

Emission of cosmic radio-waves, X - or γ -rays by moving unstable particles at late times

Krzysztof Urbanowski

University of Zielona Góra, Institute of Physics, ul. Prof. Z. Szafrana 4a, 65-516 Zielona Góra, Poland.

E-mail: K.Urbanowski@if.uz.zgora.pl

Abstract. A new quantum effect connected with the late time behavior of decaying states is described and its possible observational consequences are analyzed: It is shown that charged unstable particles as well as neutral unstable particles with non-zero magnetic moment which live sufficiently long may emit electromagnetic radiation. This mechanism is due to the nonclassical behavior of unstable particles at late times (at the post exponential time region). Analyzing the transition times region between exponential and non-exponential form of the survival amplitude it is found that the instantaneous energy of the unstable particle can take very large values, much larger than the energy of this state at times from the exponential time region. Based on the results obtained for the model considered, it is shown that this new purely quantum mechanical effect may be responsible for causing unstable particles produced by astrophysical sources and moving with relativistic velocities to emit electromagnetic-, X - or γ -rays at some time intervals from the transition time regions.

1. Introduction

Late time properties of unstable states arouse interest since the theoretical predictions of deviations from the exponential form of the decay law at suitably long times. The problem was that the predicted effect is almost negligible small and therefore very difficult to its confirmations in laboratory experiments. On the other hand, numbers of created unstable particles during some astrophysical processes are so large that some of them can survive up to times t at which the survival probability depending on t transforms from the exponential form into the inverse power-like form. It appears that at this time region a new quantum effect is observed: A very rapid fluctuations of the instantaneous energy of unstable particles take place. These fluctuations of the instantaneous energy should manifest itself as fluctuations of the velocity of the particle. Based on results presented in [1] we show in this report that this effect may cause unstable particles to emit electromagnetic radiation of a very wide spectrum: from radio frequencies through X -rays to γ -rays of very high energies and that the astrophysical processes are the place where this effect should be observed.

2. Preliminaries

Analyzing properties of unstable states $|\phi\rangle \in \mathcal{H}$ (where \mathcal{H} is the Hilbert space of states of the considered system) one usually starts from studies of their decay law. The decay law, $\mathcal{P}_\phi(t)$ of



an unstable state $|\phi\rangle$ decaying in vacuum is defined as follows

$$\mathcal{P}_\phi(t) = |a(t)|^2, \quad (1)$$

where $a(t)$ is the probability amplitude of finding the system at the time t in the initial state $|\phi\rangle$ prepared at time $t_0 = 0$,

$$a(t) = \langle \phi | \phi(t) \rangle. \quad (2)$$

and $|\phi(t)\rangle$ is the solution of the Schrödinger equation for the initial condition $|\phi(0)\rangle = |\phi\rangle$:

$$i\hbar \frac{\partial}{\partial t} |\phi(t)\rangle = H |\phi(t)\rangle, \quad (3)$$

where H denotes the total self-adjoint Hamiltonian for the system considered.

From basic principles of quantum theory it is known that the amplitude $a(t)$, and thus the decay law $\mathcal{P}_\phi(t)$ of the unstable state $|\phi\rangle$, are completely determined by the density of the energy distribution $\omega(\mathcal{E})$ for the system in this state [2]

$$a(t) = \int_{Spec.(H)} \omega(\mathcal{E}) e^{-\frac{i}{\hbar} \mathcal{E} t} d\mathcal{E}. \quad (4)$$

where $\omega(\mathcal{E}) \geq 0$ and $a(0) = 1$.

In [3] assuming that the spectrum of H must be bounded from below, ($Spec.(H) = [E_{min}, \infty)$ and $E_{min} > -\infty$), and using the Paley–Wiener Theorem [4] it was proved that in the case of unstable states there must be $|a(t)| \geq A \exp[-bt^q]$, for $|t| \rightarrow \infty$. Here $A > 0$, $b > 0$ and $0 < q < 1$. This means that the decay law $\mathcal{P}_\phi(t)$ of unstable states decaying in the vacuum, (1), can not be described by an exponential function of time t if time t is suitably long, $t \rightarrow \infty$, and that for these lengths of time $\mathcal{P}_\phi(t)$ tends to zero as $t \rightarrow \infty$ more slowly than any exponential function of t . This effect was confirmed in experiment described in the Rothe paper [5], where the experimental evidence of deviations from the exponential decay law at long times was reported. In the light of this result the following problem seems to be worth considering: If (and how) deviations from the exponential decay law at long times affect the energy of the unstable state and its decay rate at this time region.

