

Assessment of a Multiple Model Based Parametric Method for Output-Only Vibration-Based Damage Detection for a Population of Like Structures

Kyriakos J. Vamvoudakis-Stefanou, John S. Sakellariou and Spilios D. Fassois
Stochastic Mechanical Systems & Automation (SMSA) Laboratory, Department of
Mechanical & Aeronautical Engineering, University of Patras, Patras, Greece

E-mail: vamvoudakisk@mech.upatras.gr, sakj@mech.upatras.gr,
fassois@mech.upatras.gr

Abstract. This study focuses on the problem of vibration-based damage detection for a population of like structures. Although nominally identical, like structures exhibit variability in their characteristics due to variability in the materials and manufacturing. This inevitably leads to variability in the dynamics, which may be so significant as to mask deviations due to damage. Damage detection via conventional vibration-based methods, using a common threshold in the decision making mechanism thus becomes highly challenging. The study presents a detailed assessment of a recently introduced Multiple Model (MM) based AutoRegressive (AR) model parameter method aiming at addressing this problem. The assessment is based on high numbers of experimental test/inspection cases using composite beams damaged via impact, as well as comparisons with the corresponding conventional (single model based) method. The results confirm significant improvement over the method's conventional counterpart. A sensitivity analysis additionally indicates that the method is relatively insensitive to the model order, but sensitive to the specific beams selected as baseline (training) ones; in fact their selection may lead to excellent results.

1. Introduction

When dealing with a population of like structures there are two possibilities for vibration-based Structural Health Monitoring: Either a *common* threshold is adopted in the decision making mechanism [1] for all structures, or an *individual* threshold is established for each individual structure. The former approach has some obvious advantages in terms of set-up and cost, and is of high interest. Yet, the problem is that like structures are never identical due to inevitable variability associated with materials and manufacturing, plus of course the fact that they may operate under somewhat different conditions. This inevitably leads to variability in the dynamics, which may be so significant as to mask deviations due to damage.

Unlike other uncertainty factors, such as those associated with environmental and operational variability [2], the handling of the variability associated with a population of like structures and its effects on vibration-based SHM is, thus far, effectively unexplored. In two recent studies [3,4] by the present authors, a Multiple Model (MM) vibration-based framework for SHM for a population of like structures (composite beams subject to delamination) was introduced, and very interesting results and favourable comparisons with corresponding conventional methods were obtained. The



main idea behind this framework is on the use of vibration data obtained from *several* - instead of a single - like structures for modelling the healthy dynamics during the methods' training (baseline phase) and then decision making based on the simultaneous use of all baseline (healthy) models based on a minimal distance type criterion. The results of these studies have illustrated the poor performance of conventional methods and the significantly improved performance of the Multiple Model methods. Yet, the assessments presented are only preliminary and *not* comprehensive, in the sense that neither the sensitivity of the methods with respect to the specific structures selected for inclusion in the baseline, nor that referring to the selected model order have been studied.

Thus the aim of the present study is exactly to address these issues and provide a comprehensive assessment of one of the methods - the MM based AutoRegressive (AR) model parameter method. The laboratory-scale composite beams subject to delamination and other types of damage induced by impact, used in the previous studies [3,4], are presently employed as well, along with a *high number* of test/inspection cases. This is made possible by "rotating" the baseline beams within the set of all healthy beams. Moreover, the sensitivity of the method with respect to the *number* and *specific* structures selected for inclusion in the baseline phase, as well as with respect to *model order* employed, are studied in detail.

The rest of this article is organized as follows: The experimental set-up and the damage scenarios are briefly reviewed in section 2, the MM based AutoRegressive (AR) model parameter method is elaborated upon in section 3, the assessment of the method is performed in section 4, and the conclusions are summarized in section 5.

2. The experimental set-up and the damage scenarios

The study is based on 26 nominally identical, composite beams consisting of several layers of woven and unidirectional (UD) fabric. They have square hollow cross section, uniform along their length with dimensions 600x65x65 mm (LxWxH), wall thickness of 3mm and corner radius of 8mm (figure 1). Each beam represents the topology of the main part of a commercial UAV boom. In the experimental setup each beam is tightly clamped at one end, simulating its connection to the fuselage, while its free end is attached to an aluminum mass representing the aircraft tail (figure 1). The beam is excited at its free end with a random, approximately white, Gaussian force applied vertically at Point X via an electromechanical shaker, while vibration acceleration response signals are measured at point Y1 via a lightweight accelerometer.

