Location of the corrosion damage in rectangular plates

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Abstract. The presence of corrosion influences the dynamic behaviour of plates, by modifying its natural frequencies and mode shapes. Many methods to assess damages in structural elements are known, but they mainly address to the geometric discontinuities, where mass loss is insignificant. In some of the prior research, we have got success in achieving a mathematical relation between damage geometry and frequency shifts. Since the mass decrease is also associated with numerous corrosion mechanisms, the idea of this paper is to establish the influence of mass changes in the natural frequencies of plates and including this information in a baseline, to apply it in processes for corrosion assessment. Thus, by research the evaluation of frequency changes due to mass loss and stiffness decrease has been followed. The finite element analysis was performed, considering corroded area as being simulated by a constantly decreasing plate thickness. In this way, is highlighted the correlation between frequency changes in the transversal vibration modes and mass loss. The dependency between mass loss and frequencies is accomplished, as a mathematical relation as well. This relation is associated with another relation, who predicts the frequency decreases due to stiffness reduction, in order to find the curves of frequency shifts and promote them as a baseline for corrosion assessment. Also, a corrosion assessment method, based on the contrived frequency shift curves, has been developed and proved as reliable and easily applicable.

1. Introduction
Degradation of structural elements due to wear or corrosion affects the safety of the structures by reducing their cross-section or/and mechanical strength. In literature numerous methods developed to assessing damages using vibration signals are presented: these methods base on changes of modal parameters, most common being the frequency shifts; comprehensive reviews are provided for instance in [1]. Most methods dedicated to assess the damage location and severity are model-based. The main disadvantage of these methods is that it's difficult to distinguish between model errors and changes of the modal parameters [2]. Consequently, the process of finding the modal parameters associated with the damaged system in order to fit its answers to that obtained by measurement is complex and often inadequate results are obtained [3, 4].

In our previous research we contrived mathematical relations that precise predict, for slender beams, the frequency decrease due to a transversal discontinuity [5-7] or change due to mass variation [8]. These relations are the base of an original damage detection method applicable for breathing and open cracks as well. It consists of comparing a sequence of normalized frequency shifts for the weak-axis bending vibration modes, measured at different time points, with patterns derived from the mode shapes and their derivatives.

In this paper we present investigations, performed on rectangular plates clamped at all ends, in which both the stiffness changes and loss of mass due to corrosion are considered, aiming to fit the method for the case of plates.

2. Frequency changes in beam-type structures due to mass and rigidity variation

The vibration of beams with geometrical discontinuities and/or not uniformly distributed mass was earlier studied and permitted the enunciation of following inferences:

- if the stiffness reduction is placed on slices not subjected to bending, i.e. regions 1 and 4 in figure 1, the frequency drop is insignificant;
- if the stiffness reduction is placed on slices achieving important bending, i.e. regions 2 and 3 in figure 1, the frequency drop is important;
- if the mass loss is placed on slices subjected large displacement, i.e. regions 1 and 3 in figure 1, the frequency increase is important;
- if the mass loss is placed on slices achieving no displacement, i.e. regions 2 and 4 in figure 1, the frequency change is insignificant.

As a consequence, the kinetic and potential energy distribution for the bending modes are altered from the healthy state with respect to the modal curvature and mode shape respectively. Moreover, the natural frequencies of the bending modes change in relation to the stored energy alteration. By occurrence of a discontinuity at location x, the frequencies drop $\Delta f_{i-S}(x)$ is:

$$\Delta f_{i-S}(x) = f_{i-U} - f_{i-S}(x) = f_{i-U} \cdot \frac{\sqrt{\delta_D} - \sqrt{\delta_U}}{\sqrt{\delta_D}} \cdot \left(\overline{\phi}_i''(x) \right)^2 \qquad (1)$$