Studying properties of unstable states at late times it is useful to express $a(t)$ in the following form $a(t) = a_{exp}(t) + a_{lt}(t)$, where $a_{exp}(t)$ is the exponential part of $a(t)$, that is $a_{exp}(t) = N \exp[-i\frac{t}{\hbar}(E_\phi^0 - \frac{i}{2}\Gamma_\phi^0)]$, (E_ϕ^0 is the energy of the system in the state $|\phi\rangle$ measured at the canonical decay times, i.e. when $\mathcal{P}_\phi(t)$ has the exponential form, Γ_ϕ^0 is the decay width, N is the normalization constant), and $a_{lt}(t)$ is the late time non-exponential part of $a(t)$. From the literature it is known that the characteristic feature of survival probabilities $\mathcal{P}_\phi(t)$ is the presence of sharp and frequent fluctuations at the transition times region, when contributions from $|a_{exp}(t)|^2$ and $|a_{lt}(t)|^2$ into $\mathcal{P}_\phi(t)$ are comparable and that the amplitude $a_{lt}(t)$ and thus the probability $\mathcal{P}_\phi(t)$ exhibits inverse power-law behavior at the late time region for times t much later than the crossover time T . The crossover time T can be found by solving the following equation,

$$|a_{exp}(t)|^2 = |a_{lt}(t)|^2. \quad (5)$$

In general $T \gg \tau_\phi$, where $\tau_\phi = \hbar/\Gamma_\phi^0$ is the live-time of ϕ . Formulae for T depend on the model considered (i.e. on $\omega(E)$) in general). A typical form of the decay curve, that is the form of the non-decay probability $\mathcal{P}_\phi(t)$ as a function of time t is presented in Fig. (1). The results of calculations presented in this Figure were obtained for the Breit–Wigner energy distribution function, $\omega(E) \equiv \omega_{BW}(E)$, where

$$\omega_{BW}(E) \stackrel{\text{def}}{=} \frac{N}{2\pi} \Theta(E - E_{min}) \frac{\Gamma_\phi^0}{(E - E_\phi^0)^2 + (\frac{\Gamma_\phi^0}{2})^2}, \quad (6)$$

and $\Theta(E)$ is the unit step function. The calculations leading to graphics presented in Fig. (1) were performed for $s_0 \stackrel{\text{def}}{=} (E_\phi^0 - E_{\min})/\Gamma_\phi^0 = 20$.

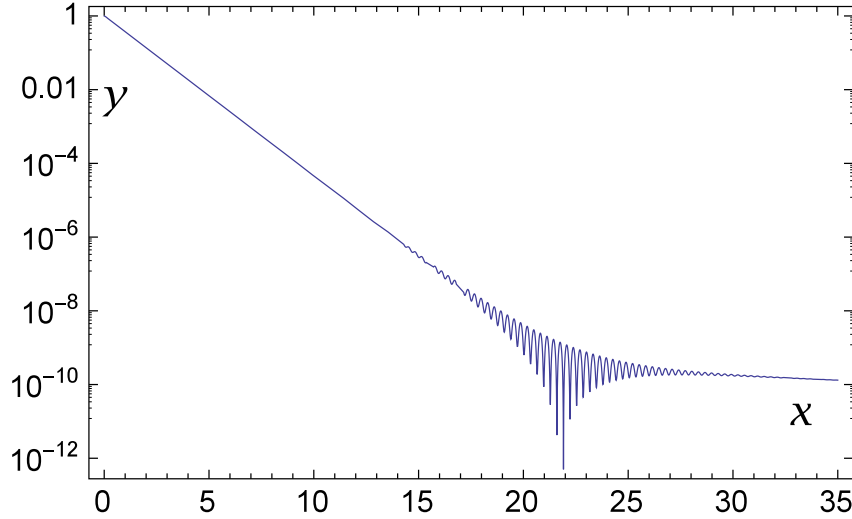


Figure 1. The general, typical form of the decay curve $\mathcal{P}_\phi(t)$. Axes: $x = t/\tau_\phi$, $y = \mathcal{P}_\phi(t)$ — the logarithmic scale.

