

# Statistical evaluation of characteristic SDDL<sub>V</sub>-induced stress resultants to discriminate between undamaged and damaged elements

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**Abstract.** The stochastic dynamic damage location vector (SDDL<sub>V</sub>) method utilizes the vectors from the kernel of a damaged-induced transfer function matrix change to localize damages in a structure. The kernel vectors associated with the lowest singular values are converted into static pseudo-loads and applied alternately to an undamaged reference model with known stiffness matrix, hereby, theoretically, yielding characteristic stress resultants approaching zero in the damaged elements. At present, the discrimination between potentially damaged elements and undamaged ones is typically conducted on the basis of modified characteristic stress resultants, which are compared to a pre-defined tolerance value, without any thorough statistical evaluation. In the present paper, it is tested whether three widely-used statistical pattern-recognition-based damage-detection methods can provide an effective statistical evaluation of the characteristic stress resultants, hence facilitating general discrimination between damaged and undamaged elements. The three detection methods in question enable outlier analysis on the basis of, respectively, Euclidian distance, Hotelling's  $T^2$  statistics, and Mahalanobis distance. The study of the applicability of these methods is based on experimentally obtained accelerations of a cantilevered residential-sized wind turbine blade subjected to an unmeasured multi-impulse load. The characteristic stress resultants are derived by applying the static pseudo-loads to a representative finite element (FE) model of the actual blade.

## 1. Introduction

Structural health monitoring (SHM) is widely employed within many systems, and the importance of its application for wind turbine blades is growing due to the increasing size and number of operating wind turbines, see, e.g., [1]. Vibration-based approaches are commonly applied in such a way that the dynamic response from the current state is compared to the representative response from the healthy reference state. In principle, the structure is potentially damaged if the current state differs significantly from the reference state namely due to changes in physical properties, typically a stiffness reduction.

The present paper concentrates on damage localization for which stochastic dynamic damage location vectors (SDDL<sub>V</sub>s) are employed. The basis of the SDDL<sub>V</sub> method is the damage location vector (DLV) method [4], in which the null space of the changes in the flexibility matrix from a pre- and post-damaged structure is utilized to locate the damage. The vectors that form the basis of the null

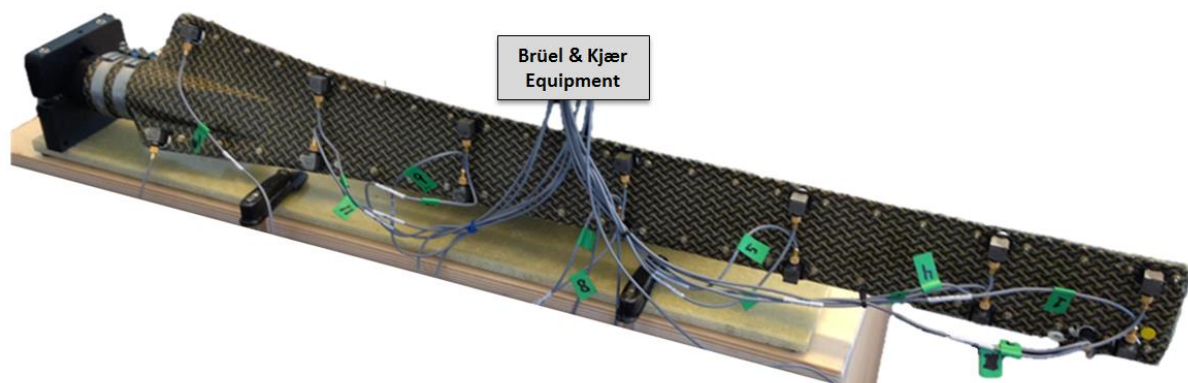


space are designated as DLVs, since that particular null vector associated with the lowest singular value is applied to an undamaged reference model as static-pseudo loads. The stresses in the model will, theoretically, approach zero in the damaged area(s).

