Journal of Physics: Conference Series **626** (2015) 012060 doi:10.1088/1742-6596/626/1/012060

Geometric phase in vacuum condensates, application to Unruh effect and to quantum thermometer

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Abstract.

We report on the recent results revealing the presence of the geometric phase in all the systems characterized by particle creation from vacuum and vacuum condensates. This fact makes the geometric phase a useful tool in the study and the understanding of disparate phenomena. Its possible application ranges from the dynamical Casimir effect to the Hawking effect, from quantum field theory in curved space to the study of CP and CPT symmetries, from the graphen physics to superconductivity and to the Bose Einstein condensate. Here, we consider the possibility of the detection of the Unruh effect and of the fabrication of a very precise quantum thermometer. We analyze the Mukunda-Simon phase for a two level atom system and consider two case: 1) atoms accelerated in electromagnetic field, and 2) atoms interacting with thermal states. The Mukunda-Simon phase generalizes the Berry phase to the case of non-cyclic and non-adiabatic evolutions; therefore it represents a more useful instrument in experimental implementations with respect to the Berry phase.

1. Introduction

Phenomena like Unruh [1], Hawking [2] and Parker effects [3, 4] and many aspects of the quantum field theory (QFT) in curved background are topics of particular interest in many field of the research. However, at the present, their detection is very difficult. In the next experiments, a key role in the attempt to reveal such phenomena could be played by the analysis of geometric phases. It has been shown that geometric phases and invariants can be used to test CPT symmetry in meson systems [5], to prove the existence of postulated particles like the axions [6], to test SUSY violation in thermal states [7], to reveal the Unruh effect [8, 9, 10] and to build a quantum thermometer [8, 11, 12]. The presence of geometric invariants has been shown also in other context [13, 14]. Here we present a more deep and general result. We show that all the systems in which vacuum condensates or particle creation from vacuum appear [15]–[20], are also characterized by the presence of the geometric phase [21]–[34] in their evolution [35]. In a previous paper [8], we have shown that in the above systems, and in general in all the phenomena represented by Bogoliubov transformations, the Aharonov–Anandan invariant (AAI) [36] is produced in their evolution. Such an invariant is however very hard to be experimentally observed. On the contrary, the geometric phase has been detected in many systems [37], [38], [39] and [40]. Therefore, the discovery of the existence of a relation between vacuum condensates and geometric phase can open a new way, based on the interferometry, to the investigation of disparate phenomena.

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We reveal the link between vacuum condensates and geometric phase [35], by using our results presented in Ref.[8], together with the results presented in [41, 42].

As example of possible application of the geometric phase produced by the systems listed above, here we study the realizability of an interferometer in which the Mukunda-Simon (MS) phase [24] can be detected in a two level system. In particular, we study the role which can be played by the MS phase in the detection of the Unruh effect and in the building of a quantum thermometer. The ideas of using geometric phases and invariants to probe the Unruh effect [8, 9, 10], and to build a thermometer [8, 11, 12] have been already presented in previous works. However our work represents a progress in several directions also in the two examples here considered. Indeed, besides the exhibition of the physically intriguing relation between boson condensation, AAI and the geometric phase, here we consider a realistic scheme for experimental implementations and a phase, the MS one which generalize the Berry phase, (used in [9, 11]) to the case of non-cyclic, non-adiabatic and non unitary evolution.

The results which we obtain are consistent with the ones presented in [9, 10] for the Unruh effect. In [10] a similar scheme, but a different definition of the geometric phase have been used [34]. The numerical results relative to the thermometer are consistent with to the ones presented in [11]. However, our approach can be used also in more wide contests.

Recently a further proposal has been presented for the realization of a quantum thermometer [12]. Such studies regard the analysis of the Berry phase and of the dynamical phase in a Bose-Einstein condensate .