3. Late times and energy of unstable states

The energy and the decay rate of the system in the state $|\phi\rangle$ under considerations, (to be more precise the instantaneous energy and the instantaneous decay rate), can be found using the following relations (for details see [6, 7, 8]).

$$\mathcal{E}_\phi \equiv \mathcal{E}_\phi(t) = \Re(h_\phi(t)), \quad \gamma_\phi \equiv \gamma_\phi(t) = -2 \Im(h_\phi(t)), \quad (7)$$

where $\Re(z)$ and $\Im(z)$ denote the real and imaginary parts of z respectively and $h_\phi(t)$ denotes the "effective Hamiltonian" for the one-dimensional subspace of states \mathcal{H}_\parallel spanned by the normalized vector $|\phi\rangle$,

$$h_\phi(t) \stackrel{\text{def}}{=} i\hbar \frac{\partial a(t)}{\partial t} \frac{1}{a(t)}. \quad (8)$$

It is easy to show that equivalently

$$h_\phi(t) \equiv \frac{\langle \phi | H | \phi(t) \rangle}{\langle \phi | \phi(t) \rangle}. \quad (9)$$

There is $\mathcal{E}_\phi(t) = E_\phi^0$ and $\Gamma_\phi(t) = \Gamma_\phi^0$ at the canonical decay times and at asymptotically late times [1, 6, 8, 9]

$$\mathcal{E}_\phi(t) \simeq E_{\min} + \frac{c_2}{t^2} + \frac{c_4}{t^4} \dots, \quad (\text{for } t \gg T), \quad (10)$$

$$\Gamma_\phi(t) \simeq \frac{c_1}{t} + \frac{c_3}{t^3} + \dots \quad (\text{for } t \gg T), \quad (11)$$

where $c_i = c_i^*$, $i = 1, 2, \dots$, ($c_1 > 0$ and the sign of c_i for $i \geq 2$ depends on the model considered), so $\lim_{t \rightarrow \infty} \mathcal{E}_\phi(t) = E_{min}$ and $\lim_{t \rightarrow \infty} \Gamma_\phi(t) = 0$. Results (10) and (11) are rigorous.

The basic physical factor forcing the amplitude $a(t)$ to exhibit inverse power law behavior at $t \gg T$ is the boundedness from below of $\sigma(H)$. This means that if this condition is satisfied and $\int_{-\infty}^{+\infty} \omega(E) dE \equiv \int_{E_{min}}^{+\infty} \omega(E) dE < \infty$, then all properties of $a(t)$, including the form of the time-dependence at times $t \gg T$, are the mathematical consequence of them both. The same applies by (8) to the properties of $h_\phi(t)$ and concerns the asymptotic form of $h_\phi(t)$ and thus of $\mathcal{E}_\phi(t)$ and $\Gamma_\phi(t)$ at $t \gg T$.

Using relations (2), (4), (8) and assuming the form of $\omega(E)$ and performing all necessary calculations numerically one can find $\mathcal{E}_\phi(t)$ for all times t including times $t \gg T$. A typical behavior of the instantaneous energy $\mathcal{E}_\phi(t)$ at the transition time region is presented in Fig (2). In this Figure calculations were performed for the Breit–Wigner energy distribution function (6) and function $\kappa(t)$ is defined as follows

$$\kappa(t) = \frac{\mathcal{E}_\phi(t) - E_{min}}{E_\phi^0 - E_{min}}. \quad (12)$$