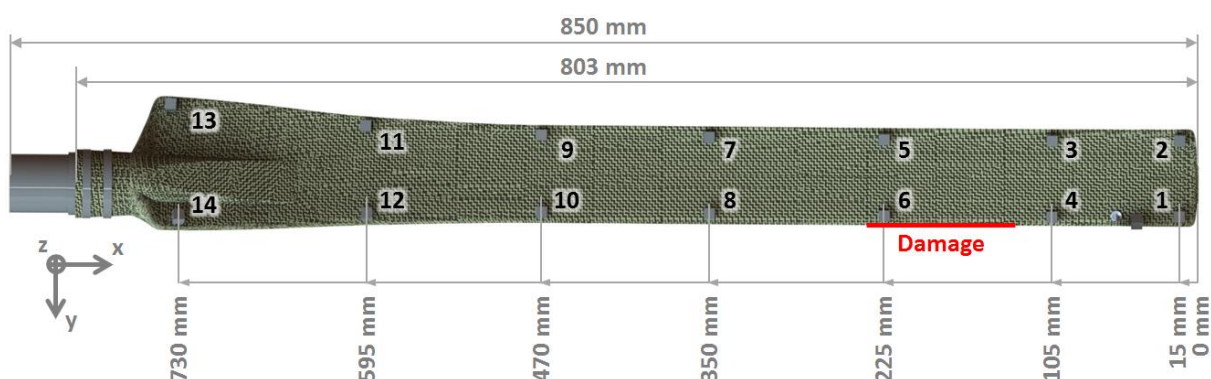
The SDDL method is an extension of the DLV method that treats output-only measurements and includes the dynamics of the system by applying the null space of the changes in the transfer function matrix instead of the changes in the flexibility matrix. In [2], D. Bernal has proved that the method is applicable for localizing structural damages in numerical models of truss systems. Additionally, the applicability of the SDDL method has, by the authors of the present paper, been demonstrated in an experimental context with the residential-sized wind turbine blade shown in figure 1 [3]. Here, it was suggested that the implementation of a statistical procedure for discrimination between actual damage-induced irregularities and noise-induced ones may significantly improve the method. This is explored in the present paper where different statistical approaches to discriminate between damage and other irregularities in the aforementioned residential-sized wind turbine blade are tested. Thus, the study is based on the same experimental setup as the one presented in [3].

## 2. Experimental setup

The approximately 800 mm polymer blade is reinforced with carbon fibre and composed of two separable shells by means of 25 bolts along the leading and trailing edges. Different damage conditions can be examined by untightening one or more bolts. In the present study, the damage indicated in figure 2 is analysed.



**Figure 1.** Experimental setup for OMA of the residential-sized wind turbine blade.



**Figure 2.** Dimensions of the blade, and location of the simulated damage.

The blade was subjected to unmeasured multi-impulse loads that were applied by hitting the structure randomly over the surface with a pencil. The response was captured by use of 14 equally spaced Brüel & Kjær uniaxial accelerometers, as illustrated in figure 2, which were measuring

perpendicularly to the surface. A sufficiently high sampling frequency of 8192 Hz was utilized throughout the experiments. The recordings were uninterrupted time series that were divided into smaller partitions in order to obtain more than one experiment from the different states.

### 3. Damage localization using the SDDL method

The recordings from the reference state and the current state are mathematically described by means of system identification techniques. Here, the dynamic properties of the system are defined in a state matrix,  $A_c$ , and the output measurements are defined in an output matrix,  $C_c$ .

Generally, the state matrix and the output matrix contribute to establish the kernel of the transfer function matrix which, as previously declared, is utilized for locating potential damages from changes between the two states. The basic form of the transfer function matrix is

$$G(s) = C_c(sI - A_c)^{-1}B_c + D_c, \quad (1)$$

where the input matrix,  $B_c$ , and the direct transmission matrix,  $D_c$ , are unknown since the system is purely stochastic with output-only measurements. An approach for estimating a fictive input is implemented from [2], hereby yielding

$$G(s) = R(s)D_c \quad (2)$$

where

$$R(s) = C_c A_c^{-2} (sI - A_c)^{-1} \begin{bmatrix} C_c A_c \\ C_c I \end{bmatrix}^\dagger \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad (3)$$

with the dagger sign designating the Moore-Penrose pseudo-inverse. The direct transmission term,  $D_c$ , is assumed to be constant, because it is unaffected by the introduction of damage. The change in the transfer function matrix is then proportional to the change in  $R(s)$ , i.e.