In this paper, we propose to use a Mach-Zehnder interferometer in which a two level atom system with a nonunitary evolution is analyzed in two different cases. In the first case, we consider atoms interacting with an electromagnetic field, which are accelerated in one branch of the interferometer, and are inertial in the other arm. The difference between the MS phases produced in the two arms is due to the Unruh effect. In the second case, we consider atoms interacting with thermal states and we show that a very precise measurement of the temperature can be obtained by means of the MS phase.

In the following, we show the relation existing between the AAI, the geometric phase and the vacuum condensates. Then we study the possibility to use the geometric phases to detect the Unruh effect and to realize a quantum thermometer.

2. Geometric phase and AAI

Let us consider the state $|\psi\rangle$ of a quantum system and define the infinitesimal "reference distance" in the projective Hilbert space P [42],

$$dD^2 = \Big[\Big\langle \frac{d}{dt}\Big(\frac{\psi}{\|\psi\|}\Big) \Big| \frac{d}{dt}\Big(\frac{\psi}{\|\psi\|}\Big)\Big\rangle + \dot{\phi}^2 - 2i\dot{\phi}\Big\langle \frac{\psi}{\|\psi\|}\Big| \frac{d}{dt}\Big(\frac{\psi}{\|\psi\|}\Big)\Big\rangle \Big] dt^2 \,,$$

where

$$\phi(t) = \frac{i}{2}\ln\left[\frac{\langle\psi(0)|\psi(t)\rangle}{\langle\psi(t)|\psi(0)\rangle}\right] .$$

Eq.(1) is gauge invariant and reparametrization invariant [42]. In a similar way, the Fubini-Study metric can be expressed as

$$dS^2 = \Big(\Big\langle \frac{d}{dt}\Big(\frac{\psi}{\|\psi\|}\Big) \Big| \frac{d}{dt}\Big(\frac{\psi}{\|\psi\|}\Big)\Big\rangle - \Big[i\Big\langle \frac{\psi}{\|\psi\|}\Big| \frac{d}{dt}\Big(\frac{\psi}{\|\psi\|}\Big)\Big\rangle \Big]^2 \Big) dt^2 \,.$$

The total length of the S-path measured using the Fubini-Study metric differs from the AAI only by a numerical factor of two. By defining the quantity

$$d\Phi^2 = dD^2 - dS^2 \tag{1}$$

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where

$$d\Phi = \left[\left\langle \frac{\psi}{\|\psi\|}\right| i\frac{d}{dt} - \dot{\phi}(t) \left| \frac{\psi}{\|\psi\|}\right\rangle\right], \tag{2}$$

the geometric phase for an arbitrary evolution of a quantum system is then given by the difference between the D-path and the S-path in the projective Hilbert space P [42],

$$\Phi(\Gamma) = \int_\Gamma \sqrt{dD^2 - dS^2} = \int_\Gamma \left[\left\langle \frac{\psi}{\|\psi\|}\right| i\frac{d}{dt} - \dot{\phi}(t) \left| \frac{\psi}{\|\psi\|}\right\rangle\right] dt\,.$$

Such a formula matches the Aitchinson-Wanelik formula for the geometric phase and relates the Aharonov-Anandan geometric invariant (which is two times the Fubini-Study metric) to the geometric phase. Moreover, we note that, since the length of the S-path is the minimum length of the path measured by the "reference distance" function D [41, 42], the presence of the AAI in the evolution of a system implies the presence of the D invariant and consequentially the existence of the geometric phase (3). Therefore, the systems where vacuum condensates appear [16, 17, 18], for which the existence of the AAI has been revealed [8], are also characterized by the presence of the geometric phase, which can be detected much easier than the AAI.