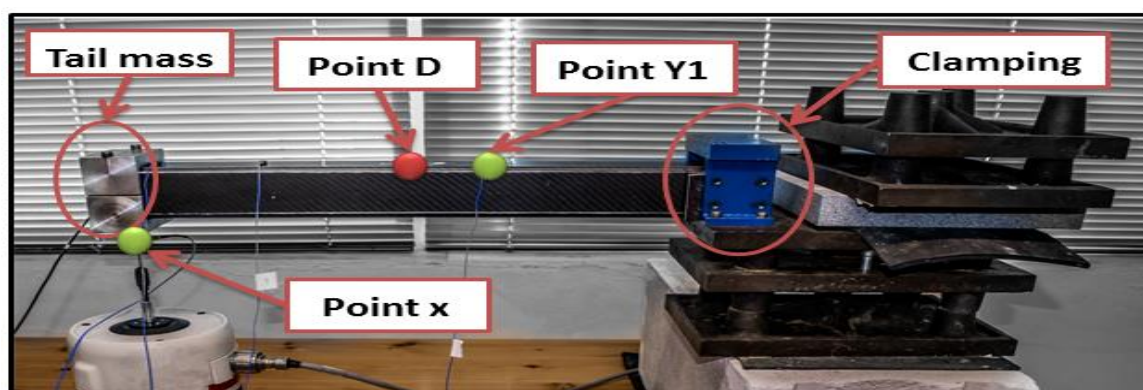


Figure 1. Photo of the experimental set-up: Point x represents the force excitation position, Point Y1 the vibration acceleration measurement position, Point D the damage position.

Three damage scenarios (table 1) that may include a combination of delamination, small cracks and broken fibres are induced in 3 of the 26 beams at point D (figure 1) by using a pendulum type impact hammer with distinct energy level for each damage. Welch-based FRF magnitude estimates [5, pp 329, 338] for the healthy and damaged beams, revealing significant variability in the dynamics

of the healthy beams, as well as the damages effects, are presented in figure 2. For a detailed presentation of the experimental set-up, the employed equipment and the damage scenarios the reader is referred to [3].

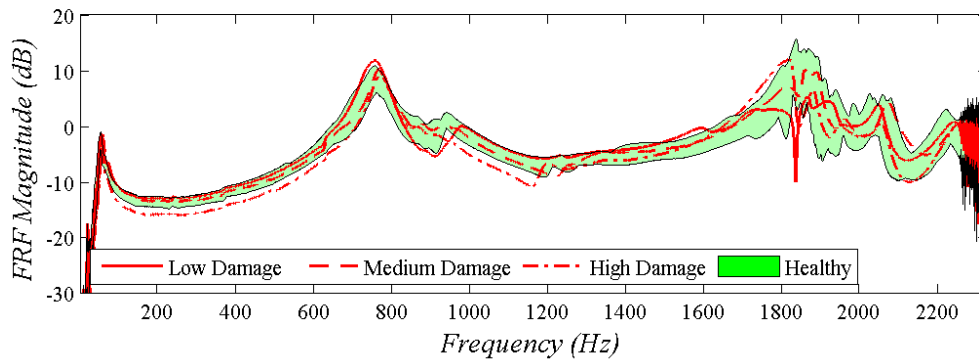


Figure 2. Welch-based FRF magnitude estimates for 23 healthy and 3 damaged beams (7 experiments per beam).

Table 1. The damage scenarios and experimental details.

Structural State	Impact Energy (J)	Number of beams	No of experiments per beam	Total number of experiments
Healthy	---	23	7	161
Damage A	5	1	7	7
Damage B	10	1	7	7
Damage C	15	1	7	7

Sampling frequency $f_s = 4\,654.5$ Hz; signal bandwidth $[3 - 2\,327.25]$ Hz.

3. The Multiple Model (MM) based AR parameter method

The method is of the *response-only type*, based on the parametric AR modelling [6] of the random vibration response signal(s) - the excitation is not required. A scalar AR model is of the form:

$$y[t] + \sum_{i=1}^{n_a} a_i y[t-i] = e[t], \quad e[t] \sim NID(0, \sigma_e^2) \quad (3)$$

with $y[t]$ designating the random vibration response signal, n_a the model order, a_i the i -th AR parameter, and $e[t]$ the model residual that is a white Gaussian zero-mean with variance σ_e^2 sequence. *NID* stands for Normally Independently Distributed. The employed characteristic quantity Q is the AR model parameter vector ($Q = \theta$) [7].

