

Trapping Cold Atoms Using Surface Plasmons with Phase Singularities Generated by Evanescent Bessel Beams

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Abstract. When a laser Bessel beam light is totally reflected internally at the planar surface of a dielectric on which a thin film has been deposited, a surface plasmons with phase singularities can be produced. This two-dimensional plasmons lead to an attractive potential with enhancement. Using this effect, we construct a model for optical trapping cold atoms. Efficiency of the trap will be presented by an ellipse with average radius r in the (x, y) directions and the distance d from planar surface in the z -direction.

1. Introduction

Trapping methods of neutral atoms are interest in both basics side as pure quantum effects and in application side in many modern technologies such as nano, bio, electro, quantum optic. Cold trapped radioactive atoms can be used in fundamental symmetry experiments, including the experiments on nuclear decay, atomic parity non-conservation and the search for parity and time-reserve violating electric dipole moment [1]. Trapping of ultracold atoms also gives us an opportunity to study the collision processes in cold atomic samples. Trapped cold atoms can be used in the formation of cold molecules [2]. Since the cold molecule trap lifetime was approximately half a second, so the production of cold molecules opens new ways of research in molecular spectroscopy. It has been shown theoretically and experimentally that neutral atoms can be captured by an optical trap [3], based on the use of an evanescent wave occurred around a quasi-one-dimensional micro-nano structure, such as optical fiber [4], thin silica fiber [5], carbon nanotube [6-8], nano pilar [9]. In those optical traps, the effective optical attractive potential created by light is inversely proportional to the radius of fiber. The condition for existence of the trap is that the fiber diameter is about two times smaller than the light wave length. In the last decade surface plasmon (SP) and surface plasmon polariton (SPP), the collective oscillations of conduction electrons and their coupled with photon at the metal/dielectric interface, receive much attention in physics, chemistry, biology and technology communities. Due to their UV-vis-wavelength and local field enhancement effects, surface plasmons and surface plasmon polaritons found vast application in many areas, near field microscopy, Forster resonance energy transfer (FRET), surface enhanced Raman spectroscopy (SERS), nonlinear optics, bio- and nano sensors



[10-15]. In this work we study the optical trap using the surface plasmon effect. Plasmon and plasmon polaritons theories always are difficult to understand for numerous experimental physicists and technicians, a simple model will be useful and needed. In this work, using a laser Bessel beam light and the Kretschmann configuration of excitation surface plasmons, we propose a simple model of optical trap. In this model the effective volume of the trap will be presented in a cylindrical form with the average radius and height of first bound state. This simple picture could be useful for imagine and design idea of optical plasmon could atom trap.

2. Optical plasmon trap and conditions for surface excitation plasmon

We use the Kretschmann configuration of excitation surface plasmons for optical surface plasmon trap design. The Kretschmann configuration is presented in the figure 1, consisting a metallic thin film M deposited on a planar dielectric substrate denoted as $(x, y, z = 0)$ plane, laser source L, and detector D. A strong laser beam in the $(x, y = 0, z)$ plane produced by the source D

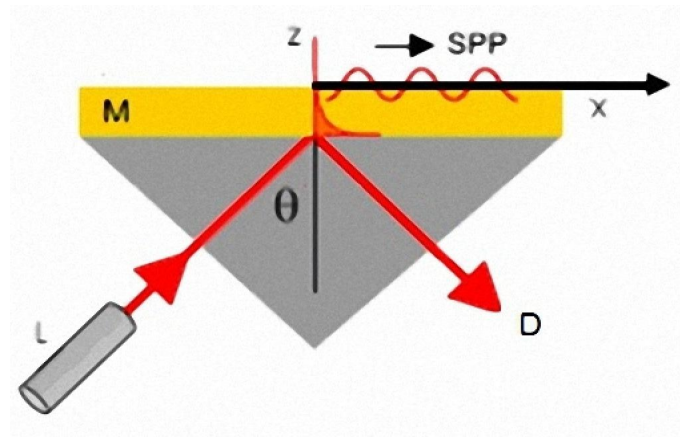


Figure 1. Kretschmann configuration of excitation surface plasmons. M: thin metal layer, L: laser beam source, D: detector.