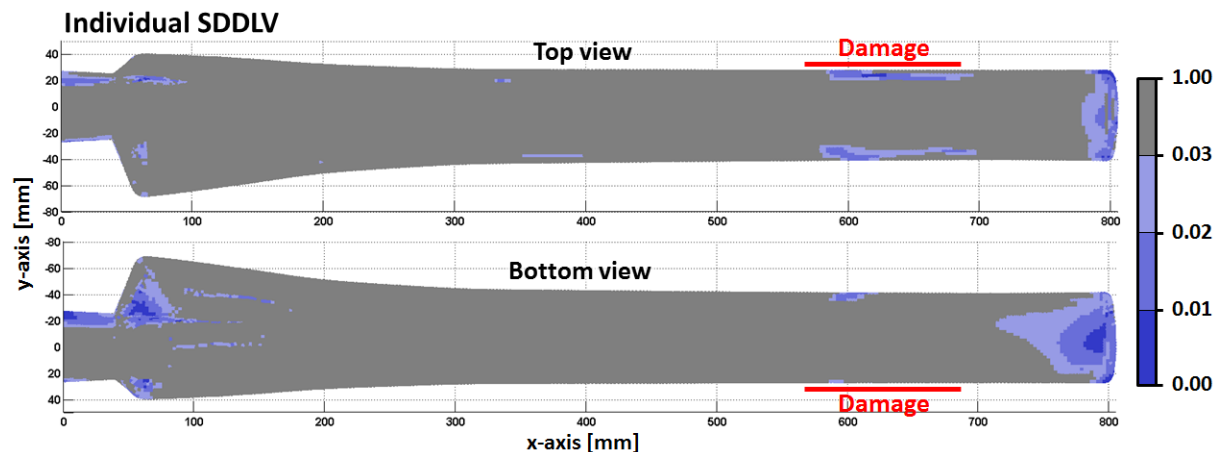
$$\Delta G(s) \propto \Delta R(s) = R_d(s) - R_u(s). \quad (4)$$

The SDDLVs, used as pseudo-loads, are estimated from the quasi-null space of  $\Delta R(s)^T$  by singular value decomposition (SVD). In particular, the right singular vector associated with the smallest singular value is chosen to constitute the SDDL.

The pseudo-loads are applied to a finite element (FE) model of the blade, which is based on an approximated 3D CAD geometry, discretized by equally sized first order shell elements. The model is calibrated against four experimentally-obtained natural eigenfrequencies and modes shapes of the undamaged blade. The pseudo-loads are applied at the same position and direction as the accelerometers.

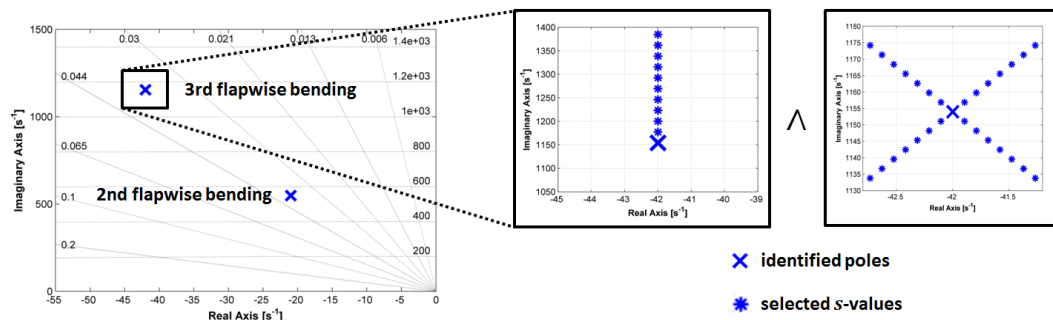
### 4. Selection of $s$ -values

The damage localization is not efficient for all  $s$ -values of the transfer function matrix. The guidance in [2] is to evaluate close, but not too close, to the poles of the system. Previous research by the authors of the present paper, see [3], reveals that a slight increase of an  $s$ -value corresponding to an identified system pole is applicable for damage localization, see figure 3.