The main idea behind the method is that AR models corresponding to *several* healthy structures, instead of just one (as in conventional methods), are used to effectively represent the healthy structural dynamics while taking variability into account. As a vibration-based method [7-9], the MM based AR parameter method is applied in two phases: (a) The baseline (training) phase, and (b) the inspection (damage detection) phase.

Baseline phase. In the baseline phase vibration signals are obtained from a number of like structures in their healthy state. Let v be the number of employed healthy structures and $Y_o = \{y_{o,k}[t]\}$ the respective response signals with $t = 1, \dots, N$ and $k = 1, \dots, v$ (the subscript o is used to designate the healthy state). Using the available signals (presently one per healthy structure) a set of AR models $m_o = \{m_{o,1}, \dots, m_{o,v}\}$ are obtained, representing the population's healthy dynamics.

Inspection phase. In this phase vibration response signal(s) are obtained from the current structure under inspection (which is in unknown state). The objective is to decide whether the current structure's dynamics are represented by the set of healthy AR models obtained in the baseline phase, in which case the current structure is declared as healthy - if this is not the case, then the structure is declared as damaged.

In doing so, let $Y_u[t]$ designate the current response signal(s) and m_u the respective AR model (the subscript u is used to designate unknown state). Then a proper distance metric in the method's characteristic quantity space (AR model parameters) is employed, and the distances between Q_u and each one of the $Q_{o,1}, \dots, Q_{o,v}$ used as baseline are obtained.

The KL divergence [10] is the distance employed, which for $Q_u, Q_{o,k}$, each following l -dimensional Gaussian distribution with means μ_u, μ_o and covariance matrices $\Sigma_{Q_u}, \Sigma_{Q_{o,k}}$, respectively, is defined as (capital/lower case bold face symbols represent matrix/vector quantities):

$$\begin{aligned} d(Q_u, Q_{o,k}) &= d_{KL}(Q_u, Q_{o,k}) \\ &= \frac{1}{2} \left(\text{tr} \left(\Sigma_{Q_u}^{-1} \Sigma_{Q_{o,k}} \right) + (\mu_u - \mu_o)^T \Sigma_{Q_u}^{-1} (\mu_u - \mu_o) - l - \ln \left(\frac{\det \Sigma_{Q_{o,k}}}{\det \Sigma_{Q_u}} \right) \right) \end{aligned} \quad (1)$$

with $\text{tr}(\cdot)$ designating trace and $\det(\cdot)$ determinant of the indicated quantity.

Once the distances between the current AR model and those of the baseline are available, damage detection is based on the following decision making mechanism that takes *all* distances to the baseline models into account through an *overall distance metric* D :

$$\begin{aligned} D = \sum_{k=1}^v d(Q_u, Q_{o,k}) \leq l_{\text{lim}} &\quad \rightarrow \quad \text{Healthy structure} \\ \text{Otherwise} &\quad \rightarrow \quad \text{Damaged structure} \end{aligned} \quad (2)$$

with l_{lim} designating a user defined (based on the baseline signals) threshold.

4. Assessment of the MM based AR parameter method

The assessment of the MM based AR parametric method is based on damage detection tests with an increasing number of beams used in the baseline phase ($v = \{1, 4, 8, 12, 16\}$). Note $v = 1$ designates the corresponding conventional method. The testing procedure for each number of beams in the baseline phase is based on the following algorithm:

Step 1: All possible combinations among the 23 healthy beams in sets of v beams are considered.

Step 2: If the total number of combinations is very large (for $v = \{4, 8, 12, 16\}$), a random subset of combinations is employed, so that each one of the 23 healthy beams is used in at least one combination. Otherwise, all possible combinations are employed (these are referred to as the baseline combinations).

Step 3: The first baseline combination and one experiment per beam (baseline experiments) are used in the baseline phase of the method. At present the remaining experiments with the baseline beams are not subsequently used.

Step 4: The remaining healthy beams ($23 - v$) are used in the inspection phase, and along with the damaged beams, formulate the set of inspection beams. Damage detection is performed for all the experiments associated with the inspection beams (inspection experiments).

Step 5: Steps 3 and 4 are repeated until all baseline combinations are exhausted.