3. Geometric phase and two level atoms

By following the discussion presented in [8, 10] and treating the atom as an open system in the reservoir of the electromagnetic field, one has a nonunitary evolution of the atom. In particular, considering the interaction of the atom with vacuum modes of the electromagnetic field in the multipolar scheme [43] and analyzing the evolution of the total density matrix in the frame of the atom (see [8] for details), one obtain the state describing the atomic system at time τ (with τ, atom proper time) [8]

$$|\phi_+(\tau)\rangle = \sin\left(\frac{\theta(\tau)}{2}\right)|+\rangle + \cos\left(\frac{\theta(\tau)}{2}\right)e^{i\Omega\tau}|-\rangle\,. \tag{3}$$

Here $\theta(\tau)$ is the angles reported in [8, 10].

We then study the Mukunda-Simon phase at time t for the state $|\phi_+(t)\rangle$. Such a geometric phase is defined as

$$\Phi(t) = \arg\langle\phi_+(0)|\phi_+(t)\rangle - \Im\int_0^t \langle\phi_+(\tau)|\dot{\phi}_+(\tau)\rangle d\tau\,; \tag{4}$$

explicitly we have

$$\Phi(t) = \arg\left[\sin\frac{\theta}{2}\sin\frac{\theta(t)}{2} + \cos\frac{\theta}{2}\cos\frac{\theta(t)}{2}e^{i\Omega t}\right] - \Omega\int_0^t \cos^2\frac{\theta(\tau)}{2}d\tau\,, \tag{5}$$

with $\theta \equiv \theta(0)$. If one consider a time interval equal to $t = 2\pi\Omega$, for which $\arg\left[\sin\frac{\theta}{2}\sin\frac{\theta(t)}{2} + \cos\frac{\theta}{2}\cos\frac{\theta(t)}{2}e^{i\Omega t}\right] = 0$, the result of Eq.(5) coincide with Eq.(15) of ref.[10], in which another definition of the geometric phase has been used [34]. Notice that the phase introduced in [34] represents the geometric phase defined for nonunitary evolution. On the other hand, the MS phase can be used for systems undergoing both a unitary evolution and a nonunitary evolution [24]. Moreover it generalizes the Berry phase to the non-cyclic and non-adiabatic evolution case, therefore, the analysis of the MS phase is advantageous also in systems such as the one presented in [9], in which the behavior of the Berry phase has been studied.

Let us now use Eq.(5) for atoms accelerated and interacting with thermal states.

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4. Unruh effect

The Unruh effect consists in the fact that for an accelerated observer the ground state of an inertial system appears at a non-zero temperature depending on the acceleration of the observer. Such a phenomenon has not yet been detected. However, the new fact is that the use of geometric phases and invariants could allow the detection of the Unruh effect in table top experiments, a strategy that has been proposed already in other physically different settings [8, 9, 10].

Here, in particular, we show the possible realizability of a Mach-Zender interferometer in which paths of slightly different lengths can be chosen in order to let the geometric phase be dominating over the relative dynamical phase. We compute the Mukunda-Simon phase for the two level system in the presence of an acceleration and in the inertial case. The atom interaction with the electromagnetic field itself produces a geometric phase, however, the difference between the two phases is due only to the atom acceleration.

For a two-level atom uniformly accelerated in the x direction with acceleration a, the function $\sin\frac{\theta(t)}{2}$ in Eq.(5) becomes

$$\sin\frac{\theta_a(t)}{2} = \pm\sqrt{\frac{1}{2} + \frac{R_a - R_a\, e^{4A_a t} + \cos\theta}{2\sqrt{e^{4A_a t}\sin^2\theta + \left(R_a - R_a\, e^{4A_a t} + \cos\theta\right)^2}}}\,, \tag{6}$$

and similar for $\cos\frac{\theta(t)}{2}$. Here $R_a = B_a/A_a$, with A_a and B_a reported in [10, 8], moreover we consider the same values of the parameter as in [8].

