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Echoes from the past

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Abstract.
Primordial inflation is regarded as the leading paradigm for the very early universe. After quickly reviewing some basic elements, I discuss the current status of inflationary theories especially in the light of effective theory approach and primordial gravitational waves. I also comment on future probes for the early universe.

1. Introduction

Currently, inflation [1] is regarded as the leading candidate to describe the very early universe before the onset of the standard hot big bang evolution. While the current observable universe requires a number of extremely finely tuned initial conditions as suggested by the cosmic microwave background (CMB), inflation can provide these conditions naturally. For example, the so-called horizon problem states that at the moment of the generation of the CMB the observable universe could not be in causal communication but there were $10^4 - 10^5$ causally disconnected regions, so that the chance of having the same temperature with the accuracy of 10^{-5} as the current observations on the CMB is extremely unlikely. According the the inflationary picture, the whole observable universe originated from a single causally connected patch and it is no surprise that the temperature of the CMB is homogeneous. Note that we need then a certain amount of expansion during inflation. This is quantified by the number of e-folds N, which is the logarithmic ratio of the scale factor $a(t)$ at the end of inflation to the initial moment, $N = \log(a_e/a_i)$. Taking into account our ignorance on certain aspects of the early universe, we require $N \gtrsim 50 - 60$.

Inflation does not only provide the necessary initial conditions for the hot big bang evolution of the universe. During inflation, quantum mechanical uncertainties are amplified following the expansion of the universe and they become the classical perturbations in the amount of expansion once the modes of our interest exit the horizon. After inflation and these modes come inside the horizon again, and then experience causal evolution such as gravitational instability. These small inhomogeneities later become the temperature anisotropy of the CMB, the seed of galaxies, and so on – all the observable structure in the universe. Thus, according to inflation, we can probe quantum mechanical signatures relevant for the speculative physics in the early universe by accurate observations on cosmic scales [2].

The perturbations are usually described by the metric fluctuations. Writing the spatial component of the flat Friedmann-Robertson-Walker metric with most general and physical fluctuations,

$$dl^2 = a^2(t) \left\{ [1 + 2\mathcal{R}(t, \boldsymbol{x})] \delta_{ij} + h_{ij}(t, \boldsymbol{x}) + \cdots \right\} dx^i dx^j \,, \tag{1}$$

where h_{ij} is pure tensor so that it is transverse and traceless. $\mathcal{R}(t, \boldsymbol{x})$ is proportional to the background metric δ_{ij} and is thus scalar in nature. Because it describes the perturbation in the spatial hypersurface, it is also called the curvature perturbation. It is related to the isotropic scaling of space and thus relevant to the temperature fluctuation $\delta T/T_0$ of the CMB. Meanwhile, $h_{ij}(t, \boldsymbol{x})$ describes area-conserving anisotropic stretching of space as can be inferred by its transverse and traceless nature, and possesses two physical degrees of freedom. Thus it is identified as the primordial gravitational waves, leading to quadruple anisotropy in the temperature fluctuation of the CMB and is thus relevant to the polarization of the CMB.

The properties of these perturbations are described by the correlation functions. The inflationary predictions are such that their power spectra are nearly scale invariant with respect to k and almost perfectly Gaussian. Indeed, the running of the power spectrum of the curvature perturbation is described by the spectral index $n_{\mathcal{R}}$,

$$P_{\mathcal{R}}(k) = \left\langle |\mathcal{R}(k)|^2 \right\rangle \propto k^{n_{\mathcal{R}}-4} \tag{2}$$

with k^{-3} factor being just dimensional. Recent observation by Planck constrains $n_{\mathcal{R}} = 0.960 \pm 0.007$ at 1σ level [3]. The departure from Gaussian nature, non-Gaussianity, is frequently parametrized by the so-called non-linear parameter $f_{\rm NL}$. In the squeezed configuration of the 3-point correlation function, viz. bispectrum of the curvature perturbation, it is given by

$$f_{\rm NL} = \frac{5}{12} \lim_{k_3 \to 0} \frac{B_{\mathcal{R}}(k_1, k_2, k_3)}{P_{\mathcal{R}}(k_1) P_{\mathcal{R}}(k_3)} = \frac{5}{12}(1 - n_{\mathcal{R}}) \,, \tag{3}$$