At the end, a total number of damage detection tests for all the inspection experiments and all the baseline combinations is obtained, forming the set of inspection cases.

This algorithm is a modified cross validation eliminating the effects of baseline beam selection, plus leading to a drastically higher number of inspection cases to assess the method with. For each

number of baseline beams considered, the number of inspection beams, baseline experiments, inspection experiments, baseline combinations, and inspection cases are presented in table 2.

Table 2. Inspection beams, inspection experiments, & inspection cases for various values of v .

No. of baseline beams (v)	No. of inspection beams (i)	Baseline experiments	Inspection experiments ($z = 7 \cdot i$)	Baseline combinations (h)	Inspection cases ($h \cdot z$)
1	25	1	175	23	4 025
4	22	4	154	500	77 000
8	18	8	126	500	63 000
12	14	12	98	500	49 000
16	10	16	70	500	35 000

4.1. Baseline phase

The estimation of the parametric AR models is based on $N = 10\,000$ sample long (≈ 2.15 s) data records using Ordinary Least Squares with QR implementation [11, pp 318-320]. The model order selection is based on the Bayesian Information Criterion (BIC) and the Residual Sum of Squares / Signal Sum of Squares (RSS/SSS) [11, pp 498-514]. The selected model order for all models is presented in table 3, along with a summary of estimation details (the performance characteristics correspond to an individual baseline model).

Table 3. AR model estimation details

Model	Measur. Point	Selected model	No. of estimated parameters	SPP*	BIC**	RSS/SSS(%)**
AR	Y2	AR(47)	47	~ 212	-1.271	28.17

Estimation method: Ordinary Least Squares (OLS), QR implementation (MATLAB function `arx.m`)

Signals normalized via sample mean removal and division by own sample standard deviation.

Signal length used: $N = 10\,000$ samples (2.15 s)

* Samples Per Parameter ** Refers to a single baseline (healthy) model

4.2. Inspection phase

Damage detection results for $v = \{1, 12\}$ and the corresponding number of inspection cases (table 2) are presented in figure 3. The distance metric D from the current model to the baseline model set is provided per inspection case. As depicted, for $v = 1$ (conventional approach) several distance metrics related to damaged beams are lower than those related to their healthy counterparts, thus rendering damage detection ineffective. Improved results are obtained by the MM based method as v is increased to 12.

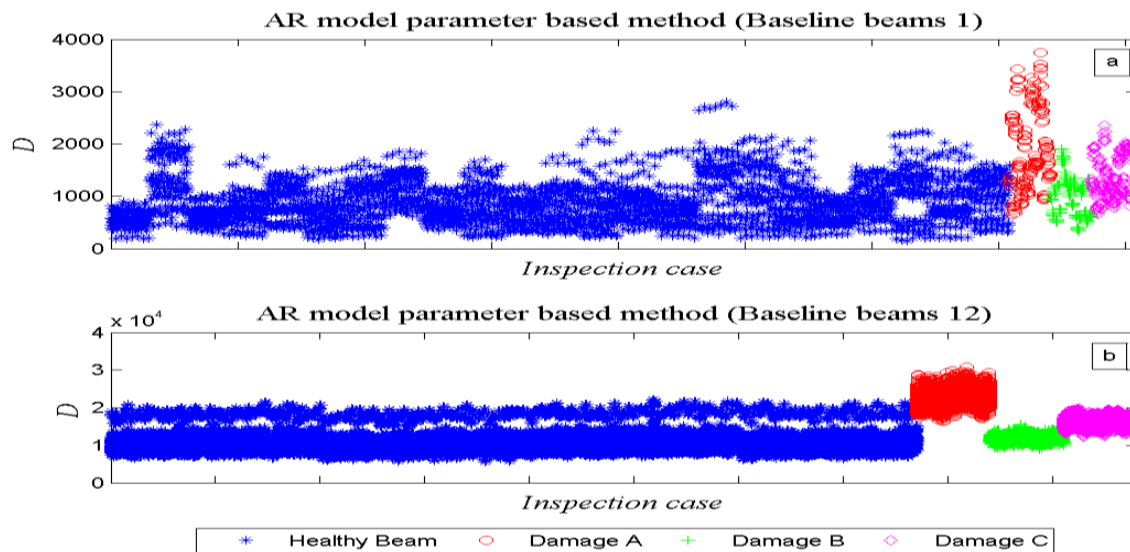


Figure 3. Damage detection by the MM based AR model parameter method: Overall distance metric D for each inspection case. (a) Single-model baseline (4 025 inspection cases); (b) 12-model baseline (49 000 inspection cases).

