hankel matrix let an identical dimension of the past and future submatrices to be obtained. Thus, the two submatrices result in the extended observability and controllability matrices with an equal number of rows and columns, respectively. However, the separation line can be shifted as long as the resulting past and future submatrices have a number of time lags greater than the number of system states. This approach allows multiple subspaces to be attained and multiple sets of modal parameters to be compared.

2.4 State-space Realization

These dynamic characteristics, such as natural frequencies, damping, and mode shapes, are implied in the stochastic state-space representation of the structure. Because the projected subspace represents the product of the extended observability and controllability matrices, SVD is utilized to separate these two matrices. Then, the system and measurement matrices, $\mathbf{A}_{\mathrm{d}}$ and $\mathbf{C}_{\mathrm{d}}$, are calculated by

$$\begin{gathered}\overline{\mathbf{Y}}_{\mathrm{f}}\mathbf{V}_{\mathrm{p,m}}=\mathbf{U}_{\mathrm{id,m}}\mathbf{\Lambda}_{\mathrm{id,m}}\mathbf{V}_{\mathrm{id,m}}+\mathbf{U}_{\mathrm{id,0}}\mathbf{\Lambda}_{\mathrm{id,0}}\mathbf{V}_{\mathrm{id,0}}\\ \mathbf{\Gamma}=\mathbf{U}_{\mathrm{id,m}}=\begin{bmatrix}\mathbf{C}_{\mathrm{d}}\\ \mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}\\ \mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}^{2}\\ \vdots\\ \mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}^{n_s}\end{bmatrix},\ \mathbf{A}_{\mathrm{d}}=\begin{bmatrix}\mathbf{C}_{\mathrm{d}}\\ \mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}\\ \mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}^{2}\\ \vdots\\ \mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}^{n_s-1}\end{bmatrix}^{\dagger}\begin{bmatrix}\mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}\\ \mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}^{2}\\ \mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}^{3}\\ \vdots\\ \mathbf{C}_{\mathrm{d}}\mathbf{A}_{\mathrm{d}}^{n_s}\end{bmatrix}\end{gathered} \tag{13}$$

where the subscripts, "m" and "0", denote the main and redundant components in the product of the extended observability and controllability matrices; $\mathbf{\Gamma}$ represents the extended observability matrix; n_{s} is a given number of states; and $(\ )^{\dagger}$ denotes the pseudo inverse. By applying the eigen analysis to $\mathbf{A}_{\mathrm{d}}$, the eigenvalues are the complex-valued natural frequencies, and the eigenvectors represents the mode shapes with respect to states. In each mode, these modal parameters can be obtained by

$$\begin{gathered}\mathbf{A}_{\mathrm{d}}\boldsymbol{\eta}=\lambda_{\mathrm{d}}\boldsymbol{\eta}\\ \frac{1}{\Delta t}\ln\left(\lambda_{\mathrm{d}},\lambda_{\mathrm{d}}^{*}\right)=-\xi\omega_{\mathrm{n}}\pm j\omega_{\mathrm{n}}\sqrt{1-\xi^{2}}\\ \boldsymbol{\varphi}=\mathbf{C}_{\mathrm{d}}\boldsymbol{\eta}\end{gathered} \tag{14}$$

where λ_{d} is one of the eigenvalues in $\mathbf{A}_{\mathrm{d}}$; λ_{d}^{*} is the complex conjugated λ_{d}; $\boldsymbol{\eta}$ is the eigenvector with respect to the state vector; ω_{n} and ξ are the natural frequency and damping ratio; Δt is the sample time; and $\boldsymbol{\varphi}$ is the mode shapes with respect to the output. By calculating all eigenvalues and eigenvectors from $\mathbf{A}_{\mathrm{d}}$, the modal parameters can be extracted.

3. Application to Field Test Measurements

The last example is to employ the proposed system identification method in a structural health monitoring problem of a cable-stayed bridge in Taiwan. The detailed instrumentation on this bridge are available from [8]. In this study, the focus is to examine the proposed system identification method using the ambient vibration responses, collected from the deck of the cable-stayed bridge in the vertical direction. As compared to the methods in [8], this study centers on the efficiency and qualitative results of system identification to be achieved.