where f_{i-U} is the frequency of the undamaged beam and $f_{i-S}(x)$ for the ith transverse mode, and $\overline{\phi}_i''(x)$ is the value of the dimensionless mode shape curvature at location x. The terms δ_U and δ_D are the static deflections at the free end of the similar cantilever beam for the healthy state and if the discontinuity is located at the fixed end respectively. These deflections, being in direct relation with the beam's global rigidity EI and EI_{eq}, reflect the stored energy in the two mentioned states, thus the fraction in equation (1) represent the damage severity.
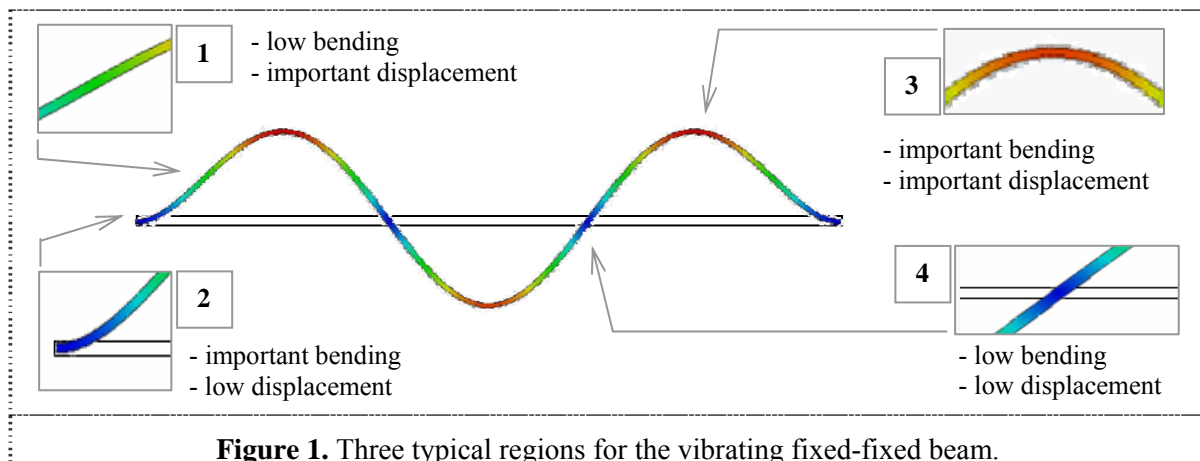

Figure 1. Three typical regions for the vibrating fixed-fixed beam.

If a loss of mass ΔM at location x is observed, the bending frequency increase $\Delta f_{i-M}(x)$ is:

$$\Delta f_{i-M}(x) = f_{i-M}(x) - f_{i-U} = f_{i-U} \cdot \frac{\sqrt{M} - \sqrt{M - 4\Delta M}}{\sqrt{M - 4\Delta M}} \cdot \left(\overline{\phi}_i(x) \right)^2 \qquad (2)$$

where M is the healthy beam's mass and $\overline{\phi}_i(x)$ is the dimensionless mode shape value at location x.

If a discontinuity associated with a mass loss is present, the natural frequency shift becomes:

$$\Delta f_{i-D}(x) = f_{i-U} \cdot \left[\frac{\sqrt{M} - \sqrt{M - 4\Delta M}}{\sqrt{M - 4\Delta M}} \cdot \left(\bar{\phi}_i(x)\right)^2 - \frac{\sqrt{\delta_D} - \sqrt{\delta_U}}{\sqrt{\delta_D}} \cdot \left(\bar{\phi}_i''(x)\right)^2 \right] \quad (3)$$

By dividing the equations (1) and (2) with f_{i-U} relative frequency shifts are accomplished. A sequence of relative frequency shifts, lets say $i = 1\ldots n$, constitute patterns characterizing the damage location. In next sections the possibility to contrive patterns for plate-type structures is studied.