**Figure 3.** Stress field illustrating the normalized elemental mean von Mises stress for one SDDLTV and showing the appearance of the damage.

As shown in figure 3 and concluded in [3], the results of applying individual SDDLTVs imply that statistical evaluation can be included in the methodology to improve the damage localization. Such evaluation can be based on several experiments and/or several  $s$ -values. The  $s$ -value selection process is still a quite unexplored area of the SDDLTV method, hence no strictly generalized guidance exists. The  $s$ -values can be chosen from infinite combinations of increments/decrements of the real and imaginary parts of an identified pole. In the present study, the two selection approaches depicted in figure 4 are tested, i.e. 10  $s$ -values arranged in intervals of 2 to 20 % increment of the imaginary part and 28  $s$ -values arranged in four branches of seven equally-spaced points from combinations of increments /decrements of the real and imaginary parts. It is noticed that the third flapwise bending mode is chosen, since it was most consistently excited during the experiments.



**Figure 4.** Selection of  $s$ -values based on the identified poles.

All the examined  $s$ -values are applicable for damage localization. However, the disturbance in the individual SDDLTVs is larger close to the pole.

The stresses evaluated along the four branches are not found to vary in any specific manner with regards to direction. Consequently, all experiments are evaluated for the 10  $s$ -values to establish the basis of the statistical evaluation.

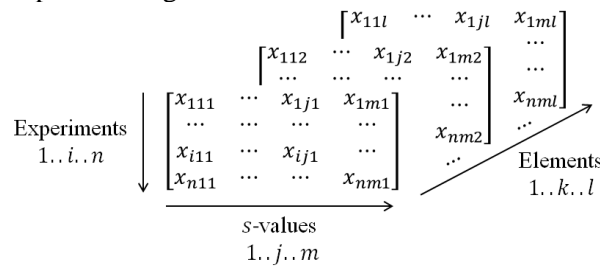
## 5. Methodology of statistical evaluation

Statistical evaluation using pattern recognition works by training a statistical baseline model from several healthy states followed by testing the current state against the baseline in order to capture any significant changes. For multivariate data, several outlier analysis methods based on, e.g.,  $T^2$  statistics,  $Q$  statistics and the Mahalanobis distance have been successfully applied for damage detection, see

e.g., [6] and [7]. The methods have the following of steps in common with the process of declaring the health of the structure:

- Train a baseline from the data measured in the healthy state.
- Determine a threshold defined by an appropriate quantile of the baseline.
- Test the current data using a pattern recognition technique for estimation of outliers.

In the present paper, it is tested whether some of these damage detection methods can be implemented in the SDDL method to provide an effective statistical evaluation of the characteristic stress resultants obtained by applying the SDDLs to the FE model of the undamaged blade. The elements of the FE model are evaluated one by one, and the results are gathered in a three-dimensional matrix, with the structure depicted in figure 5.



**Figure 5.** Suggestion for organizing the three-dimensional SDDL data.

The outlier analysis methods used are based on Euclidean distance,  $T^2$  statistics, and the Mahalanobis distance. The Euclidean distance is the distance between the current data from the experiment and the mean data from the experiments used for training, i.e.

$$d(x_{ik}) = \sqrt{\sum_{j=1}^m (x_{ijk} - \mu_{ik})^2}, \quad (5)$$

where  $x_{ijk} \in \mathbb{R}$  is the stress resultant from the current experiment and  $\mu_{ik} \in \mathbb{R}$  is the mean of the training data.