For an inertial atom, $a = 0$, the geometric phase $\Phi_{a=0}$ assumes the identical expression of Eq. (5), with $\sin\frac{\theta_a(t)}{2}$ and $\cos\frac{\theta_a(t)}{2}$, replaced by $\sin\frac{\theta_0(t)}{2}$ and $\cos\frac{\theta_0(t)}{2}$, in which the coefficients A_a, B_a, R_a are replaced by $A_0 = B_0 = \gamma_0/4$, with γ_0 spontaneous emission rate, and $R_0 = 1$, respectively.

The phase difference between the accelerated and inertial atoms, $\Delta\Phi_U(t) = \Phi_a(t) - \Phi_{a=0}(t)$, gives the geometric phase in terms of the acceleration of the atom. We consider an angle $\theta(0) = \pi/5$ and transition frequencies of the atom in the microwave regime, $\omega_0 \sim 10^9 s^{-1}$. We have $\Delta\Phi_U \sim 10^{-4}\pi$ for accelerations of order of $10^{17} - 10^{18} m/s^2$ and time scale of order of $10^{-10}s$. Such values are accessible with the current technology. The dynamic phases can be made negligible compared to the geometric ones, if the branches of the interferometer are built in order that the two dynamical phases are almost equal. This result can be obtained for example, for atoms with accelerations of order of $10^{17} m/s^2$ and $\omega_0 = 4\times 10^9 s^{-1}$, with a Mach-Zehnder interferometer of length of 2 cm and a difference in the branch lengths of 10^{-1}mm.

5. Quantum thermometer

We show now that a very precise thermometer could be built by means an interferometer in which the Mukunda-Simon phase is analyzed. Indeed a phase similar to that in Eq.(5) appears also in the interactions of the atom with a thermal state. In this case the coefficients A_a and B_a are replaced by the coefficients A_T and B_T depending on the temperature and reported in [8]. Thus an interferometer in which a single atom follows two different paths and interacts with two thermal states at different temperatures can represent a very precise quantum thermometer. Indeed, if it is known the temperature of one sample, the temperature of the other one can be defined by measuring the difference between the geometric phases in the two paths.

For example, if one knows the temperature T_h of the hotter source, which can be used as reference temperature, by measuring $\Delta\Phi_T$, one can derive precise estimations of the temperature T_c of the colder cavity (in [8] the AAI invariant has been analyzed, which is very hard to be detected in an interferometer).

If one considers time intervals of the order of $t = 20\times 2\pi/\omega_0$, values of ω_0 of the order of the GHz and $T_h \sim 1K$, one can measure temperatures of the cold source of $\sim$ 2 orders of magnitude below the reference temperature of the hot source. If one could use atoms with energy gaps smaller than GHz, for example considering hyperfine structure of particular atoms and lower values of the reference temperature, one can have estimation T_c very low. Also in the case of the thermometer, paths of slightly different lengths can be chosen in order to let the geometric phase be dominating over the relative dynamical phase. A similar result has been obtained in Ref.[11], where the behavior of the Berry phase has been analyzed for an atom, described as an harmonic oscillator, interacting with a scalar field in a cavity, representing the thermal source. In ref.[12] a different kind of measurement has been proposed, consisting in revealing the dynamical phase in a Bose-Einstein condensate in a Ramsey interferometer, .

6. Conclusions

We have shown that since Aharonov-Anandan invariants are present in all the phenomena where vacuum condensates appear, then these phenomena are characterized by the presence also of geometric phases. Thus, the geometric phases appear to be a useful tool in the study and the understanding of phenomena in different fields of physics (see also [5]-[6]). Here we have studied the Mukunda-Simon phase for a two level atom system. In particular, we have shown that atoms with transition frequencies in the microwave regime accelerated in an Mach-Zender interferometer of length of $2\,cm$ could represent an efficient tool in the laboratory detection of the Unruh effect. On the other hand, we have shown that a two level atom system, interacting with two different thermal states can be utilized in an interferometer to build a very precise quantum thermometer.

Acknowledgement

Partial financial support from MIUR and INFN is acknowledged.

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