with frequency ω and radius R_0 is incident at angle θ to surface metallic thin film. Laser beam is reflected at the interface $z = 0$ between the dielectric substrate and the metallic thin film. Consequently a surface plasmon evanescent mode is produced at the vacuum/metal interface. This evanescent mode creates an effective attractive potential, leads to a bound state of a cold atom at the vacuum/metal interface. We can use that effect as working principle of optical plasmon trap for cold atom. The matching conditions for excitation surface plasmon are

- i) wave vectors of photon γ and surface plasmon in the x-direction are coincided $k_{photon,x} = k_{Sp,x}$,
- ii) incident and the total reflection angles θ_0 at are coincided $\theta = \theta_0$,
- iii) metal thickness smaller than skin depth. These surface plasmon excitation requirements are presented in the figure 2.

Denote ε_0 and ε_m are the dielectric constants of dielectric and metal, c is the speed of light, the dispersion laws are

$$\omega = ck, \quad (1)$$

for photon in vacuum or air, and

$$\omega = ck\sqrt{(\varepsilon_m + 1)/\varepsilon_m}, \quad (2)$$

for surface plasmon. The total reflection angle satisfies the relation

$$\sin(\theta_0) = \frac{ck}{\omega\sqrt{\varepsilon_0}}. \quad (3)$$

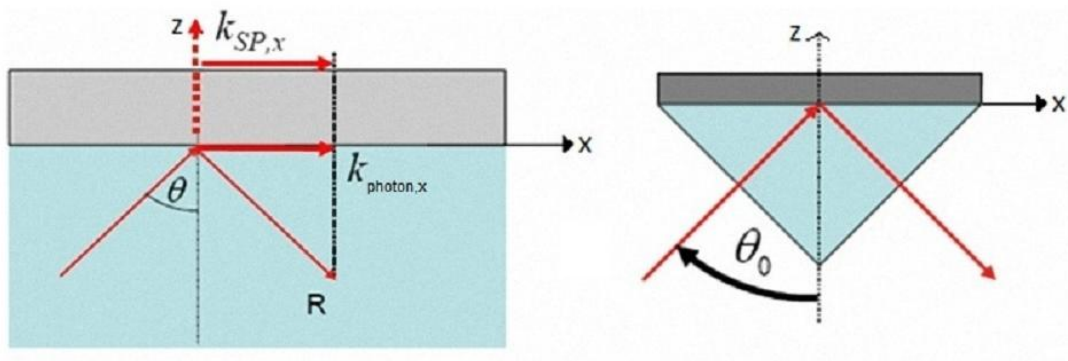


Figure 2. Matching conditions for excitation surface plasmon.

From the condition of equal wave vectors of photon γ at total reflection angle and surface plasmon we have the resonance condition for surface plasmon excitations

$$c\sqrt{(\varepsilon_m + 1)/\varepsilon_m} = c/(\sqrt{\varepsilon_0} \sin \theta_0) = \omega_0/k_0, \quad (4)$$

where ω_0 and k_0 are the resonant frequency and wave vector (see the figure 3).

3. Theory of optical plasmon trap model

Consider a Bessel laser beam is totally reflected at the planar dielectric/metal interface and consequently a surface plasmon evanescent mode is produced at the vacuum/metal interface. The envelop function of a Bessel laser beam order l traveling along z -direction and having electrical vector in the y -direction can written as [15]

$$F_{kl}(r, z) = CI_B e^{ik_z z} (z/z_0)^{l+1/2} \exp(-z^2/z_0^2) J_l(r/R), \quad (5)$$

where C is a factor, I_B is the intensity of Bessel laser beam, $r = \sqrt{x^2 + y^2}$ is radial vector (parallel to the interface plane), R is effective radius of Bessel laser beam, z_0 is the typical ring spacing of Bessel laser beam, $J_l(r/R)$ is the Bessel function of order l .