so that $f_{\rm NL}$ is vanishingly small. Indeed, the current bound on $f_{\rm NL}$ is $f_{\rm NL} = 2.7 \pm 5.8$ at 1σ level, consistent with 0 [4]. For gravitational waves, likewise we can anticipate the power-law form of the power spectrum,

$$P_T(k) = \sum_{s=+,\times} \left\langle |h_s(k)|^2 \right\rangle \propto k^{n_T - 3} \,, \tag{4}$$

where s runs over the two independent polarization states denoted by $+$ and $\times$. An important quantity of interest is the tensor-to-scalar ratio

$$r = \frac{P_T}{P_{\mathcal{R}}} \,, \tag{5}$$

which Planck put an upper bound $r \lesssim 0.11$ [3] while BICEP2 reported $r = 0.20^{+0.07}_{-0.05}$ [5].

So it seems that we are entering a new regime of observations using the primordial gravitational waves, although it is yet to be seen as of now how large is the fraction of the dust polarization in the BICEP2 result. If BICEP2 is mostly due to the primordial origin, the most favoured model is the simplest one with the potential $V(\phi) = m^2\phi^2/2$. Thus the energy scale of inflation is as high as 10^{16} GeV, with the field excursion being larger than $m_{\rm Pl}$. Moreover, general relativity and quantum field theory seem to remain valid up to such a high energy scale, leading to the first clue of quantized gravity. But at the same time there seems no compelling hint beyond this simplest case. Thus the question we may naturally ask is what are the remaining windows for new physics beyond this simple picture.

2. Running of the primordial spectra

We first notice that there are a number of anomalies in the CMB. For example, there seems to exist hemispherical asymmetry between northern and southern hemisphere. On gravitational waves side, obviously there is a tension between Planck bound and BICEP2 detected value of r.

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Among them, let us consider the low multipole regime of the TT power spectrum. Obviously the central values are at least marginally lower than the standard ΛCDM prediction [6]. Normalized at $k = 0.05$ Mpc^{-1}, the power on larger scales is suppressed than theoretical prediction. A possible solution is that the "running" of the scalar spectral index,

$$\alpha_{\mathcal{R}} \equiv \frac{dn_{\mathcal{R}}}{d\log k}\,, \tag{6}$$

is negative. This negative running is also supported to accommodate the large value of r: for Planck alone, the upper bound on r is relaxed to $r < 0.26$ with $\alpha_{\mathcal{R}} = -0.022 \pm 0.010$ [3]. To reconcile Planck and BICEP2, $\alpha_{\mathcal{R}} = -0.028 \pm 0.009$ [5] can do the job.

But in the standard, simplest model of inflation, $\alpha_{\mathcal{R}}$ is much suppressed. Expressed in terms of the slow-roll parameters,

$$\alpha_{\mathcal{R}} = -2\xi_V^2 - 24\epsilon_V^2 + 16\epsilon_V\eta_V\,, \tag{7}$$

where

$$\epsilon_V \equiv \frac{m_{\text{Pl}}^2}{2}\left(\frac{V'}{V}\right)^2\,, \quad \eta_V \equiv m_{\text{Pl}}^2\frac{V''}{V}\,, \quad \xi_V^2 \equiv m_{\text{Pl}}^4\frac{V'V'''}{V^2}\,. \tag{8}$$

With $\epsilon_V = \mathcal{O}(0.01)$, $|\alpha_{\mathcal{R}}| = \mathcal{O}(\epsilon_V^2)$ and thus cannot accommodate the expected large value of $\alpha_{\mathcal{R}}$.

Because the scalar perturbation is dependent on the dynamics in the matter sector, we can obviously generalize to the case where inflation is driven by multiple number of fields to relax this difficulty. In the context of the so-called δN formalism [7], where the perturbation in the number of e-folds between the initial flat hypersurface during inflation and the final comoving one after inflation is equivalent to the final curvature perturbation, we can find [8]

$$\alpha_{\mathcal{R}} = 4\epsilon^2 - 2\epsilon\eta + \frac{N_aN_b}{N_dN^d}\left(8\epsilon w^{ab} + 4w^a{}_c w^{bc} - 2D_N w^{ab}\right) - (n_{\mathcal{R}} - 1)^2\,, \tag{9}$$

where $N_a = \partial N/\partial\phi^a$, the slow-roll parameters are given in terms of the Hubble parameter as