In the  $T^2$  statistics-based method, the dimensionality is reduced to dimension  $r$  by use of principal component analysis (PCA). Hereby, the similarity between  $m$  characteristic stress resultants from the current experiment and the training data is derived through

$$T^2(x_{ik}) = (x_{ik} - \mu_{jk})P\Lambda^{-1}P^T(x_{ik} - \mu_{jk})^T \quad (6)$$

in which  $P \in \mathbb{R}^{m \times r}$  and  $\Lambda \in \mathbb{R}^{r \times r}$  contain, respectively, the eigenvectors and eigenvalues of the covariance matrix computed from the trained data. In this context, it is found that the dimensionality can be reduced to one-tenth of original size without losing any significant information.

The Mahalanobis-squared distance employs the inverse covariance matrix of the data, i.e.

$$D^2(x_{ik}) = (x_{ik} - \mu_{jk})\Sigma^{-1}(x_{ik} - \mu_{jk})^T, \quad (7)$$

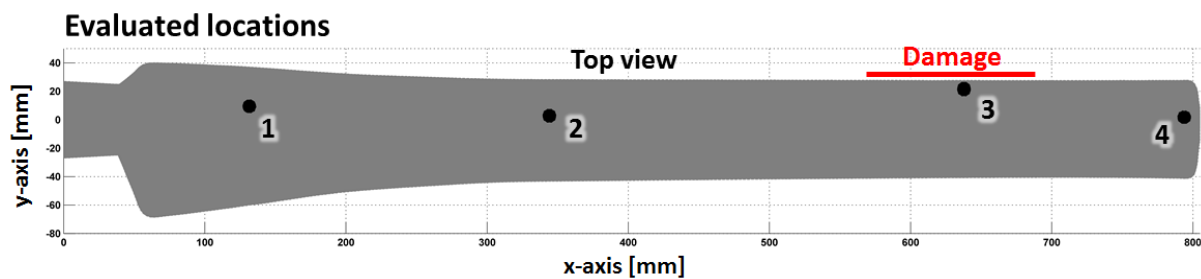
where  $\Sigma \in \mathbb{R}^{m \times m}$  is the covariance matrix of the training data.

The similarity measures calculated from any of the three above-mentioned methods need to be compared to a threshold calculated from data from the healthy structure. It has been chosen to calculate the threshold for each element matrix  $l$  based on the following steps:

- Test the first row of the undamaged data matrix and calculate the similarity using one of the three methods while applying the rest of the undamaged data matrix as the base.
- Repeat this for all vectors in the training matrix and sort the distances in descending order.
- The threshold is then equal to the value exceeded by 5 % of the similarity measures.