The remarks made above are also depicted by figure 4 in terms of Receiver Operating Characteristic (ROC) curves [12, pp 34-35], for different values of v . These curves represent the true positive rate (percentage of correct damage detections) versus the false positive rate (percentage of false alarms) for varying decision thresholds. The potential improvement in damage detection is evident when moving from the conventional ($v = 1$) to the Multiple Model ($v > 1$) based method.

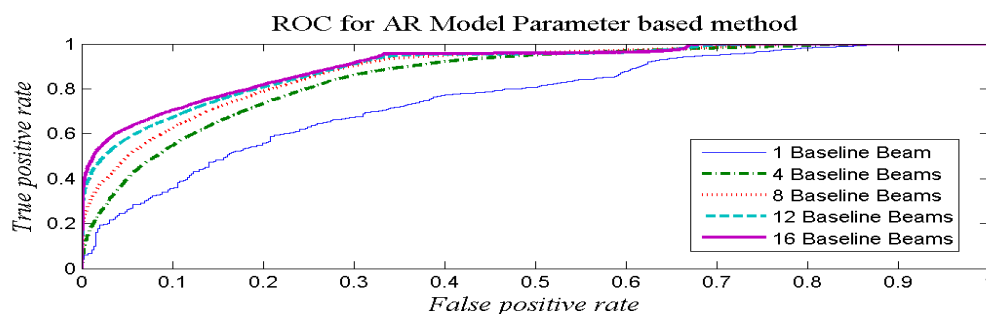


Figure 4. ROC curves for different values of v ($\{1, 4, 8, 12, 16\}$) and 500 combinations of baseline beams.

Despite the significant improvement, the method is still not effective as shown in figure 4, which is reasonable as the fundamental difficulty stemming from the high variability in the healthy beam dynamics *cannot* be fully overcome. Then a sensitivity analysis is performed based first on the AR model order and secondly on the selection of specific baseline beam combinations. Thus, tests with 5 different AR orders ($n_a = \{27, 37, 47, 57, 67\}$) and 12 baseline beams are made, examining whether the selection of a different model order affects detection performance. The results of these tests are presented in figure 5 in terms of ROC curves, depicting limited sensitivity of the MM based method.

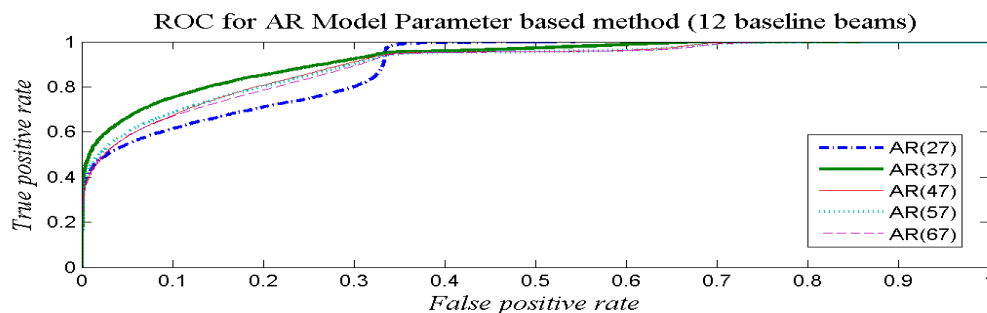


Figure 5. ROC curves for different AR model orders, 12 baseline beams and 500 combinations of baseline beams.

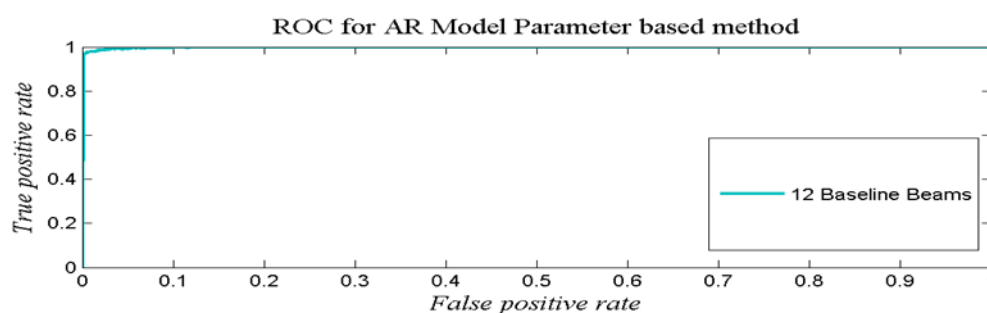


Figure 6. ROC curve for $v = 12$ and 50 properly selected combinations of baseline beams for achieving effective damage detection.