$$\epsilon \equiv -\frac{\dot{H}}{H^2}\,, \quad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon}\,, \tag{10}$$

and [9]

$$w_{ab} = u_{(a;b)} + \frac{R_{c(ab)d}}{3}\frac{\dot{\phi}_0^c}{H}\frac{\dot{\phi}_0^d}{H} \quad \text{with} \quad u_a = -\frac{V_{;a}}{3H^2}\,, \tag{11}$$

$$D_N w^{ab} = w^{ab}{}_{;c}\frac{\dot{\phi}_0^c}{H}\,. \tag{12}$$

In principle one can hope to accommodate large negative value of r in this case because the above general expression includes geometric information on the field space, like $R_{abcd;e}$ and $V_{;abc}$. But it seems to remain an open challenge to construct such a realistic model.

For the gravitational waves, there seem a number of anomalies, such as too high C_ℓ^{EE} and too low C_ℓ^{BB} at low multipoles. These may all be due to statistical uncertainty, but they may be real as well and we should wait for more accurate observations to be made very soon. But nevertheless these anomalies look quite interesting, and one of them is a positive bump at around $150 \lesssim \ell \lesssim 300$ beyond 2σ significance. This means the excess of the primordial gravitational waves more than $r = 0.20$ naively so that the spectral index of the tensor spectrum n_T is positive. But for standard case $n_T < 0$ always,

$$n_T = -2\epsilon\,. \tag{13}$$

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Since gravitational waves are after all gravity and thus only care the total energy density: they are blind to detailed dynamics during inflation, so considering multi-field inflation as we did for the scalar running does not help. Instead, we extend the notion of the slow-roll approximation where we implicitly assume hierarchies between a slow-roll parameter and its derivatives.

In this context of the "general slow-roll" formalism [10], the departure from perfect de Sitter phase is parametrized by a function $p(\log\tau) = -2\pi\tau a$, where $d\tau \equiv dt/a$ is the conformal time. Then the solution of the mode function can be written as a perturbative series of the Green's function solution [11]. Then, up to second order corrections n_T is written as [12]

$$n_T = 2\frac{p'}{p} + 2\alpha\left(\frac{p'}{p}\right)' + 2(4-\pi)\frac{p'p''}{p^2}\,, \tag{14}$$

where $p' \equiv dp/d\log(-\tau)$ and $\alpha \equiv 2 - \log 2 - \gamma \approx 0.577216$ with γ being the Euler-Mascheroni constant. Keeping up to this order, $n_T > 0$ as suggested by the bump in the BICEP2 result gives the condition $p'/p \gtrsim 1$ [13]. This can be translated into the slow-roll parameters as

$$\frac{d\log\epsilon}{d\log a} \lesssim -1\,, \tag{15}$$

so that ϵ is rapidly decaying as $1/a$ or even faster, so that η is negatively large. Of course, this period during which the above condition is satisfied cannot last too long, otherwise inflation does not terminate. Typical form of the potential in this case looks flat over a certain range, like ultra-slow-roll inflation [14] or punctuated inflation [15]. Of course, we may find a blue tilt of the tensor spectrum in different context, e.g. inflation driven by Galilion-type field [16] or string gas cosmology [17]. Also we may build other consistency checks, like the running of the tensor-to-scalar ratio [18], where we can eliminate ϵ contribution.

3. Effective single field inflation and features

In the previous section, we noticed the current observational data may suggest momentary deviations from otherwise vanilla predictions. Then one question remains ahead: the deviations, or more generally "features", mean there is a structure between smooth theory that spans super-Planckian field values. This is not surprising in the context of effective field theory: naively we expect smooth slow-roll inflation with large field excursion. But effective theory with a cutoff scale $\Lambda \lesssim m_{\rm Pl}$ allows in general higher dimensional operators $\mathcal{O}_n$ with the dimension $n > 4$ suppressed powers of Λ, such that

$$\mathcal{L}_{\rm eff}[\phi] = \mathcal{L}_0[\phi] + \sum_n c_n\frac{\mathcal{O}_n[\phi]}{\Lambda^{n-4}}\,, \tag{16}$$

where the coefficients c_n's are in general $\mathcal{O}(1)$. These higher dimensional terms bring sub-Planckian structure that generally prevents long enough slow-roll inflation without anything else, which gives deviations from otherwise smooth spectrum, viz. features. These higher dimensional operators can be obtained by two different approaches. One is to write down all the possible terms allowed by presumed symmetry principles, such as the Lorentz symmetry and gauge symmetry. In the popular approach of effective field theory of inflation, time translational symmetry is broken while spatial differmorphism is kept [19]. The other is, starting from a mother theory which contains degrees of freedom heavier than the scale of our interest, we integrate out those heavy modes. Here we concentrate on the latter, and see if there are any universal features of the integrated out heavy physics.