The parameters used in the proposed system identification method are first presented. The structural responses collected are velocities, and the total duration of the measurements is 150 seconds. The sensor locations with respect to the bridge dimension are exhibited in Figure 2. The vertical velocity responses are used in the system identification. These measurements are recorded at 100 Hz. Before employing the recorded data in system identification, the constant offsets in the measured signals are removed, and the data are resampled to 50 Hz. In MSSA, the parameters, l and h, are set to be 50 and 5, respectively. The resulting number of orders by Eq. (8) is 30. In SSI, the window length, l, remains the same, and the locations of separation lines are {320 336 352 368 384 400 416 432 448 464 480}. By exploring the power density functions of all measurements, the preliminary guess of modes is 9; however, the 8th and 9th modes have lower energy so that the adjusted number of modes is 7. Then, the range of numbers of states is 12~30 with an increment of 2. Because of these parameters, the total number of runs using the proposed system identification method is 110.

Figure 3 shows the identified natural frequencies with the range of numbers of states between 12 and 26. Likewise, the magenta curves represent the summation of power spectral density functions, while the “star” markers indicate the identified natural frequencies with respect to a separation line and number of states. The vertical lines are the finalized natural frequencies after cluster analysis. As shown in this figure, the identified modes are concentrated in low frequencies. The first peak at a very frequency in the power spectral density is interpreted as a suspicious mode. In the identification results, only a few runs show this suspicious natural frequency. The number of successfully identified modes is 6. To increase the success number, a higher number of states may be used. In addition, the identified mode shapes are drawn in Figure 4. The results are quite comparable to the study in [8], though two of modes are missing in this study. In comparison of computational time, 110 runs performed using the proposed method is about 40 seconds, while the conventional approach including the stabilization diagram needs at least 10 minutes using the same computer. For system identification, the proposed method is able to attain the computational efficiency and qualitative results.

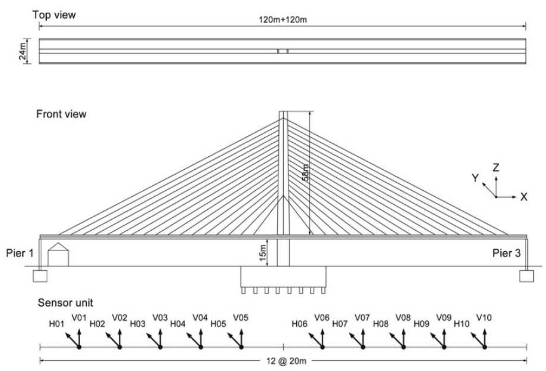

Figure 2. Sensor setup for bridge deck measurements.

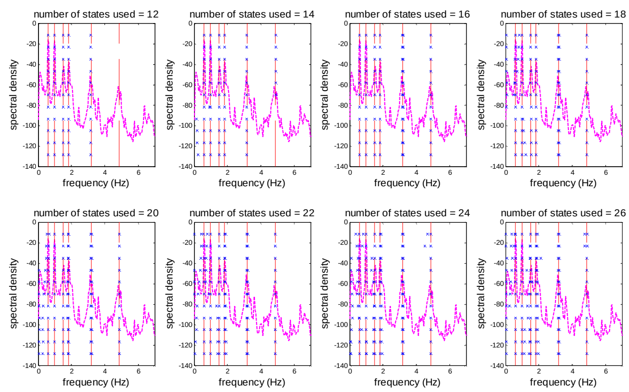

Figure 3. Identified natural frequencies from the field measurements of a cable-stayed bridge.

4. Conclusions

This study proposed an improved stochastic subspace system identification method that can enhance the efficiency and quality in operational modal analysis. In this method, multi-channel singular spectrum analysis was employed to remove noise and uncertainties in measurements, and to estimate the number of states that can be used in the stochastic state-space realization. When forming a Hankel matrix for the stochastic subspace system identification, the low-energy components were removed by a proposed criterion. An efficient projection method was also developed to expedite the process of generating a subspace. By shifting separation lines, multiple sets of the past and future matrices can be obtained, and multiple subspaces were then derived. These subspaces can yield different sets of modal parameters to be obtained. The finalized modal parameters were determined from these sets of results with respect to separation line and number of states.