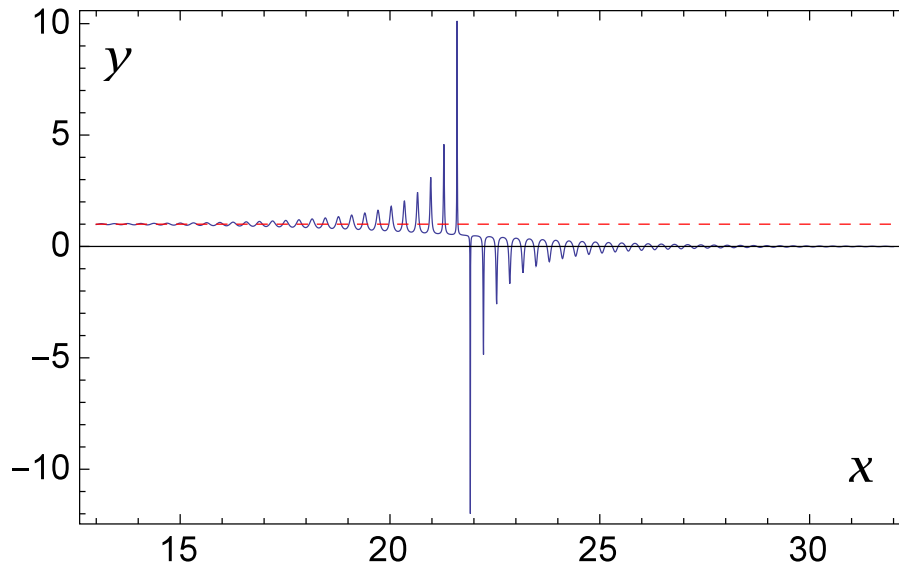


Figure 2. The instantaneous energy $\mathcal{E}_\phi(t)$ in the transitions time region: The case $s_0 = 20$. Axes: $y = \kappa(t)$, $x = t/\tau_\phi$. The dashed line denotes the straight line $y = 1$.

As it can be seen in Fig. (1) the characteristic feature decay curves is the presence of sharp and frequent oscillations at the transition times region. This means that derivatives of the amplitude $a(t)$ may reach extremely large values for some times from the transition time region and the modulus of these derivatives is much larger than the modulus of $a(t)$, which is very small for these times. This explains why in this time region the real and imaginary parts of $h_\phi(t) \equiv \mathcal{E}_\phi(t) - \frac{i}{2} \gamma_\phi(t)$, which can be expressed by the relation (8), ie. by a large derivative of $a(t)$ divided by a very small $a(t)$, reach values much larger than the energy \mathcal{E}_ϕ^0 of the the unstable state measured at times for which the decay curve has the exponential form.

4. Possible observable effects

From the point of view of a frame of reference in which the time evolution of the unstable system was calculated the Rothe experiment as well as the picture presented in Figs (1), (2) refer to the rest coordinate system of the unstable system considered.

Properties of the ratio $\kappa(t)$ taking place for some time intervals mean that the instantaneous energy $\mathcal{E}_\phi(t)$ at these time intervals differs from the energy E_ϕ^0 measured at times from the exponential decay time region (i.e. at the canonical decay times). The relation (9) explains why such an effect can occur. Indeed, if to rewrite the numerator of the righthand side of (9) as follows,

$$\langle \phi | H | \phi(t) \rangle \equiv \langle \phi | H | \phi \rangle a(t) + \langle \phi | H | \phi(t) \rangle_\perp, \quad (13)$$

where $|\phi(t)\rangle_\perp = Q|\phi(t)\rangle$, $Q = \mathbb{I} - P$ is the projector onto the subspace of decay products, $P = |\phi\rangle\langle\phi|$ and $\langle\phi|\phi(t)\rangle_\perp = 0$, then one can see that decay products described by $|\phi(t)\rangle_\perp$ contribute permanently to the energy of the unstable state considered with the intensity depending on time t . This contribution into the instantaneous energy is practically constant in time at canonical decay times whereas at the transition times, when $t \sim T$, it is fluctuating function of time and the amplitude of these fluctuations may be significant.