The recipe of obtaining effective single field theory is simple [20]. In the field space, the departure from the homogeneous and isotropic background solution $\phi_0^a(t)$ is represented by two

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fields $\pi = \pi(t, \boldsymbol{x})$ and $\mathcal{F} = \mathcal{F}(t, \boldsymbol{x})$ in such a way that

$$\phi^a(t, \boldsymbol{x}) = \phi_0^a(t + \pi) + N^a(t + \pi)\mathcal{F}\,, \tag{17}$$

where $N^a(t + \pi)$ stands for the normal vector to the background trajectory N^a evaluated at $t+\pi$. Notice that π represents deviations from $\phi_0^a(t)$ along the background trajectory, whereas $\mathcal{F}$ parametrizes deviations off the trajectory. For gravitational sector, we write the spatial metric γ_{ij} as

$$\gamma_{ij} = a^2(t + \pi)e^{2\mathcal{R}}\delta_{ij}\,. \tag{18}$$

Given that we will be eventually interested in the curvature perturbation $\mathcal{R}$ by integrating out $\mathcal{F}$, it is convenient to work in the comoving gauge where $\pi = 0$. Then, by performing the path integral over $\mathcal{F}$ we integrate out $\mathcal{F}$ and obtain the effective single field action of $\mathcal{R}$,

$$e^{S_{\text{eff}}[\mathcal{R}]} = \int [D\mathcal{F}]e^{S[\mathcal{R},\mathcal{F}]}\,, \tag{19}$$

which is at cubic order equivalent to plugging back the linear solution of $\mathcal{F}$ into the action [21],

$$\mathcal{F} = \left(-\Box + M_{\text{eff}}^2\right)^{-1} \frac{-2\dot{\theta}\dot{\phi}_0}{H}\dot{\mathcal{R}}\,, \tag{20}$$

where M_{eft} is the heavy mass scale and $\dot{\theta}$ is the angular velocity of the trajectory. The result is that the effects of heavy physics are described by the "speed of sound" given by

$$c_s^{-1} \equiv 1 + \frac{4\dot{\theta}^2}{M_{\text{eff}}^2}\,. \tag{21}$$

This is very intuitive: if the trajectory is not turning but straight where $\dot{\theta} = 0$, we can simply change the field space coordinates to have a single field description. If the mass scale M_{eff} is extremely heavy, $\mathcal{F}$ does not move at all and it can simply be ignored.

Being said "effective field theory", there must be a small parameter upon which we make the expansion of the theory. As can be read from the solution of $\mathcal{F}$, in the above prescription we have truncated at leading order in the expansion of $\Box/M_{\text{eff}}^2$,

$$\mathcal{F} = \frac{1}{M_{\text{eff}}^2}\left(1 + \frac{\Box}{M_{\text{eff}}^2} + \cdots\right)\frac{-2\dot{\theta}\dot{\phi}_0}{H}\dot{\mathcal{R}}\,. \tag{22}$$

Thus, the effective theory obtained by plugging back the leading solution remains valid for "adiabatic" trajectory for which $|\Box\mathcal{F}| \ll M_{\text{eff}}^2\mathcal{F}$, or [22]

$$\left|\frac{\ddot{\theta}}{\dot{\theta}}\right| \ll M_{\text{eff}}\,. \tag{23}$$

That is, the acceleration of $\mathcal{F}$ is small compared to the mass term so that the creation of heavy quanta is suppressed. Note that this means even for a extreme case where $c_s^{-2} \gg 1$ the effective theory is legitimate as long as the trajectory is adiabatic [23]. Of course, as the mass scale M approaches H the next-to-leading expansion term becomes important and cannot be ignored [24].

An interesting observational consequence is that if c_s sources features solely, the bispectrum is completely specified by the power spectrum and its first two derivatives, i.e. $n_{\mathcal{R}}$ and $\alpha_{\mathcal{R}}$.

