Axion-photon mixing and geometric phase

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Abstract. We report on recent results revealing the presence of a geometric phase in the axion-photon mixing. In laboratory observations its detection can reveal such mixing phenomenon and therefore prove the existence of axion like particles.

1. Introduction

The studies of ultra-light and weakly coupled particles is one of the main research fields of physics beyond the Standard Model. Particular attention has been devoted to the axion [1, 2] which is a hypothetical pseudo-Nambu-Goldstone boson with no electric charge and a mass expected to range between 10^{-6} and 10^{-2} eV. Cosmological analysis have shown that axions and axion like particles can contribute to the cold dark matter if their mass is $m_a \sim 80\mu$eV [3]–[5]. Another important property of such particles is their interaction with the electromagnetic field in the presence of strong magnetic fields. Such a coupling is responsible of the mixing and oscillation with photons [6].

Many experiments have attempted, in different ways, to reveal the presence of the axion [7]–[20], however without positive results.

On the other hand, great attention have attracted the studies of the geometric phases [21]–[35]. These have been detected in the evolution of many physical systems [36]–[40] and their possible applications in particle physics [41]–[44] and cosmology [45]–[49] have also been analyzed in detail.

In this paper we report the recent result according to which the axion-photon mixing phenomenon generate a geometric phase [50]. Such a phase appears during the time evolution of photons and axions only if the mixing between these two particles is present. Indeed, it is absent when the external magnetic field vanishes and the mixing vanishes. In such cases, the total phase coincides with the dynamical one. Then the geometric phase could be used to reveal the axions produced by photons passing through a magnetic field. By means of an interferometer, one in fact could measure the difference between the geometric phases associated to two rays of light propagating in the two arms of the interferometer, one crossing a strong magnetic field B, the other one passing through a region where $B = 0$. The $B = 0$ ray has geometric phase equal to zero, then the presence of a difference between the geometric phases of the two rays represents the manifestation of the axion-photon mixing phenomenon in the $B \neq 0$ ray.

In a recent paper, it was proposed an experimental method based on the study of the difference of phase in the interferometry to detect ALPs [51]. However no consideration concerning the geometric phase has been done in [51]. Here we consider a different physical interference phenomenon occurring only if the axion-photon mixing occurs, thus revealing the core of the origin of the interference phenomenon, since it survives even in the absence of the dynamical phase difference. The geometric phase difference represents a completely new tool for the analysis of the axion-photon mixing.

In our study we neglect the contribution to the geometric phase due to the birefringence of the vacuum produced by the presence of magnetic field [52]-[54]. Such a contribution is much smaller than the one due to axion-photon mixing. In the following we recall the formalism describing the axion-photon mixing and we compute the Mukunda-Simon phase [33] associated to the time evolution of photons in the presence of the mixing with the axion.

2. Axion-photon mixing

The lagrangian density of the axion-photon system [55] is $L = L_\gamma + L_a + L_{a\gamma\gamma}$, where L_γ and L_a are the lagrangian of free photon and axion, respectively, and $L_{a\gamma\gamma} = \frac{1}{4}\, g_{a\gamma\gamma}\, \epsilon_{\mu\nu\rho\sigma}\, a\, F^{\mu\nu} F^{\rho\sigma}$, is the interaction of two photons with the axion, which is responsible of the axion-photon mixing. $g_{a\gamma\gamma} \equiv g$ is the axion-photon coupling constant, $F^{\mu\nu}$ is the electromagnetic field strength and a is the axion field.

By considering a monochromatic light beam propagating along the z-direction in the presence of an arbitrary magnetic field and choosing the y-axis along the projection of $\mathbf{B}$ perpendicular to the z-axes, the propagation equations become

$$(\omega - i\partial_z + M)\begin{pmatrix} \gamma_{\|} \\ a \end{pmatrix} = 0\,. \tag{1}$$

Here M is the mixing matrix,

$$M = -\frac{1}{2\omega}\begin{pmatrix} \omega_P^2 & -g\omega B_T \\ -g\omega B_T & m_a^2 \end{pmatrix}, \tag{2}$$

with $\mathbf{B} = \mathbf{B}_T$ transverse field, ω_P plasma frequency and m_a axion mass. M is diagonalized by a rotation,

$$\begin{pmatrix} \gamma'_{\|}(z) \\ a'(z) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \gamma_{\|}(z) \\ a(z) \end{pmatrix}, \tag{3}$$

where $\theta = \frac{1}{2}\arctan\left(\frac{2g\omega B_T}{m_a^2 - \omega_P^2}\right)$ is the mixing angle and $\gamma_{\|}(z) = \gamma_{\|}(0)e^{-i\omega_\gamma z}$, $a(z) = a(0)e^{-i\omega_a z}$, with

$$\omega_a = \omega - \frac{\mu_-}{\omega}\,, \quad \omega_\gamma = \omega - \frac{\mu_+}{\omega}\,,$$

and

$$\mu_\pm = \frac{\omega_P^2 + m_a^2}{2} \pm \frac{1}{2}\sqrt{(\omega_P^2 - m_a^2)^2 + (2g\omega B_T)^2}\,.$$

The conversion rate of photon in axion can be increased by introducing in the conversion region a buffer gas that induces a plasma frequency [56].