Consider light as a vector, when incident angle of Bessel laser beam is θ , a projective operator $P(\theta)$ for separation the out-of plane z direction and in-plane (x, y) directions can be defined by

$$P(\theta)F(\vec{r}, z) = F_z(\theta) + F_r(\theta). \quad (6)$$

Assuming Bessel laser beam is in (yOz) plane, and the incident angle is θ relative to the z -direction, the projections can be expressed by transforms

$$x \rightarrow x/\cos \theta, \quad y \rightarrow y, \quad z \rightarrow z \cos \theta. \quad (7)$$

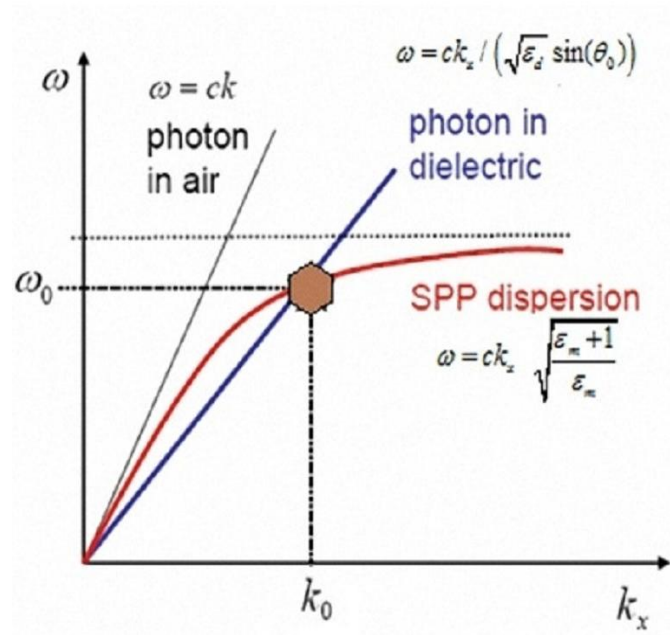


Figure 3. The resonant frequency ω_0 and wave vector k_0 for surface plasmon polariton SPP.

Apply the boundary conditions for electric and magnetic field of Bessel laser beam at the interfaces, we can realize that an evanescent light is exponential decay with distance z in vacuum $\exp(-z/\Lambda)$ where Λ is characteristic decay length of the evanescent field, and provides a two-dimensional surface plasmons with phase singularities and attractive enhancements [15]. The effective attractive optical potential in the z -direction introduced by evanescent effect can be written as

$$V_l(z; \theta) = -gI_B(z \cos \theta / z_0)^{l+1/2} \exp(-z \cos \theta / \Lambda) \cdot \exp(-z^2 \cos^2 \theta / z_0^2), \quad (8)$$

where g is effective coupling constant for interaction between evanescent wave and cold atom [3-9]. The effective attractive optical potential in the interface (x, y) plane introduced by evanescent effect is

$$U_l(x, y; \theta) = -gI_B J_l \left(\sqrt{(x^2 / \cos^2 \theta) + y^2 / R} \right). \quad (9)$$

Matching conditions for surface plasmon excitation require that the incident angle must be closed to total reflection angle $\theta = \theta_0$. For many metals, example Au $\theta_0 \sim 41 - 43^\circ \approx \pi/4$ so a good approximate to put in the expressions of effective potentials (8-9) that $\cos \theta \approx 1/\sqrt{2}$. The effective attractive optical potential in the z -direction introduced by evanescent effect now has a form

$$V_l(z) = -gI_B(z/z_0\sqrt{2})^{l+1/2} \exp(-z/\Lambda\sqrt{2}) \cdot \exp(-z^2/2z_0^2), \quad (10)$$

The effective attractive optical potential in the interface (x, y) plane introduced by evanescent effect is

$$U_l(x, y) = -gI_B J_l \left(\sqrt{2x^2 + y^2 / R} \right). \quad (11)$$

For the S state ($l = 0$), we have

$$V_0(z) = -gI_B(z/z_0\sqrt{2})^{1/2} \exp(-z/\Lambda\sqrt{2}) \cdot \exp(-z^2/2z_0^2), \quad (12)$$

$$U_0(r) = -gI_B J_0 \left(\sqrt{2x^2 + y^2}/R \right). \quad (13)$$

Define $z' = z/z_0\sqrt{2}$ and $t = z_0/\Lambda$. The values of effective potentials V_0 in z -direction as a function of z' and t , and U_0 in (x, y) -plane for the S state $l = 0$ are presented in the figure 4.

