

Proposing new experiments to test the quantum-to-classical transition

M Bahrami and A Bassi

Department of Physics, University of Trieste, Strada Costiera 11, 34014 Trieste, Italy
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Via Valerio 2, 34127 Trieste

E-mail: Mohammad.Bahrami@ts.infn.it; Bassi@ts.infn.it

Abstract. An open problem in modern physics is why microscopic quantum objects can be at two places at once (i.e. a superposed quantum state) while macroscopic classical object never show such a behaviour. Collapse models provides a quantitative answer for this problem and explain how macroscopic classical world emerges out of microscopic quantum world. A universal noise field is postulated in collapse models, inducing appropriate Brownian-motion corrections to standard quantum dynamics. The strength of collapse-driven Brownian fluctuations depend on: (i) the parameters characterizing the system (e.g., mass, size, density), and (ii) two phenomenological parameters defining the statistical properties of the collapse noise. The collapse-driven Brownian motion works such that microscopic systems behave quantum mechanically, while macroscopic objects are classical. At the intermediate mesoscopic scale, collapse models predict deviations from standard quantum predictions. This issue has been subject of experimental tests. All experiments to date have been at the scales where collapse effects are negligible for all practical purposes. However, recent experimental progress in revealing quantum features of larger objects, increases the hope for testing at unprecedented scales where collapse models can be falsified. Current experiments are mainly focused on the preparation of macroscopic systems in a spatial quantum superposition state. The collapse effects would then manifest as loss of visibility in the observed interference pattern. However, one needs a quantum interference with single particles of mass $\sim 10^{10}$ amu for a decisive test of collapse models. Creating such massive superpositions is quite challenging, and beyond current state-of-the-art. Quite recently, an alternative approach has been proposed where the collapse manifests in the fluctuating properties of light interacting with the quantum system. The great advantage of this new approach is that here there is no need for the preparation of a quantum superposed state. It has been discussed that promising results can be revealed in the spectrum of light interacting with a radiation pressure-driven mechanical oscillator in a cavity optomechanics setting. Here, we review the theoretical modelling of the above optomechanical proposal. We discuss how collapse-driven Brownian motion modifies the spectrum. We quantify the collapse effect and explain how it depends on the parameters of the mechanical oscillator (e.g., mass, density, geometry).

1. Introduction

Physics at the quantum-classical border is one of the crucial fields of today's research, both theoretical and experimental [1, 3, 2, 4, 5, 6, 7]. One of the challenging questions is to understand if, and under which conditions, quantum linearity fails when the size and complexity of the system increases. The exploration of this question has substantial consequences because if the quantum superposition principle fails beyond a certain scale (e.g., a mass scale), then it



necessitates the modification of quantum theory [8, 9]. This might result in a correct scenario for unifying quantum theory with gravity [10].

Collapse models are one possible way to modify the standard quantum theory in a fully-consistent way [8, 9, 11, 12, 13, 14, 15, 16]. Collapse model assumes a universal noise field that, when acting on matter, introduces non-linear effects on the dynamics, which explain the collapse of the wave function. In other words, one can derive the random localization of the wave function at the end of a measurement, of course with the correct quantum probabilities. The strength of the collapse process scales with the size of the system, thus the wave function of microscopic systems can be superimposed, while macroscopic objects are always well-localized. To summarize, the collapse noise breaks the validity of the quantum superposition principle beyond a certain scale, and explains how the classical macroscopic world emerges from the quantum microscopic one.

Recently, there has been rapid experimental progress in revealing quantum features such as particle-wave duality for large objects with tiny de Broglie wavelength of only a few hundred femtometer [17]. Such objects were successfully decoupled from environmental noises, thus overcoming the technological limit and thereby extending the realm of quantum theory to new regimes. This progress provides the possibility to search for test of collapse models, as well. In this regard, quite a few new experimental schemes have been proposed [9]. All these proposals are based on the natural idea of creating a macroscopic quantum superposition in space, in order to test the superposition principle. Creating macroscopic superpositions is very difficult, source of formidable technological challenges.

Quite recently, an alternative approach has been proposed, based on measuring light emitted by radiative transitions of excited states of matter [18]. Here, the collapse manifests as an extra broadening and shift in lineshapes of the spectral density. The most important advantage of this new approach is that here there is no need for the preparation of a quantum superposed state. Later, it was shown that more promising results can be revealed in the spectrum of light interacting with a radiation pressure-driven mechanical oscillator in a cavity optomechanical setting [19]. The collapse results in an increase of the intensity of the noise spectrum of the light driving the mechanical oscillator. A Similar result has been obtained in [20].