3. Model description

In our concern to demonstrate that stiffness and mass loss influence the plate frequencies with respect to the locally stored energy (i.e. the squared of the mode shape curvatures) and achieved accelerations (or displacements), we performed numerical analysis using the *SolidWorks* software. A plate, presented in figure 2, with dimensions $a = 1$ m, $b = 0.5$ m and $h = 0.002$ m was the subject of our analysis, both in healthy state and with a simulated corroded area. The corroded region has a constant thickness $h_C = 0.001$ m and a square shape with edges d; herein we present the case of damage extension $d_1 = 0.03$ m, $d_2 = 0.04$ m and $d_3 = 0.05$ m. For simplicity, we consider the corrosion in the plate centre, as depicted in figures 2 and 3.
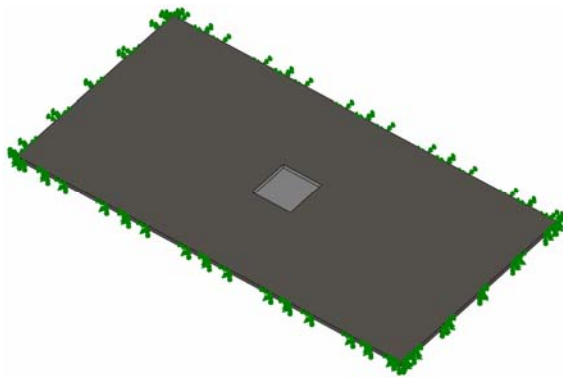

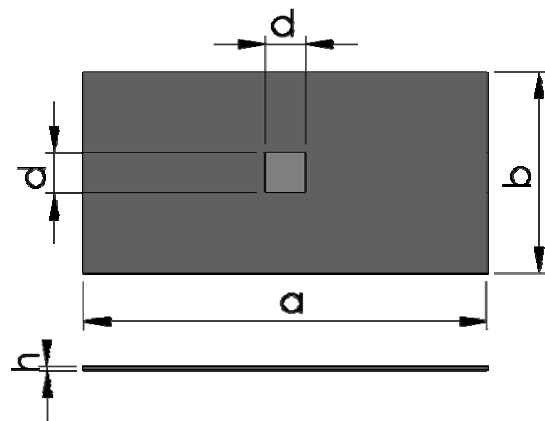**Figure 2.** Corroded plate with fixed edges.

Figure 3. Plate and corroded area dimensions.

The material related from the *SolidWorks* software library is the **AISI 1045 Steel**, cold drawn, having the mechanical properties indicated in Table 1.

Table 1. Mechanical properties of the plate material

Yield strength [N/mm^2]	Tensile strength [N/mm^2]	Mass density [kg/m^3]	Elastic modulus [N/mm^2]	Poisson's ratio [-]	Volumetric mass density [kg/m^3]
630	580	7,850	200,000	0.3	7850

In this analysis, the plates are fixed on all four contour edges and the mesh was generated by tetrahedral elements with 21 characteristic points and the maximum size of 2 mm.

4. Results and discussions

The natural frequencies for both the healthy and corroded plates were evaluated by means of FEA simulations; in all cases 30 vibration modes are considered. Table 2 present these frequencies grouped after the *i-j* indices of the vibration modes, this providing a clear overview of the plate behavior.

To find out if it can be made a parallelism between the dynamic behavior of corroded beams and plates, we derived the relative frequency shift $\Delta f_{ij}{}^{*}$ of the plate's mode *i-j*, after the relation used for beams [9]. This relation can be written in the form presented below:

$$\Delta f_{ij\_U}^{*} = \frac{\Delta f_{ij}}{f_{ij-U}} = \frac{f_{ij-U} - f_{ij-D}}{f_{ij-U}} \tag{4}$$

In the relation above, f_{ij-U} is the natural frequencies of mode *i-j* of the healthy plate and f_{ij-U} is the natural frequencies of mode *i-j* for the damaged plate. To express the relative frequency shift in percents, equation (4) has to be multiplied by 100.

Table 2. Frequency values for four type of plate grouped into m-n and relative shift for three scenarios damage