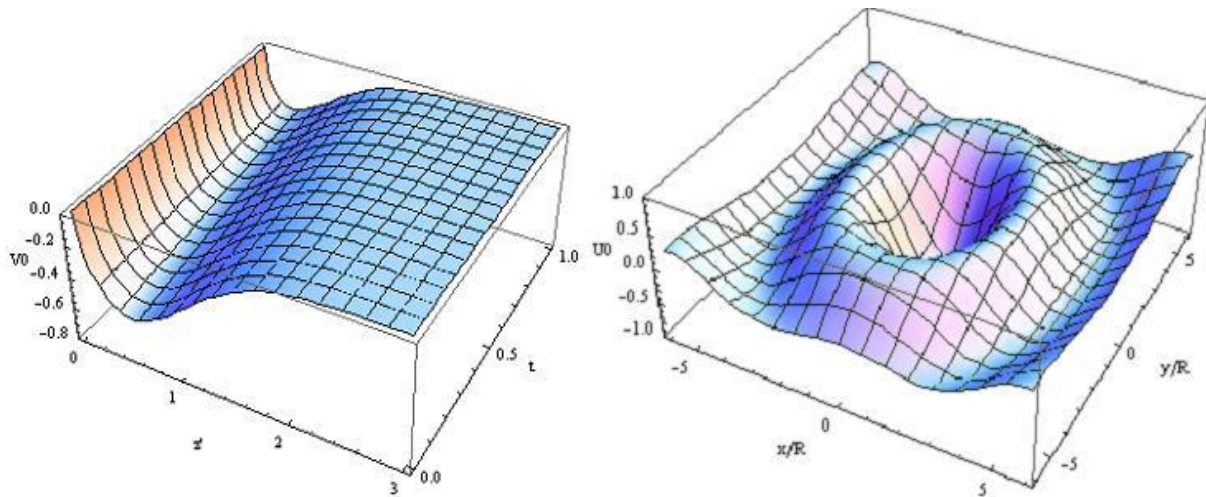


Figure 4. The values of effective potentials V_0 in z -direction as a function of z' and t , and U_0 in (x, y) -plane for the S state $l = 0$.

For the P state ($l = 1$), we have

$$V_1(z) = -gI_B (z/z_0\sqrt{2})^{3/2} \exp(-z/\Lambda\sqrt{2}) \cdot \exp(-z^2/2z_0^2), \quad (14)$$

$$U_1(r) = -gI_B J_1 \left(\sqrt{2x^2 + y^2}/R \right), \quad (15)$$

The values of effective potentials V_1 in z -direction as a function of z' and t , and U_1 in (x, y) -plane for the P state $l = 1$ are presented in the figure 5.

Note that the trapping efficiency is proportional to the intensity of Bessel laser beam and effective coupling constant for interaction between evanescent wave and cold atom. Between the two case of $l = 0$ and $l = 1$, the S state give more trapping efficiency.

4. Simple pictorial model of optical plasmon trap

Plasmon and plasmon polariton theories always are difficult to understand for numerous experimental physicists and technicians, so in this part, we develop a simple pictorial model optical plasmon trap cold atom. Using the well known series expansion of Bessel function

$$J_0(x) \approx 1 - \frac{x^2}{4} + \frac{x^4}{64} + \dots, \quad (16)$$

and neglecting a constant, we can approximate the effective potential in the z -direction as

$$U_0(x, y) \approx \frac{gI_B}{4R^2} (2x^2 + y^2), \quad (17)$$

The Hamiltonian for motion in (x, y) -directions is

$$H_{0in}(x, y) = \frac{p_x^2 + p_y^2}{2m} + \frac{gI_B}{4R^2} (2x^2 + y^2), \quad (18)$$