In this paper, we will review the theoretical modelling of the above optomechanical proposal. The structure of this paper is as follows. We first derive the collapse dynamical equation for the center-of-mass of a complex system. We use the mass-proportional Continuous Spontaneous Localization (CSL) model [15], which is the most-studied collapse model in the literature. Then in the narrow wave function limit, we discuss how statistical effects of the CSL collapse noise can be reproduced by adding a linear random potential into the Schrödinger equation. We discuss how this random potential modifies the dynamic of an optomechanical oscillator. In our setting, the oscillator is the moving mirror of a Fabry-Perot cavity, that couples to an external laser field and is immersed in a finite-temperature bath. Therefore, the noise sources are: the thermal-driven Brownian motion of the oscillator interacting with the bath, the input laser noise and the CSL collapse noise. We use the Langevin formalism to account for the dynamics of the oscillator, where the above noises are introduced by using suitable noise operators. Any observable quantity is obtained after averaging over the noises. We deduce the thermal-driven Brownian noise- and the input laser noise-operators by using standard techniques of open quantum systems theory. In this way we derive an expression for the spectrum of the output light, which shows modifications due to all noises, in particular from the CSL collapse noise. We finally discuss how the collapse frequency scales with the size and the shape of the mechanical oscillator.

2. The collapse effect on the center-of-mass motion

In this section, we show how the motion of center-of-mass is modified due to the collapse noise that interacts with a many-body system, as prescribed in collapse models. We shall focus on

the mass proportional Continuous Spontaneous Localization (CSL) collapse model, which is the most studied collapse model in the literature. The CSL evolution of the wave function reads [15]:

$$d|\Psi_t\rangle = \left[-\frac{i}{\hbar} \hat{H}_0 dt + \sqrt{\gamma} \int d^3\mathbf{x} \left(\hat{L}(\mathbf{x}) - \langle \hat{L}(\mathbf{x}) \rangle \right) dW(t, \mathbf{x}) - \frac{\gamma}{2} \int d^3\mathbf{x} \left(\hat{L}(\mathbf{x}) - \langle \hat{L}(\mathbf{x}) \rangle \right)^2 dt \right] |\Psi_t\rangle, \quad (1)$$

which is a non-linear stochastic differential equation in Itô form. Here $\gamma \simeq 10^{-28} \text{ m}^3 \cdot \text{s}^{-1}$ is the localization parameter which is a new phenomenological parameter of collapse models, $W(t, \mathbf{x})$ is standard Wiener process giving a noise $\xi(t, \mathbf{x}) = dW(t, \mathbf{x})/dt$, which is white both in space and time (i.e. $\mathbb{E}(\xi(t, \mathbf{x})) = 0$ and $\mathbb{E}(\xi(t_1, \mathbf{x})\xi(t_2, \mathbf{y})) = \delta(t_1 - t_2) \delta(\mathbf{x} - \mathbf{y})$, with $\mathbb{E}(\dots)$ the stochastic average), $\langle \hat{L}(\mathbf{x}) \rangle = \langle \Psi_t | \hat{L}(\mathbf{x}) | \Psi_t \rangle$ which induces the nonlinearity in the dynamics, and $\hat{L}(\mathbf{x})$ is the coarse-grained mass density operator:

$$\hat{L}(\mathbf{x}) = \int d^3\mathbf{y} g(\mathbf{x} - \mathbf{y}) \sum_j \frac{m_j}{m_0} \sum_s \hat{a}_j^\dagger(s, \mathbf{y}) \hat{a}_j(s, \mathbf{y}), \quad (2)$$

where $m_0 = 1 \text{ amu}$, $\hat{a}_j(s, \mathbf{y})$ is the annihilation operator of particle of type- j with mass m_j and the spin s at position \mathbf{y} ; and

$$g(\mathbf{r}) = \exp(-\mathbf{r}^2/2r_C^2)/(\sqrt{2\pi}r_C)^3, \quad (3)$$

with $r_C \simeq 10^{-7} \text{ m}$ the correlation length, which is the second phenomenological parameter of collapse models. In the CSL model a system is well-localized when its position spread is smaller than r_C .