Mode *i-j*	Without defect	Corroded area 30x30x 1 mm	Corroded area 40x40x1 mm	Corroded area 50x50x 1 mm	Corroded area 30x30x 1 mm	Corroded area 40x40x1 mm	Corroded area 50x50x 1 mm
	Frequency [Hz]				Relative frequency shift [%]		
1-1	47.803	47.770	47.749	47.731	0.208	0.317	0.396
1-2	61.898	61.903	61.899	61.892	-0.006	0.008	0.037
1-3	87.066	86.957	86.884	86.807	0.332	0.511	0.661
1-4	123.154	123.160	123.133	123.076	0.013	0.072	0.192
1-5	169.660	169.398	169.230	169.059	0.352	0.508	0.605
1-6	226.232	226.232	226.122	225.903	0.037	0.164	0.407
1-7	292.687	292.268	292.008	291.762	0.293	0.386	0.392
1-8	368.883	368.858	368.568	368.000	0.065	0.267	0.639
1-9	454.768	454.214	453.897	453.614	0.226	0.246	0.181
2-1	124.429	124.436	124.400	124.329	0.010	0.089	0.238
2-2	138.218	138.149	138.076	137.984	0.082	0.168	0.274
2-3	161.926	161.946	161.910	161.835	0.002	0.062	0.177
2-4	195.980	195.794	195.619	195.400	0.160	0.300	0.476
2-5	240.470	240.521	240.472	240.370	-0.009	0.041	0.136
2-6	295.290	295.083	294.859	294.577	0.127	0.249	0.402
2-7	360.278	360.393	360.324	360.169	-0.022	0.024	0.114
2-8	435.287	435.138	434.894	434.565	0.076	0.171	0.297
2-9	520.223	520.466	520.396	520.209	-0.043	-0.011	0.044
3-1	239.634	239.221	238.943	238.671	0.361	0.524	0.611
3-2	253.438	253.518	253.508	253.482	-0.039	-0.025	-0.002
3-3	276.787	276.481	276.290	276.125	0.248	0.312	0.291
3-4	310.031	310.131	310.073	309.954	-0.025	0.018	0.099
3-5	353.362	353.170	353.041	352.959	0.141	0.134	0.031
3-6	406.869	406.997	406.852	406.571	-0.006	0.077	0.219
3-7	470.537	470.396	470.295	470.251	0.082	0.029	-0.124
4-1	393.100	393.101	392.738	392.055	0.057	0.296	0.733
4-2	406.970	407.084	407.028	406.955	-0.033	0.000	0.037
4-3	430.281	430.317	429.982	429.361	0.046	0.238	0.555
4-4	463.239	463.211	463.008	462.757	0.034	0.106	0.195
4-5	505.995	506.116	505.761	505.113	0.029	0.199	0.476