Presuming that the reduction in the speed of sound is sufficiently small, by introducing a small parameter

$$u \equiv 1 - \frac{1}{c_s^2}\,, \tag{24}$$

we can find the change in the power spectrum generated by the speed of sound to first order in u as

$$\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}(k) = k\int_{-\infty}^{0} d\tau u(\tau)\sin(2k\tau)\,, \tag{25}$$

where $\mathcal{P}_\mathcal{R} = 2\pi^2 P_\mathcal{R}/k^3$. By inverting this relation u can be written as a function of $\Delta\mathcal{P}_\mathcal{R}/\mathcal{P}_\mathcal{R}$. With the $f_{\rm NL}$ ansatz in mind, we define a dimensionless shape function as

$$f_{\rm NL}(k_1,k_2,k_3) \equiv \frac{10}{3}\frac{k_1k_2k_3}{k_1^3+k_2^3+k_3^3}\frac{(k_1k_2k_3)^2 B_\mathcal{R}}{(2\pi)^4\mathcal{P}_\mathcal{R}^2}\,. \tag{26}$$

Evaluating this shape function in certain interesting configurations, we find [25]

$$f_{\rm NL} = \begin{cases} \dfrac{5}{54}\left[-7\dfrac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}} - 3\dfrac{d}{d\log k}\left(\dfrac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\right) + \dfrac{d^2}{d\log k^2}\left(\dfrac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\right)\right] & \left(\dfrac{k_2}{k_1} = \dfrac{k_3}{k_1} = 1: \text{ equilateral}\right) \\ -\dfrac{5}{12}\dfrac{d}{d\log k}\left(\dfrac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\right) & \left(\dfrac{k_2}{k_1} = 1\,, \dfrac{k_3}{k_1} \to 0: \text{ squeezed}\right) \\ \dfrac{1}{8}\left[-\dfrac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}} - \dfrac{5}{2}\dfrac{d}{d\log k}\left(\dfrac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\right) + \tfrac{1}{2}\dfrac{d^2}{d\log k^2}\left(\dfrac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\right)\right] & \left(\dfrac{k_2}{k_1} = 1\,, \dfrac{k_3}{k_1} = 2: \text{ folded}\right) \end{cases}, \tag{27}$$

where $k = (k_1+k_2+k_3)/2$. This approach can be extended more generally when the features may be sourced by other origins [26], but still the bispectrum is specified by $\mathcal{P}_\mathcal{R}$, $n_\mathcal{R}$ and $\alpha_\mathcal{R}$.

4. Running of galaxy bias

Large scale structure (LSS) of the universe is, along with the CMB, yet another powerful cosmological probe, and its importance has ever been increasing with galaxy surveys such as SDSS. The LSS observations can provide the measurement of geometrical distances, growth of structures, and shape of primordial correlation functions. These lower redshift information combined with the CMB data can break down the degeneracies among cosmological parameters that yields better constraints than CMB alone. Furthermore, the full three-dimensional information with a huge redshift coverage available for the LSS observations naturally yields measurement of properties of dark energy, neutrino properties as well as physics of the early universe. A number of future observations such as MS-DESI, LSST and Euclid are planned to observe LSS with improved accuracy in near future.

What we measure in large scale survey are galaxies. But as is well known, significant fraction of matter in the universe is dark matter while galaxies consist of baryons. Thus, the distribution of galaxies is not precisely the same as that of dark matter. Thus, the galaxy density field, $\delta_g \equiv (n_g - \bar{n}_g)/\bar{n}_g$, is different from the density perturbation of dark matter, $\delta \equiv (\rho_m - \bar{\rho}_m)/\bar{\rho}_m$. This discrepancy is parametrized by the so-called bias factor b,

$$\delta_h = b\delta\,. \tag{28}$$

Thus the question is how to improve our theoretical prediction of b, i.e. refining our arguments of identifying the locations of the galaxy formation, i.e. density peaks in the matter density distribution, which follows nearly Gaussian probability distribution. An obvious approach is that we identify peaks as the regions where δ exceeds a certain threshold density $\delta_c = \nu\sigma_0$ with $\nu \gg 1$, where σ_0 is the first spectral moment,

$$\sigma_n^2 \equiv \int \frac{d^3k}{(2\pi)^3} P(k) k^{2n} \,. \tag{29}$$

The bias factor can be computed by functional integration and is found to be a constant. Including primordial non-Gaussianity we find a non-trivial scale dependence proportional to $k^{-2} f_{\rm NL}$ [27] and $k^{-4}\left[\tau_{\rm NL} - (6f_{\rm NL}/5)^2\right]$ [28].