Subsequently, several baseline beam combinations are tested in order to examine whether some specific combinations may lead to effective detection. Indicative results for 50 properly selected combinations with $v = 12$, are presented in figure 6 in terms of the ROC curve. It is evident that with proper selection of the baseline beams the method's performance improves significantly; in fact reaching excellence.

5. Concluding remarks

A detailed assessment of a recently introduced Multiple Model (MM) based AutoRegressive (AR) model parameter method for Structural Health Monitoring of a population of like structures using a *single*, common, threshold in the decision making mechanism has been presented. The assessment has been based on high numbers of experimental test/inspection cases using composite beams subject to damage via impact. Comparisons with the corresponding conventional (single model based) method have been also made. The results achieved demonstrate the MM method's significant improvement over that of the corresponding conventional method, but also that it is not possible to generally (that is in all cases, *irrespective* of the beams selected as baseline) and fully overcome the fundamental difficulty associated with significant variations in the healthy population dynamics. A sensitivity analysis has additionally indicated that the method is relatively insensitive to the selected model order, but sensitive to the specific beams selected as baseline (training) ones, with proper selection potentially leading to excellent results.

Acknowledgements

The support of this study by the European Commission (FP7 Project 605549 on “Global in Flight Healthy Monitoring Platform for Composite Aerostructures Based on Advanced Vibration Based

Methods” – VIBRATION) is gratefully acknowledged. The authors wish to thank all project partners for their support and contributions.

References

- [1] Fassois S and Sakellariou J 2009 Statistical time series methods for structural health monitoring, in: *Encyclopedia of Structural Health Monitoring*, ed C Boller, F Chang, Y Fujino (John Wiley & Sons Ltd.) pp 443-472
- [2] Sohn H 2007 Effects of environmental and operational variability on structural health monitoring *The Royal Society - Philosophical Transactions: Mathematical, Physical and Engineering Sciences* **365** 539-560
- [3] Vamvoudakis-Stefanou K, Sakellariou J and Fassois S Random vibration response-only damage detection for a set of composite beams: *Proceedings of the ISMA International Conference on Noise and Vibration Engineering (Leuven, Belgium 2014)*
- [4] Vamvoudakis-Stefanou K, Fassois S, Vibration-based damage detection for a population of like structures via a multiple model framework: *Proceedings of the 6th International Symposium on NDT in Aerospace (Madrid, Spain 2014)*
- [5] Bendat J and Piersol A 2000 *Random Data: Analysis and Measurement Procedures* 3rd edition (New York: Wiley-Interscience)
- [6] Fassois S 2001 Parametric identification of vibrating structures, in: *the Encyclopedia of Vibration*, ed S Braun, D Ewins, S Rao (Academic Press) pp 673-685
- [7] Fassois S and Sakellariou J 2007 Time series methods for fault detection and identification in vibrating structures *The Royal Society - Philosophical Transactions: Mathematical, Physical and Engineering Sciences* **365** 411–448
- [8] Avendaño-Valencia L and Fassois S 2014 Robust fault detection based on multiple functional series tar models for structures with time-dependent dynamics: *Proceedings of the 7th European Workshop on Structural Health Monitoring (EWSHM) (Nantes, France 2014)* pp 804-811
- [9] Fassois S and Kopsaftopoulos F 2013 Statistical time series methods for vibration based structural health monitoring, in: *New Trends in Structural Health Monitoring*, ed W Ostachowicz and A Guemes (Springer) pp 209-264
- [10] Press W, Teukolsky S, Vetterling W and Flannery B 2007 Kullback–Leibler distance, in: *Numerical Recipes: The Art of Scientific Computing* (3rd ed.) (New York: Cambridge University Press)
- [11] Ljung L 1999 *System Identification: Theory for the User* 2nd Edition (Prentice-Hall)
- [12] Duda R, Hart P and Stork D 2000 *Pattern Classification* 2nd Edition (Wiley)