Taking the system as a rigid body and averaging over the relative coordinates, we can derive the CSL motion for the center-of-mass, where the spread of center-of-mass's position is smaller than the CSL correlation length $r_C = 10^{-7} \text{ m}$. Since we work in the non-relativistic regime, in Eq. (2) the index j runs over electrons and nucleons where the number of particles is also a constant of motion. Since masses are divided by the mass of a nucleon m_0 (see Eq.(2)), we can also neglect electrons. Accordingly, in the subspace with a fixed number of particles, we can write: $\hat{L}(\mathbf{x}) \approx \sum_{k=1}^N A_k g(\mathbf{x} - \hat{\mathbf{x}}_k)$, where N is the number of atomic nuclei, A_k is the atomic mass number, and $\hat{\mathbf{x}}_k$ is the nuclear position. We assume that the system is a rigid body where nuclei have a fixed relative distance from the center-of-mass. We also assume that the spatial fluctuating around the equilibrium relative coordinates are much smaller than r_C , implying that relative coordinates are classical variables with their equilibrium values. Accordingly, the Lindblad operator reads as:

$$\hat{L}(\mathbf{x}) \approx \hat{L}_{\text{com}}(\mathbf{x}) = \sum_{k=1}^N A_k g(\mathbf{x} - \mathbf{r}_k - \hat{\mathbf{q}}) = \int d^3\mathbf{r} \varrho(\mathbf{r}) g(\mathbf{x} - \mathbf{r} - \hat{\mathbf{q}}), \quad (4)$$

with $\hat{\mathbf{q}}$ the center-of-mass position operator, \mathbf{r}_k the equilibrium position of k -th nucleus, and $\varrho(\mathbf{r})$ the number density of the system where the origin is the center-of-mass: $\varrho(\mathbf{r}) = \sum_{k=1}^N (m_k/m_0) \delta(\mathbf{r} - \mathbf{r}_k)$. If we assume that the system is in a separable state $|\Psi\rangle = |\psi\rangle \otimes |\phi_{\text{rel}}\rangle$ with $|\psi\rangle$ the center-of-mass state and $|\phi_{\text{rel}}\rangle$ the relative coordinates state, then the CSL dynamics for the center-of-mass reads as:

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} \hat{H}_{\text{com}} dt + \sqrt{\gamma} \int d^3\mathbf{x} \left(\hat{L}_{\text{com}}(\mathbf{x}) - \langle \hat{L}_{\text{com}}(\mathbf{x}) \rangle \right) dW(t, \mathbf{x}) - \frac{\gamma}{2} \int d^3\mathbf{x} \left(\hat{L}_{\text{com}}(\mathbf{x}) - \langle \hat{L}_{\text{com}}(\mathbf{x}) \rangle \right)^2 dt \right] |\psi_t\rangle, \quad (5)$$

where \hat{H}_{com} contain only the center-of-mass degrees of freedom. In fact, we have: $\hat{H} = \hat{H}_{\text{com}} + \hat{H}_{\text{rel}}$ where \hat{H}_{rel} is the Hamiltonian of relative coordinates and $[\hat{H}_{\text{com}}, \hat{H}_{\text{rel}}] = 0$. In other words, the CSL collapse field only acts on the center-of-mass and the relative coordinates follow standard quantum dynamics determined by the Hamiltonian \hat{H}_{rel} . For cases where the center-of-mass is distributed around a time-dependent mean value $\langle \hat{\mathbf{q}} \rangle = \langle \psi(t) | \hat{\mathbf{q}} | \psi(t) \rangle$ with the spread much smaller than r_C , one can Taylor-expand $g(\mathbf{x} - \mathbf{r} - \hat{\mathbf{q}})$ to the first order around $\langle \hat{\mathbf{q}} \rangle$. Accordingly, $\hat{L}_{\text{com}}(\mathbf{x})$ can be approximated as follows:

$$\hat{L}_{\text{com}}(\mathbf{x}) \approx \int d^3\mathbf{r} \varrho(\mathbf{r}) g(\mathbf{x} - \mathbf{r} - \langle \hat{\mathbf{q}} \rangle) \left(1 + \frac{1}{r_C^2} (\mathbf{x} - \mathbf{r} - \langle \hat{\mathbf{q}} \rangle) \cdot (\hat{\mathbf{q}} - \langle \hat{\mathbf{q}} \rangle) \right). \quad (6)$$

Putting above equation into Eq.(5) and doing some lengthy calculations, one eventually obtain:

$$\frac{d}{dt} |\psi_t\rangle = \left[-\frac{i}{\hbar} \hat{H}_{\text{com}} + \sqrt{\gamma} \sum_{k=1}^3 (\hat{q}_k - \langle \hat{q}_k \rangle) dW_k(t) - \frac{\gamma}{2} \sum_{k,l=1}^3 \eta_{k,l} (\hat{q}_k - \langle \hat{q}_k \rangle) (\hat{q}_l - \langle \hat{q}_l \rangle) \right] |\psi_t\rangle \quad (7)$$

where $\hat{H}_{\mathbf{q}}$ is the standard quantum Hamiltonian of the center-of-mass,