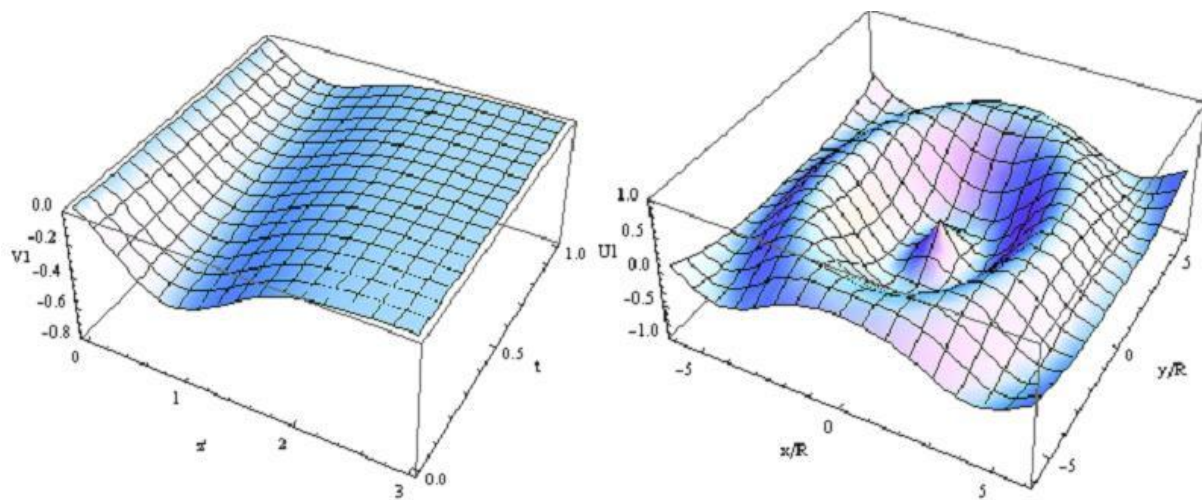


Figure 5. The values of effective potentials V_1 in z -direction as a function of z' and t , and U_1 in (x, y) -plane for the P state $l = 1$.

where m is the mass of cold atom. This Hamiltonian has the form of the Hamiltonian for harmonic oscillator, is easy to solve and gives bound state energy spectrum

$$E_m = \frac{\omega^*}{2}(2n + 1), n = 0, 1, 2, \dots \quad (19)$$

where ω^* is the characteristic frequency of the optical plasmon trap

$$\omega^* = \left(1 + \frac{1}{\sqrt{2}}\right) \sqrt{\frac{gI_B}{mR^2}} \quad (20)$$

The energy of ground bound state is

$$E_{\gamma 0} = \frac{3}{2}\omega^* = \frac{3}{2} \left(1 + \frac{1}{\sqrt{2}}\right) \sqrt{\frac{gI_B}{mR^2}} \quad (21)$$

The trapping effect by evanescent wave in (x, y) -directions can be modeled as an ellipse disc expressed by the equation

$$T_{in}(x, y) = \frac{gI_B}{4R^2}(2x^2 + y^2) \quad (22)$$

In the general case, when the incident angle of Bessel laser beam equals to the total reflection angle θ_0 we have following expressions for energy of ground bound state

$$E_{r0} = \frac{3}{2\sqrt{2}} \left(1 + \frac{1}{\cos \theta_0}\right) \sqrt{\frac{gI_B}{mR^2}}, \quad (23)$$

and for ellipse disc

$$T_{in}(x, y) = \frac{gI_B}{4R^2} \left(\frac{x^2}{\cos^2 \theta_0} + y^2 \right). \quad (24)$$

From figure 4, we can see that the effective radius of the effective attractive optical potential in the (x, y) plane introduced by evanescent effect is approximate $R_D \approx 3.8R$.

The t -dependence minimum line z'_{\min} of the effective attractive optical plasmon potential in the z -direction introduced by evanescent effect is defined by equation

$$z'_{\min} = \left(-t + \sqrt{4 + t^2} \right) / 4, \quad (25)$$

and presented in the figure 6. With this z'_{\min} later we define the distance d of our trap model