Mode 1-1

Mode 1-2

Mode 1-3

Mode 1-4

Mode 1-5

Mode 1-6
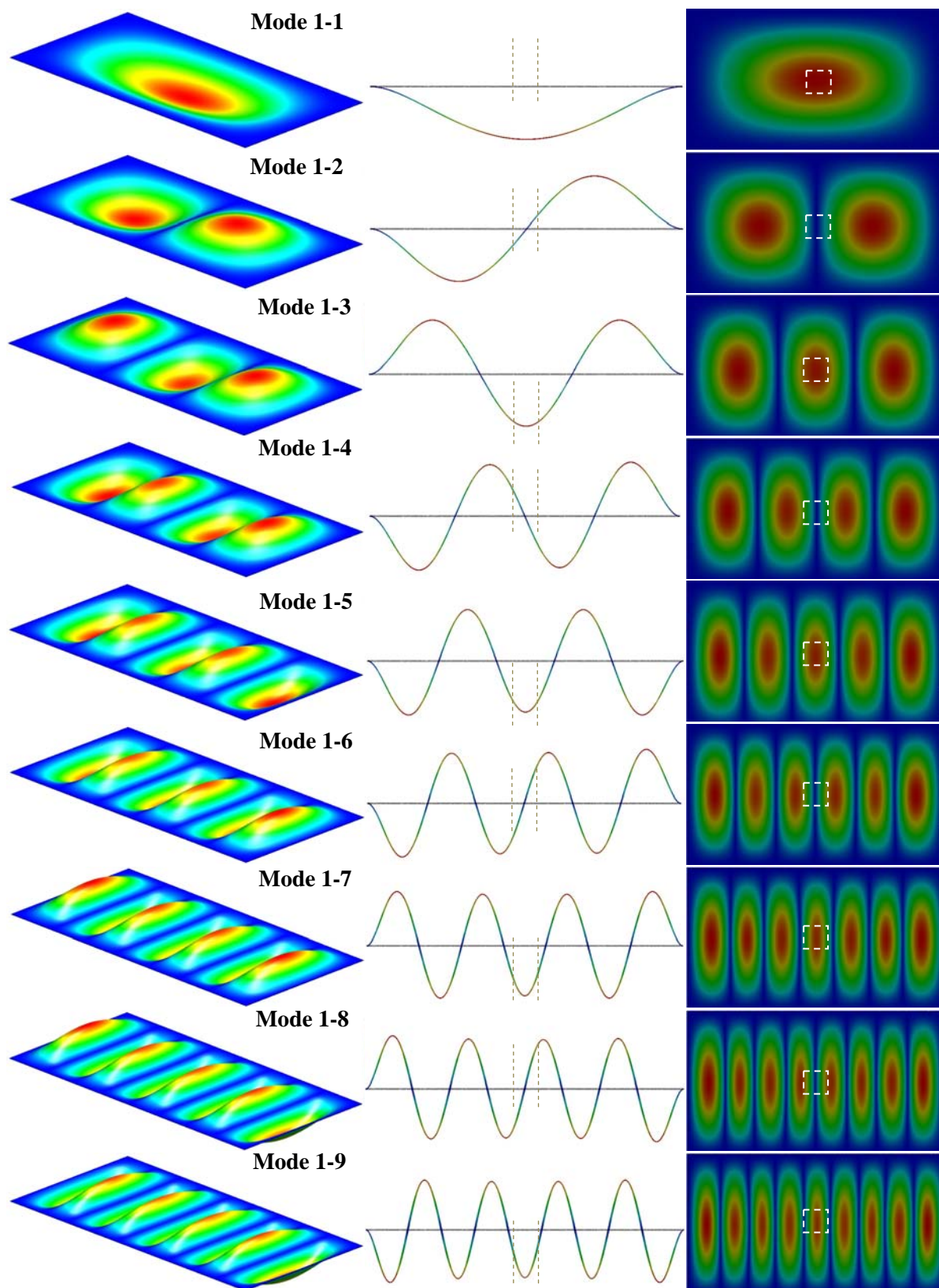
Mode 1-7

Mode 1-8

Mode 1-9

Figure 4. Vibration modes 1-1 to 1-9 (isometric and profile view) and detail of the central region.
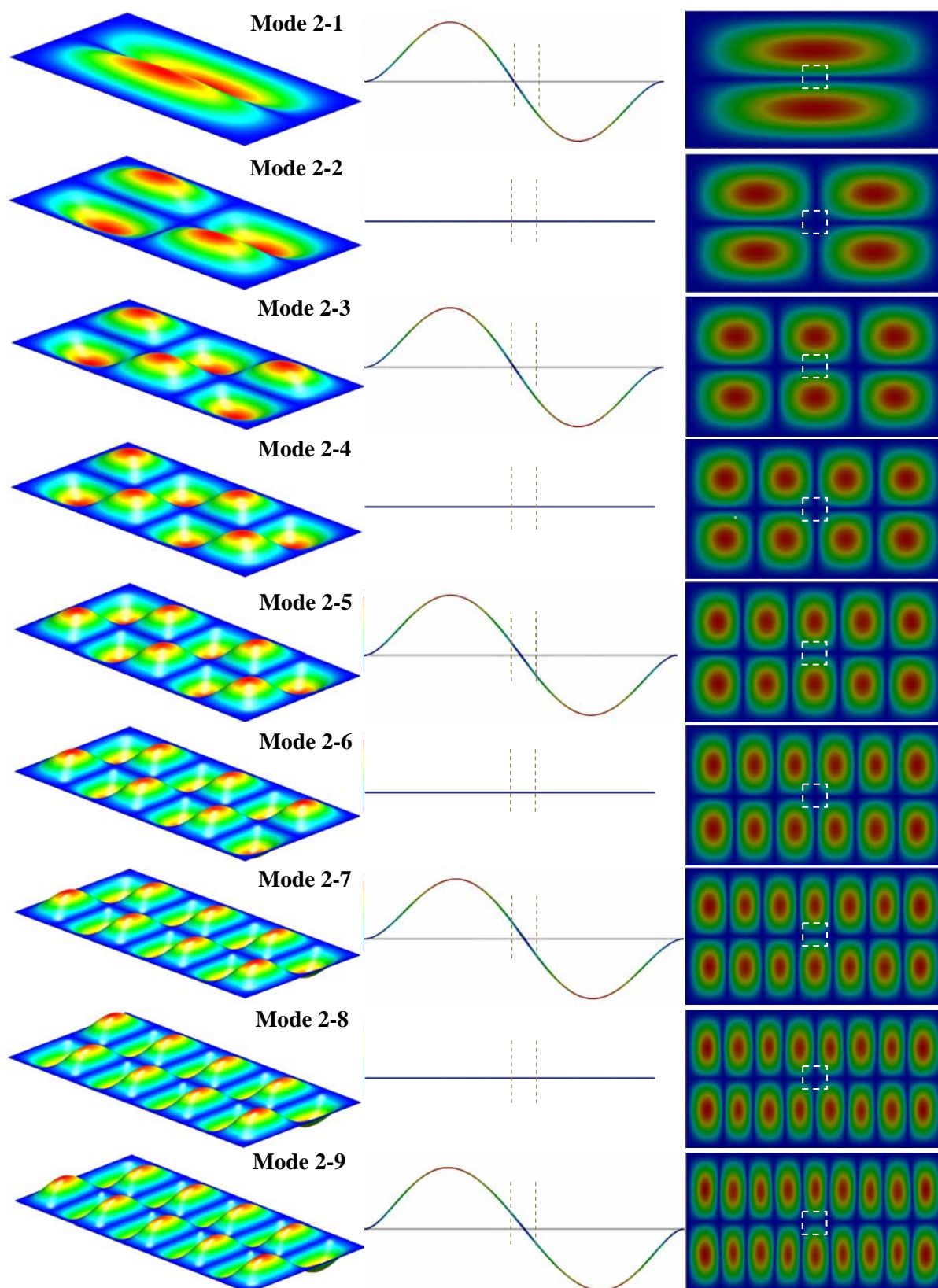
Figure 5. Vibration modes 2-1 to 2-9 (isometric and profile view) and detail of the central region.

Typical mode shapes for modes $i = 1$ and $j = 1\dots9$ are presented in figure 4. On the left column, the axonometric views are presented, while in the central column the profile view is depicted. It is remarkable that, for the even modes j, the displacement of the centrally located corrosion area is inexistent. Opposite, for the odd modes j the displacement is significant. Similar is the plate behavior for all combinations including odd modes i.

In contrary, for the combination of even modes i, the odd modes j induce important displacement, while for the even modes j the plate's central area displacement is null. This is depicted in figure 5 for the modes $i = 2$ and $j = 1\dots4$.