A more refined approach is that we identify peaks as local maxima of δ [29]. This is reasonable since after non-linear evolution such local peaks will be enhanced and will eventually lead to the formation of galaxies. To define a peak, we need to specify the density field itself and its first and second derivatives,

$$\nu \equiv \frac{\delta}{\sigma_0} \,, \quad \eta_i \equiv \frac{\partial_i\delta}{\sigma_1} \,, \quad \zeta_{ij} \equiv \frac{\partial_i\partial_j\delta}{\sigma_2} \,, \tag{30}$$

where the minimum eigenvalue of the 3×3 matrix $-\zeta_{ij}$ is greater than 0. Then we can proceed further to include corrections from the Gaussian cross-correlation between two points $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, and non-Gaussian corrections from 3-point correlation function. The former is obtained by simply expanding the 20×20 covariance matrix [30]

$$\mathcal{M} = \begin{array}{c c} & \begin{array}{cc} \boldsymbol{y}(\boldsymbol{x}_1) & \boldsymbol{y}(\boldsymbol{x}_2) \end{array} \\ \begin{array}{c} \boldsymbol{y}(\boldsymbol{x}_1) \\ \boldsymbol{y}(\boldsymbol{x}_2) \end{array} & \begin{pmatrix} M & B \\ B^T & M \end{pmatrix} \end{array} , \tag{31}$$

where M and B are 10×10 self- and cross-correlation matrices respectively and at each point we construct a 10-dimensional vector $\boldsymbol{y}^T = (\eta_i \; \nu \; \zeta_{ij})$. Then the Gaussian exponent can be expanded as a series of B as

$$\exp\left(-\frac{1}{2}\boldsymbol{y}^T\mathcal{M}^{-1}\boldsymbol{y}\right) = \left[1 + \boldsymbol{y}^T(\boldsymbol{x}_1)M^{-1}BM^{-1}\boldsymbol{y}(\boldsymbol{x}_2) + \cdots\right] \exp\left(-\sum_i \frac{1}{2}\boldsymbol{y}^T(\boldsymbol{x}_i)M^{-1}\boldsymbol{y}(\boldsymbol{x}_i)\right) . \tag{32}$$

The non-Gaussian corrections consist of 8 non-zero contributions of 3-point correlation functions,

$$\begin{gathered} \langle\nu(1)\nu(1)\nu(2)\rangle \,, \quad \langle\nu(1)\nu(1)\zeta_{ij}(2)\rangle \,, \quad \langle\nu(1)\zeta_{ij}(1)\nu(2)\rangle \,, \quad \langle\nu(1)\zeta_{ij}(1)\zeta_{kl}(2)\rangle \,, \\ \langle\eta_i(1)\eta_j(1)\nu(2)\rangle \,, \quad \langle\eta_i(1)\eta_j(1)\zeta_{kl}(2)\rangle \,, \quad \langle\zeta_{ij}(1)\zeta_{kl}(1)\nu(2)\rangle \,, \quad \langle\zeta_{ij}(1)\zeta_{kl}(1)\zeta_{mn}(2)\rangle \,. \end{gathered} \tag{33}$$

The combined non-Gaussian corrections lead to non-monotonic running of the bias factor [31].

5. Conclusions

To summarize, most recent observations are consistent with simplest models of inflation, such as large field models with power-law type potential. But nevertheless there are various windows to probe beyond this simplest picture: we have listed the scale of ultraviolet physics through the correlated features in the correlation functions, the dynamics of inflation via various runnings of the primordial spectra, and non-linear processes by large scale galaxy clustering. More importantly, we have a number of ongoing and planned observations ahead. Very soon we will have more accurate data on the CMB polarization from Planck, Keck Array and BICEP3. Furthermore, next generation experiments of the galaxy survey such as DESI and Euclid will be operational within next decade. Thus we are indeed enjoying the most exciting moment in physical cosmology and we must be expecting the unexpected.

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