From astronomical observations it is known that there are sources of unstable particles in space that emit them with relativistic or ultra-relativistic velocities in relation to an external observer so many of these particles move in space with ultra high energies. In such cases one has to consider the following problem: What effects can be observed by an external observer when the unstable particle, say ϕ , which survived up to the transition times region, $t \sim T$, or longer is moving with a relativistic velocity in relation to this observer. The distance d from the source reached by this particle is of order $d \sim d_T$, where $d_T = v^\phi \cdot T'$ and $T' = \gamma T$, $\gamma \equiv \gamma(v^\phi) = (\sqrt{1 - \beta^2})^{-1}$, $\beta = v^\phi/c$, v^ϕ is the velocity of the particle ϕ . (For simplicity we assume that there is a frame of reference common for the source and observer both and that they do not move with respect to this frame of reference). So, in the case of moving particles created in astrophysical processes one should consider the effect shown in Fig (2) together with the fact that the particle gains extremely huge energy, W^ϕ , which have to be conserved.

Let us assume that the unstable particle under considerations has a constant momentum \vec{p} , $|\vec{p}| = p > 0$. Next, let us assume for simplicity that $\vec{v} = (v_1, 0, 0) \equiv (v^\phi, 0, 0)$ then there is $\vec{p} = (p, 0, 0)$. Now, let Λ_p be the Lorentz transformation from the reference frame \mathcal{O} , where the momentum of unstable particle considered is zero, $\vec{p} = 0$, into the frame \mathcal{O}' where the momentum of this particle is $\vec{p} \equiv (p, 0, 0) \neq 0$ or, equivalently, where its velocity \vec{v}^ϕ equals $\vec{v}^\phi = \vec{v}_p^\phi \equiv \vec{p}/m\gamma$, and let E be the energy of the particle in the reference frame \mathcal{O} and E' be the energy of the moving particle and having the momentum $\vec{p} \neq 0$. In this case the corresponding 4-vectors are: $\wp = (E/c, 0, 0, 0) \in \mathcal{O}$ and $\wp' = (E'/c, p, 0, 0) = \Lambda_p \wp \in \mathcal{O}'$. There is $\wp' \cdot \wp' \equiv (\Lambda_p \wp) \cdot (\Lambda_p \wp) = \wp \cdot \wp$ in Minkowski space, which is an effect of the Lorentz invariance. (Here the dot “ \cdot ” denotes the scalar product in Minkowski space). Hence, in our case: $\wp' \cdot \wp' \equiv (E'/c)^2 - p^2 = (E/c)^2$ because $\wp \cdot \wp \equiv (E/c)^2$ and thus $(E')^2 = c^2 p^2 + E^2$. One can show that in the case of a free particle $p \equiv \frac{E}{c} \gamma \beta$. Therefore $c^2 p^2 + E^2 \equiv E^2 \gamma^2 \beta^2 + E^2 = E^2 (1 + \beta^2 \gamma^2) \equiv E^2 \gamma^2$. The last relation means that the energy, E' , of moving particles equals

$$E' = \sqrt{E^2 + c^2 p^2} \equiv E \gamma \stackrel{\text{def}}{=} W^\phi. \quad (14)$$

So in the case of the instantaneous energy, $\mathcal{E}_\phi(t)$ considered earlier we can write that

$$W^\phi \equiv \mathcal{E}_\phi(t_k) \gamma_k = W^\phi(t_k), \quad (15)$$

at the instant t_k (where $\gamma_k = \gamma(t_k)$).

We have $\mathcal{E}_\phi(t) \equiv E_\phi^0$ at canonical decay times. Thus, $W^\phi \equiv E_\phi^0 \gamma$, at these times (where $\gamma = \gamma(t \sim \tau_\phi)$) and at times $t \gg \tau_\phi$, $t \sim T$ we have $\mathcal{E}_\phi(t) \neq E_\phi^0$. Invoking now the principle of conservation of energy, we can conclude that in the case illustrated in Fig (3), we have

$$W_1^\phi = W_2^\phi = W_3^\phi, \quad (16)$$

where $W_i^\phi = W^\phi(t_i)$.