An example provided was the proposed method applied to the field measurements of a cable-stayed bridge. The results were quite comparable to the previous study, and the computation efficiency was significantly improved in comparison to the conventional stochastic subspace system identification method. Because of these assessments, the proposed method was applicable to be used in most structural health monitoring problems in terms of operational modal analysis.

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References

[1] Xie F and Levinson D 2011 Evaluating the effects of the I-35W bridge collapse on road-users in the twin cities metropolitan region Transportation Planning and Technology. 34(7) 691-703
[2] Farrar CR and Worden K 2007 An introduction to structural health monitoring Philosophical Transactions of the Royal Society of London A. 373 303-315
[3] Sim SH and Spencer BF Jr. 2009 Decentralized strategies for monitoring structures using wireless smart sensor networks Newmark Structural Engineering Laboratory (NSEL) Report Series, No. 19, University of Illinois at Urbana-Champaign, Urbana, Illinois (http://hdl.handle.net/2142/14280)
[4] Farrar CR and James GH 1997 System identification from ambient vibration measurements on a bridge Journal of Sound and Vibration 205(1) 1-18
[5] Brownjohn JMW 2003 Ambient vibration studies for system identification of tall buildings Earthquake Engineering and Structural Dynamics 32 71-95
[6] Lynch JP, Wang Y, Loh KJ, Yi J-H and Yun C-B 2006 Performance monitoring of the Geumdang Bridge using a dense network of high-resolution wireless sensors Smart Materials and Structures 15 1561-1575
[7] Siringoringo DM and Fujino Y 2008 System identification of a suspension bridge from ambient vibration response Engineering Structures 30 (2) 462-477
[8] Weng J-H, Loh C-H, Lynch JP, Lu K-C, Lin P-Y, and Wang Y 2008 Output-only modal identification of a cable-stayed bridge using wireless monitoring systems Engineering Structures 30 1820-1830
[9] Jang SA, Jo H, Cho S, Mechitov KA, Rice JA, Sim SJ, Jung HJ, Yun CB, Spencer BF and Agha G 2010 Structural health monitoring of a cable-stayed bridge using smart sensor technology: deployment and evaluation Journal of Smart Structures and Systems 6(5-6) 439-459
[10] Van Overschee P and De Moor B 1991 Subspace algorithms for the stochastic identification problem Proceedings of the 30th IEEE Conference on Decision and Control, Brighton, UK, 1321-1326
[11] Peeters B and De Roeck G 1999 Reference-based stochastic subspace identification for output-only modal analysis Mechanical Systems and Signal Processing 13(6) 855-878
[12] Peeters B and De Roeck G 2001 Stochastic system identification for operational modal analysis: a review Journal of Dynamic Systems, Measurement, and Control 123(4) 659-667
[13] Broomhead DS and King GP 1986 Extracting qualitative dynamics from experimental data Physica D 20 217-236
[14] Broomhead DS, Jones R and King GP 1987 Singular system analysis with application to dynamical systems In E.R. Pike & L.A. Lugaito (Eds.) Chaos, noise, and fractals 15-27
[15] Fraedrich K 1986 Estimating dimensions of weather and climate attractors Journal of the Atmospheric Sciences 43 419-432
[16] Elsner JB and Tsonis AA 1996 Singular spectrum analysis: a new tool in time series analysis Plenum Press, New York
[17] Yang Y and Nagarajaiah S 2014 Blind denoising of structural vibration responses with outlier via principal component pursuit Journal of Structural Control and Health Monitoring 21(6) 962-978
[18] Akaike H 1974 Anew look at the statistical identification model IEEE Trans. Autom. Control 716-723
[19] Schwarz G 1978 Estimating the dimension of a model Ann. Stat. 6 461-464
[20] Koehler AB and Murphree ES 1988 A comparison of the Akaike and Schwarz of criteria selecting model order J. R. Stat. Soc. Ser. C: Appl. Stat. 37(2) 187-195