3. Mukunda-Simon phase and mixing

The Mukunda-Simon phase [33] for the mixed photons is

$$\Phi_\gamma(t) = \arg\langle\gamma'_{\|}(0)|\gamma'_{\|}(t)\rangle \, - \Im\int_0^t \langle\gamma'_{\|}(t')|\dot{\gamma}'_{\|}(t')\rangle dt'\,, \tag{4}$$

which explicitly is given by

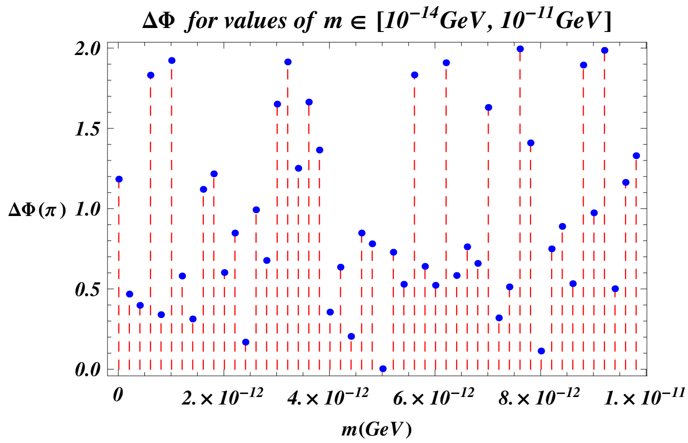

Figure 1. Plot of $\Delta\Phi = \Phi_\gamma$ as function of the axion masses m_a, for a time interval $t = 240h$ and for values of the coupling constant $g \in [10^{-12}, 10^{-10}]$ GeV^{-1} and $\omega_P = 0.9m_a$.

$$\Phi_\gamma(t) \quad = \quad \arg\left(\cos^2\theta e^{-i\omega_\gamma t} + \sin^2\theta e^{-i\omega_a t}\right) + \left(\omega_\gamma \cos^2\theta + \omega_a \sin^2\theta\right) t\,.$$

Non trivial geometric phase Φ_γ is obtained only in the presence the axion-photon mixing. Indeed, $\Phi_\gamma(t) = 0$, when $\theta = 0$.

Notice that for the estimated values of the axion masses, $m_a \in [10^{-6}, 10^{-2}]$ eV, the dynamical phases in the two arms of the interferometer, (the $B \neq 0$ branch and the $B = 0$ one), are almost equal, this implies that the difference of phase obtained in the crossing point of the two rays $\Delta\Phi$ is almost equal to the geometric phase, $\Delta\Phi \sim \Phi_\gamma$ [50].

In order to add many times the geometric phase and to make it visible, one can use suitable mirrors in the interferometer which make possible the traversing of the rays many times back and forth in the branches. In this way, one can obtain effective length of the arms arbitrarily long. Moreover, in order to enhance the sensibility of the experiment, Fabry-Perot cavity and squeezed light can be used in the interferometer [57]–[60].

We consider the following parameter for our estimations: the experiment last for a time $t = 240$ hours, the magnetic field $B = 10\,\text{T} \simeq 1.95 \times 10^{-15}\,\text{GeV}^2$, energy $\omega = 10$ eV, coupling constant and axion mass belonging to the intervals $g \in [10^{-12}, 10^{-10}]\text{GeV}^{-1}$, $m_a \in [10^{-6}, 10^{-2}]$ eV, respectively, and plasma frequency $\omega_P = 0.9m_a$.

The results obtained are: a) the geometric phase is practically independent on the experimentally accepted value of the coupling constant g; b) on the contrary, Φ_γ is strongly dependent on the value of the axion mass and it changes suddenly for different mass values, as shown in Fig. 1. This fact implies not only that the presence of a non zero geometric phase of the photon demonstrates the presence of axion-photon mixing, but also that by means of the geometric phase one can achieve a precise estimation of the value of the axion mass.

4. Conclusion

We have revealed the existence of the geometric phase in the axion-photon mixing and we have proposed an interferometric measurement to reveal the occurrence of such a phenomenon and thus the existence of axion like particles. For estimated values of the axion mass and of the coupling constant g, the difference between the phases of the two rays, the one passing in the arm with $B = 0$ and that passing through the magnetic field region, is almost completely coincident with the geometric phase produced by the mixing in the $B \neq 0$ branch.

Therefore, the detection of the geometric phase associated with the time evolution of photons propagating in a magnetic field would implies the occurrence of the axion-photon mixing. We propose then the Mukunda-Simon phase as a novel tool in the detection of such a phenomena. In the system we propose and in the range of the parameter we considered, the geometric phase is practically independent on the estimated value of the coupling constant. However it is highly sensible to the axion masses. Therefore, the geometric phase could provide the demonstration of the existence of the axions and also a value of their masses, thus allowing a more deep understanding of their contributions to the cold dark matter and to the dark energy component given by the particle mixing phenomenon [61]–[66].

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