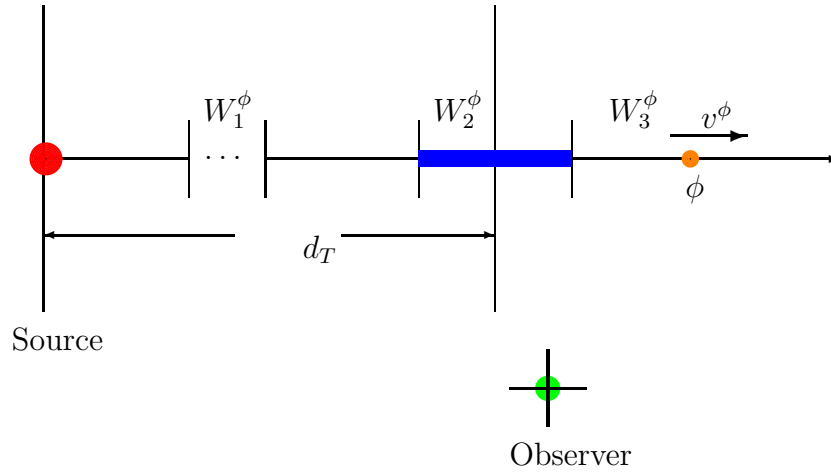


Figure 3. Here $d_T = v^\phi \cdot T'$, $T' = \gamma T$, $W_i^\phi = W^\phi(t_i)$, ($i = 1, 2, 3$) and $W_i^\phi(t_i)$ is the energy of moving relativistic particle ϕ , $t_1 \ll t_2 \ll t_3$, $W^\phi(t_i) = \mathcal{E}_\phi(t_i) \gamma(t_i)$, $\gamma = \gamma(t) = (\sqrt{1 - \beta^2})^{-1}$, $\beta = v^\phi(t)/c$.

The most interesting relation can be obtained from (16) for $t_1 \ll T'$ and $t_2 \sim T'$, (where $T' = \gamma T$): $W_1^\phi \equiv E_\phi^0 \gamma = W_2^\phi$, or the equivalent one,

$$E_\phi^0 \gamma_1 \equiv \mathcal{E}_\phi(t_2) \gamma(t_2). \quad (17)$$

It is convenient to rewrite the last relation as follows,

$$\gamma_1 = \frac{\mathcal{E}_\phi(t_2)}{E_\phi^0} \gamma(t_2). \quad (18)$$

The ratio $\frac{\mathcal{E}_\phi(t_2)}{E_\phi^0}$ can be extracted from Fig (2):

$$\frac{\mathcal{E}_\phi(t)}{E_\phi^0} = 1 + (\kappa(t) - 1) \frac{E_\phi^0 - E_{min}}{E_\phi^0}. \quad (19)$$

Thus if $\kappa(t) \neq 1$ then $\gamma_2 \neq \gamma_1$, which means that $v_2^\phi \neq v_1^\phi$ or that $a_{average} = \frac{v_2^\phi - v_1^\phi}{t_2 - t_1} \neq 0$. So, the moving charged unstable particle ϕ has to emit electromagnetic radiation at the transition time region. The same concerns neutral unstable particles with non-zero magnetic moment. The energy of the electromagnetic radiation emitted by such a charged particle in a unit of time can be found using the Larmor formula:

$$P = \frac{1}{6\pi\epsilon_0} \frac{q^2 \dot{v}^\phi{}^2}{c^2} \gamma^6, \quad (20)$$

where P is total radiation power, q is the electric charge, ϵ_0 – permittivity for free space. The known analogy from classical physics is the following one: A conservation of the angular momentum and a pirouette like effect.