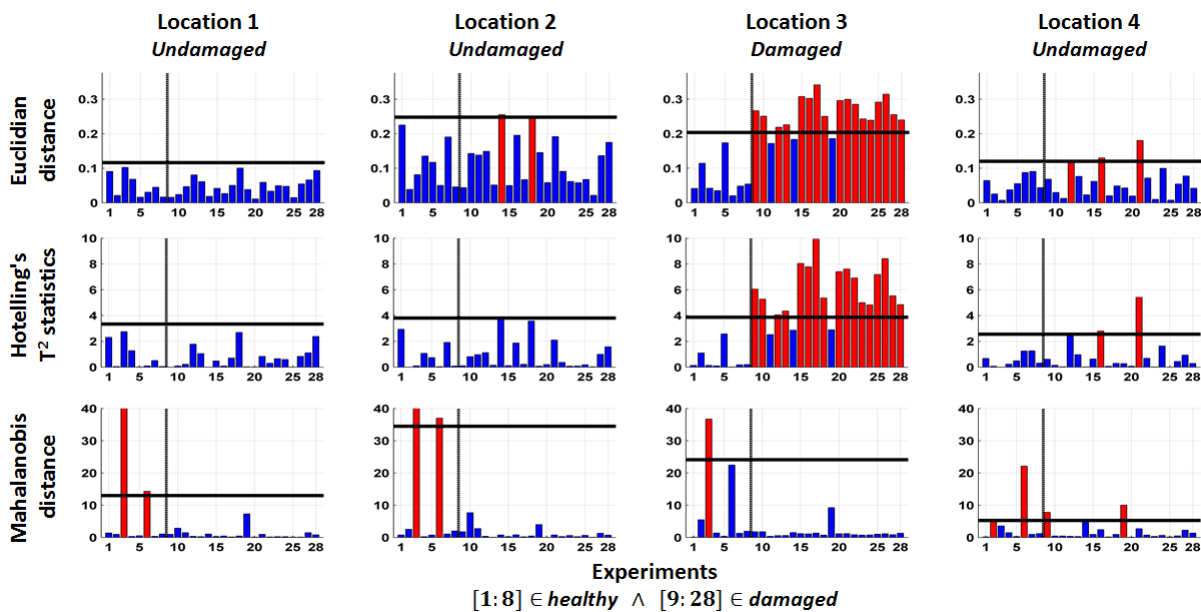
## 6. Results

The following results are based on 280 undamaged and 200 damaged experiments that are based on evaluation of 10 SDDLVs in different points along the imaginary axis, as illustrated in figure 4. The variances of the experiments are reduced by taking the mean of 10 experiments and using these new vectors for the statistical evaluation. In this way, 28 undamaged and 20 damaged experiments are obtained, where the first 20 undamaged experiments are utilized for training the baseline. The previous research in [3] states that noise often appears in the area around the tip and root of the blade. Consequently, it is chosen to demonstrate the proposed statistical evaluation procedures for the locations shown in figure 6.



**Figure 6.** Location of elements utilized for inspection of the statistical evaluation.

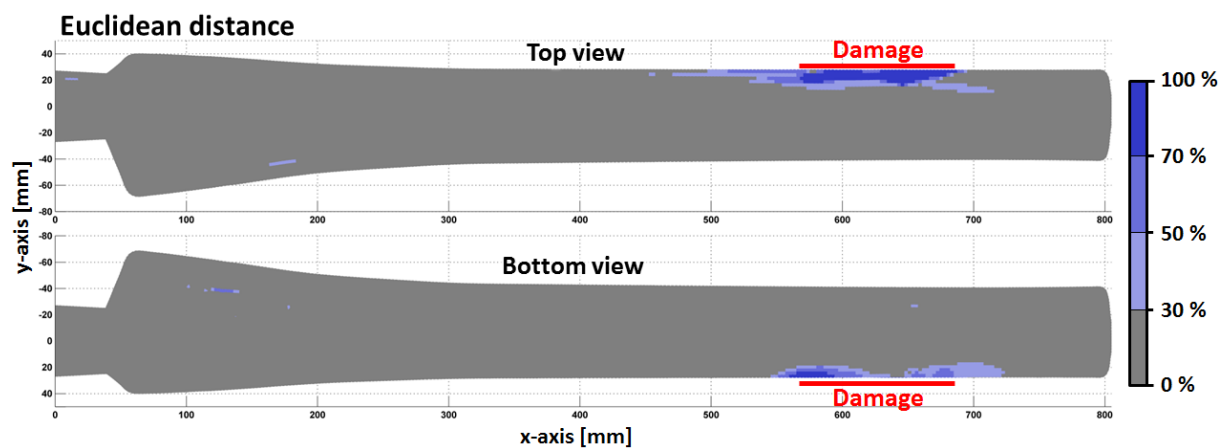
Location 3 should, theoretically, be the only one to have distances higher than the calculated threshold in the damaged experiment, while distances at the other locations should be below the threshold. The results of the locations are presented in figure 7.