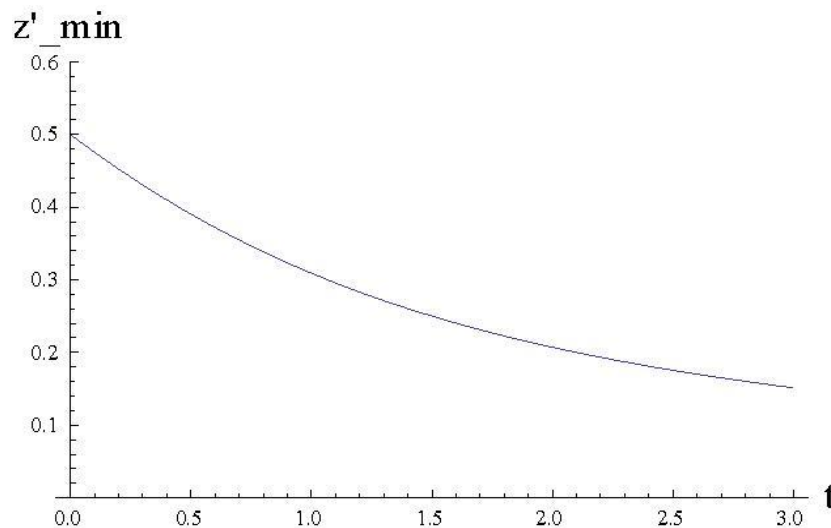


Figure 6. The t -dependence minimum line of the effective attractive optical plasmon potential in the z -direction.

Finally, we suppose a simple pictorial model for the effective attractive optical plasmon potential introduced by evanescent effect. The trap is modeled by an elliptic cylinder as presented in the figure 7. This ellipse expressed by equation (24) with average radius $R_D \approx 3.8R$, and lies above (x, y) -plane a height $d = z'_{\min} z_0 \sqrt{2}$, where z'_{\min} expressed by equation (25) at incident point of the Bessel laser beam. The strength of the trap is gI_B . This simple pictorial model of the optical plasmon trap might be useful for imagination, set up experiment, and technical design, etc.

5. Discussion

The main results of this work is simple theory describing interaction a Bessel laser beam with surface plasmon in thin film metal. Based on detail numerical investigation, we suppose a simple pictorial model for the effective attractive optical plasmon potential introduced by evanescent effect. The trap is modeled by an elliptic cylinder as presented in the figure 7. This ellipse expressed by equation (24) with average radius $R_D \approx 3.8R$, and lies above (x, y) -plane a height $d = z'_{\min} z_0 \sqrt{2}$, where z'_{\min} expressed by equation (25) at incident point of the Bessel laser beam. The strength of the trap is gI_B . This simple pictorial model of the optical plasmon trap might be useful for further investigation and practical design in cold atom trap areas.

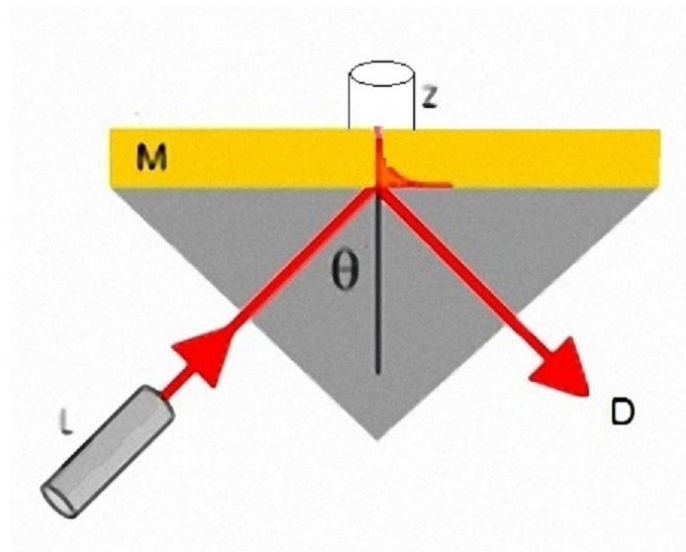


Figure 7. The pictorial model for the effective attractive optical plasmon potential introduced by evanescent effect. The trap is modeled by an elliptic cylinder. This ellipse expressed by equation (24) with average radius $R_D \approx 3.8R$ and lies above (x, y) -plane a height $d = z'_{\min} z_0 \sqrt{2}$, where z'_{\min} expressed by equation (25) at incident point of the Bessel laser beam. The strength of the trap is gI_B .

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