The right column in figure 4 illustrates the displacements achieved by the plate's central area. Analyzing the displacements in corroboration with the rotations induced in this region, one can qualitatively evaluate the attained bending moment and consequently the stored energy for the healthy plate. By occurrence of corrosion, thus thickness reduction, the frequencies will change in concordance with this locally stored energy. The relative frequency shifts, shown in figure 6, reflect this phenomenon.
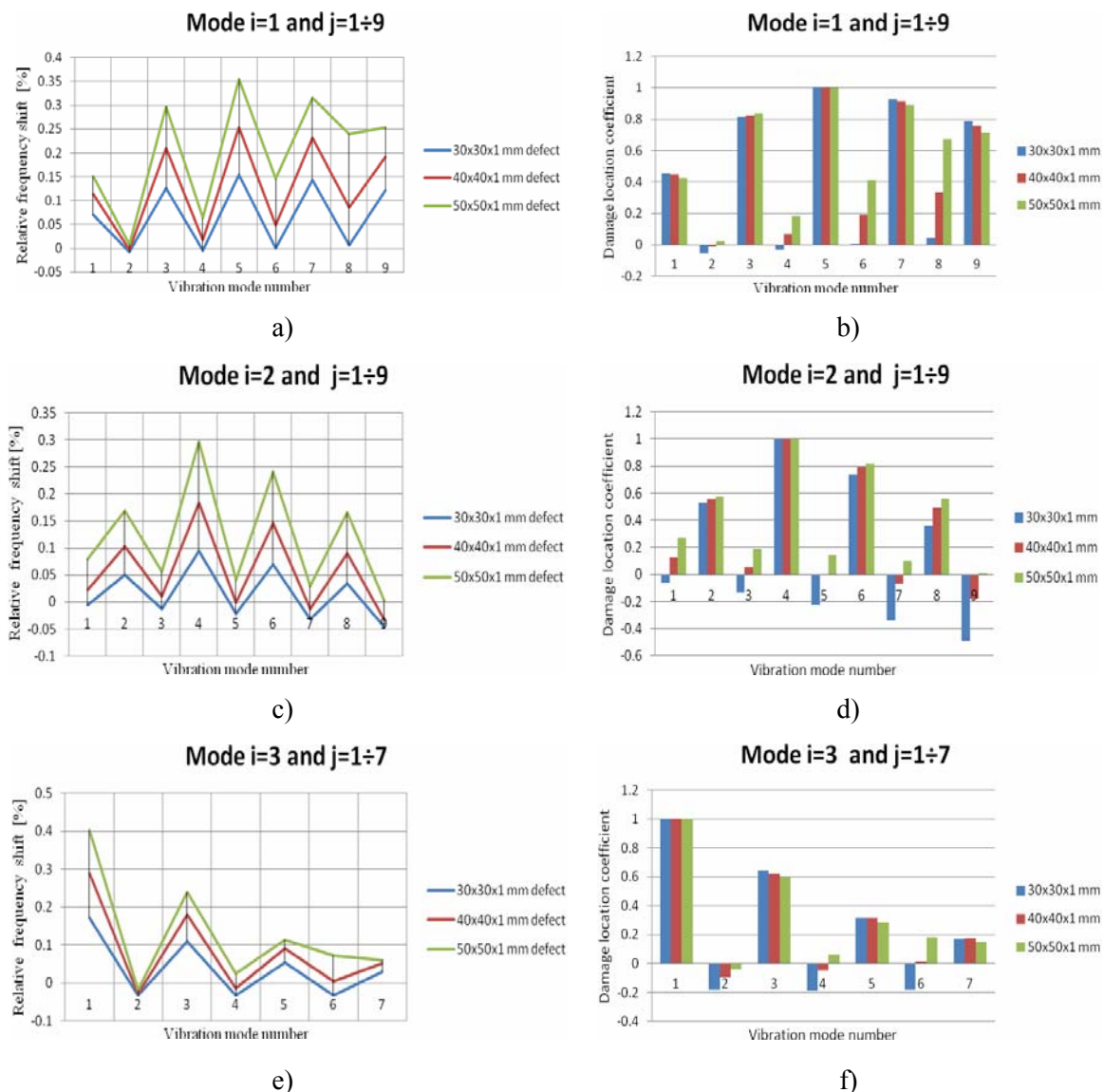

a)

b)

c)

d)

e)

f)

Figure 6. Relative frequency shifts and subsequent damage location coefficients.

Figure 7. Relative frequency shifts and subsequent damage location coefficients.

It is obvious that the frequencies admit changes with respecting to the rules imposed by the physical laws. Each corroded region produce different combination of frequency changes [10], thus these can be used as a benchmark in damage detection. One advanced solution is to normalize the sequence of relative frequency shifts, in order to obtain Damage Location Coefficients (see figure 7) that in a manner are similar with those designed for beams.

5. Conclusions

This paper analyses the frequency changes occurring in corroded rectangular plates, with a special focus on the case of a central damaged area. Interpretation of the results emphasizes two divergent tendencies: (i) the frequency decreasing due to stiffness (thickness) reduction and (ii) the frequency increasing due to loss of mass. The four definitory parameters of these changes are damage location, damaged surface, damage shape (i.e. the ratio between edges of the rectangular area) and thickness reduction. By normalization, the variables can be taken off and the number of parameters is consequently reduced, as well as in the case of defining damage location coefficients by suppressing the influence of corroded thickness. Also, it is possible to define valid patterns for any damage scenario by performing a set of simulations, and they can be assumed as reference in damage detection processes.

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