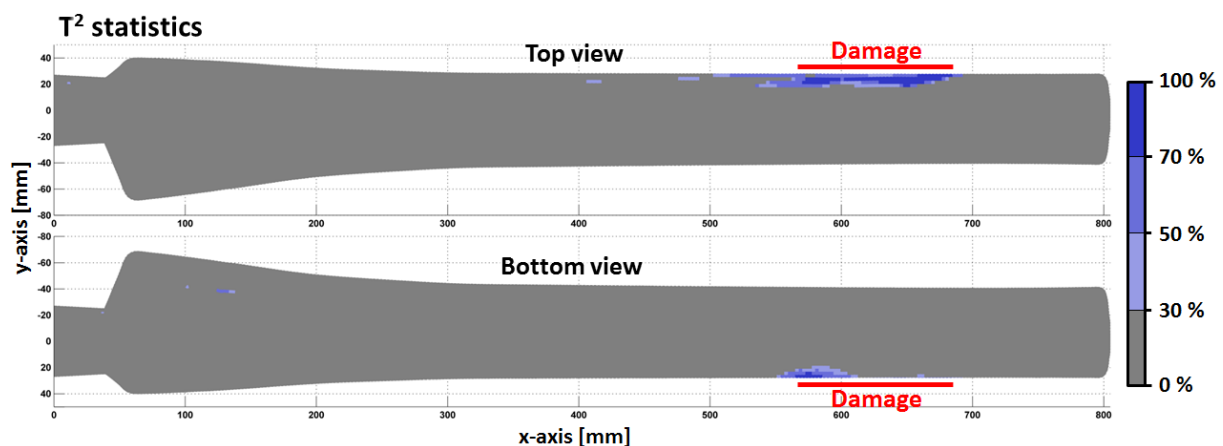
**Figure 7.** Statistical results from four locations by use of Euclidean distance,  $T^2$  statistics, and Mahalanobis distance. The horizontal lines indicate the threshold and the vertical lines discriminate between the healthy experiments to the left and the damaged experiments to the right.

It appears that the two methods employing Euclidean distance and  $T^2$  statistics detect the damage in location 3, without too many false alarms at any of the other locations. It is also noticed that the  $T^2$  statistics-based method is slightly better than the Euclidean distance-based method. On the contrary, the Mahalanobis distance does not appear to be applicable to the measured data. This has been shown to improve if the data is taken as the mean of many more experiments, but that is a problem due to the limited amount of available data.

The Euclidean distance has been calculated for all elements using all of the damaged experiments, and in figure 8, the amount of experiments that yield a distance exceeding the threshold is presented as a percentage of all experiments. Evidently, the damage is clearly located and the amount of noise present in the plot is reduced significantly compared to the results from individual SDDLVs, see figure 3. The same procedure has been applied for the results of  $T^2$  statistics. The results, which can be seen in figure 9, show how the damage is located practically perfect without any disturbances. It is noticed how the exact size of the damage is assessed, which is important information with respect to commercial applications.



**Figure 8.** Statistical evaluation for SDDLV-induced stress resultants using the Euclidean distance.



**Figure 9.** Statistical evaluation for SDDLV-induced stress resultants using  $T^2$  statistics.

## 7. Conclusion

The presented paper deals with the localization of damage in a residential-sized wind turbine blade by use of the SDDL method. The SDDL method has previously been demonstrated, by the authors, as capable of locating different damages in the aforementioned blade, although noise disturbances made it difficult to obtain unambiguous localization when using individual SDDLs. This is handled in the



present study by extending the SDDL method to contain a statistical evaluation yielding the final discrimination between damaged and undamaged areas.

The statistical evaluation is based on outlier analysis of the characteristic stress resultants, for a total of 10  $s$ -values, in each element. The statistical patterns, i.e. mean vector and covariance matrix, are established on the basis of several experiments from the healthy state. Subsequently, data from the current state are tested against this pattern by alternate use of three different similarity measures, namely the Euclidian distance,  $T^2$  statistics, and the Mahalanobis distance. While the Mahalanobis distance proves completely inapplicable, it is found that by employing either Euclidian distance or  $T^2$  statistics, the damage is located unambiguously. Of the latter two approaches,  $T^2$  statistics is preferable as it not only provides unambiguous localization but also assesses the size of the damage practically perfect.

Future research activities will deal with further improvement of the  $s$ -value selection process in order to establish a particular guidance. Moreover, a sensitivity study, in which the influence of damage size and location is examined, will be conducted.

## 8. Acknowledgement

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