5. Some estimations

From the results presented in Fig (2) one can find coordinates of, e.g., the highest maximum: $(x_{mx}, y_{mx}) = (21.60, 10.27)$. They allow one to find that $\kappa(t_{mx}) = y_{mx} = 10.27$. Coordinates of points of the intersection of this maximum with the straight line $y = 1$ are equal: $(x_1, y_1) = (21.58, 1.0)$ and $(x_2, y_2) = (21.62, 1.0)$. Thus $\Delta x = x_2 - x_1 = 0.04$. (Here $x = t/\tau_\phi$), and this Δx allows one to find $\Delta t = t_{mx} - t_1$. Let us consider now μ meson as an example. In such a case: $E_\mu^0 - E_{min} = m_{\mu^\pm} - (m_e + m_{\bar{\nu}_e} + m_{\nu_\mu}) \simeq 105$ [MeV], $(\tau_\mu = 2, 198 \times 10^{-6}[\text{s}])$ and the crossover time equals: $T^\mu \simeq 165\tau_\mu = 0, 37 \times 10^{-3}[\text{s}]$, [1]). Using these data one finds that $\gamma(t_1) \simeq 10.21\gamma(t_{mx})$. From this relation one can extract Δv^μ . Next using such obtained Δv^μ and $\Delta t = t_{mx} - t_1$ extracted from Fig (2) one can use the Larmor formula (20) and to calculate the total radiation power P . The result is $P \sim 4.6$ [eV/s].

Astrophysical and cosmological processes in which extremely huge numbers of unstable particles are created seem to be a possibility for the above discussed effect to become manifest. The fact is that the probability $\mathcal{P}_\phi(t) = |a(t)|^2$ that an unstable particle ϕ survives up to time $t \sim T$ is extremely small. According to estimations of the luminosity of some γ -rays sources the energy emitted by these sources can even reach a value of order 10^{52} [erg/s], and it is only a part of the total energy produced there. If to consider a source emitting energy 10^{50} [erg/s] then, eg., an emission of $\mathcal{N}_0 \simeq 6.25 \times 10^{47}$ [1/s] particles of energy 10^{18} [eV] is energetically allowed. The same source can emit $\mathcal{N}_0 \simeq 6.25 \times 10^{56}$ [1/s] particles of energy 10^9 [eV] and so on.

For the model defined by $\omega_{BW}(E)$ the cross-over time T is given by the following approximate relation (valid for $E_\phi^0/\Gamma_\phi^0 \gg 1$),

$$\Gamma_\phi^0 T \equiv \frac{T}{\tau_\phi} \sim 2 \ln \left[2\pi \left(\frac{E_\phi^0 - E_{min}}{\Gamma_\phi^0} \right)^2 \right], \quad (21)$$

(see, eg. [6, 8, 10]). As it was mentioned, e.g. in [11], a typical value of the ratio $(E_\phi^0 - E_{min})/\Gamma_\phi^0$ is $(E_\phi^0 - E_{min})/\Gamma_\phi^0 \geq O(10^3 - 10^6)$. Taking $(E_\phi^0 - E_{min})/\Gamma_\phi^0 = 10^6$ one obtains from (21) that $\mathcal{N}_\phi(T) \equiv \mathcal{P}_\phi(T)\mathcal{N}_0 \sim 2.53 \times 10^{-26}\mathcal{N}_0$. So there are $\mathcal{N}_\phi(T) \sim 14 \times 10^{21}$ particles per second of energy $W^\phi = 10^{18}$ [eV] or $\mathcal{N}_\phi(T) \sim 14 \times 10^{30}$ particles of energy $W^\phi = 10^9$ [eV] in the case of the considered example and T calculated using (21).

6. Conclusions

- Fluctuations of the instantaneous energy, $\mathcal{E}_\phi(t)$, of the moving unstable particles at transition times region, $t \sim T$, (where T is the crossover time — see (5) and (21)) cause fluctuations of the velocity of these particles at this time region, which forces the charged particles to emit electromagnetic radiation.
- Similarly to the charged particle, a moving neutral unstable particle with non-zero magnetic moment have to emit electromagnetic radiation in the transition time region.
- Ultra relativistic unstable charged particles or unstable particles with non-zero magnetic moment can emit X - or γ -rays in the transition time region.
- Astrophysical sources are able to create such numbers \mathcal{N}_0 of unstable particles that sufficiently large number $\mathcal{N}_\phi(T) \gg 1$ of them has to survive up to times T and therefore to emit electromagnetic radiation at transition times. The expected spectrum of this radiation can be very wide: From radio frequencies up to γ -rays.

Acknowledgments

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