$$\eta_{k,l} = \iint d^3\mathbf{r} d^3\mathbf{r}' \frac{\exp\left(-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4r_C^2}\right)}{(2\sqrt{\pi}r_C)^3} \frac{\partial \varrho(\mathbf{r})}{\partial r_k} \frac{\partial \varrho(\mathbf{r}')}{\partial r'_l}, \quad (8)$$

and $w_k(t) = dW_k(t)/dt$ are white noises:

$$w_k(t) = \int d^3\mathbf{x} \xi(t, \mathbf{x}) \int d^3\mathbf{r} g(\mathbf{x} - \mathbf{r} - \langle \hat{\mathbf{q}} \rangle_{\Psi_t}) \frac{\partial \varrho(\mathbf{r})}{\partial r_k}, \quad (9)$$

with a zero mean (i.e. $\mathbb{E}(w_k(t)) = 0$) and correlation: $\mathbb{E}(w_k(t) w_l(s)) = \delta(t-s) \eta_{k,l}$. For an object with cuboid, or disc, or spherical geometry, one finds $\eta_{k,l} = \delta_{kl} \eta_{k,k}$ which implies that white noises $w_k(t)$ are also independent. Henceforth, we will consider objects which are cuboid, or disc, or spherical.

Instead of working with the stochastic *nonlinear* dynamics given in Eq.(7), we work with the Schrödinger equation with a stochastic potential as follows:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = (\hat{H}_{\text{com}} + \hat{V}(t)) |\psi(t)\rangle, \quad \hat{V}(t) = -\hbar \sqrt{\gamma} \sum_{k=1}^3 \hat{q}_k w_k(t), \quad (10)$$

which is a linear stochastic differential equation in Stratonovich form. As discussed several times in the literature [22, 23, 24, 25, 26], the effects of nonlinear terms in Eq.(7), at the statistical level, can be mimicked also by linear random potentials. For individual realizations of the noise, the effects are very different (those of a linear dynamics vs. those of a nonlinear one), while at the statistical level they coincide, if the potential is suitably chosen.

3. Quantum Langevin equations for an optomechanical oscillator

We now discuss how the collapse mechanism modifies the dynamical equation for an optomechanical system. The approximations we used in this section are all standard approximations in optomechanics (e.g., see Ref. [27]). In fact, the final quantum Langevin equations are the standard ones in optomechanics and are only modified by a noise term induced by the collapse noise. As discussed before, to the extent that one is only concerned with the statistical observable effects, the collapse can be included as a random linear potential $V(t)$

as given in Eq.(10). We shall denote the position operator along the cavity axis by \hat{q} , and its conjugate momentum by \hat{p} . We will consider the cavity axis along the x -axis. We shall denote the components of the Hamiltonian and of the noise term along the cavity axis by the subscript q (see Eqs.(11,12)). We also assume that the Hamiltonian of the optomechanical system, in the rotating frame of the laser frequency, is given by [27]:

$$\hat{H}_q = \hbar(\omega_c - \omega_l)\hat{a}^\dagger\hat{a} - \hbar\chi\hat{a}^\dagger\hat{a}\hat{q} + \frac{1}{2}m\omega_m^2\hat{q}^2 + \frac{1}{2m}\hat{p}^2 + i\hbar\varepsilon(\hat{a}^\dagger - \hat{a}), \quad (11)$$

where m is the effective mass of the moving mirror (i.e., the mass of the mechanical oscillator), ω_l is the frequency of the external laser, ω_c is the frequency of the cavity mode driven by the laser, ω_m is the harmonic frequency of the mechanical oscillator, $\chi = \omega_c/L$ is the optomechanical coupling constant between the cavity and the mechanical oscillator with L the length of the cavity, and $\varepsilon = \sqrt{2\kappa\mathcal{P}}/\hbar\omega_l$ with \mathcal{P} the laser power and κ the cavity photon decay rate. Along the cavity axis, the random potential is given by:

$$V_q(t) = -\hbar\sqrt{\eta}w(t)\hat{q}, \quad (12)$$

where $\eta = \gamma\eta_{1,1}$ and $w(t)$ is a white noise with zero mean (i.e. $\mathbb{E}(w(t)) = 0$) and correlation $\mathbb{E}(w(t)w(s)) = \delta(t-s)$. Notice that one can separate the motion along different axes, only for the case where white noises in Eq.(9) are independent (e.g. for a cuboid object).

For convenience, we will use the dimensionless operators $\hat{Q} = \hat{q}\sqrt{m\omega_m/\hbar}$ and $\hat{P} = \hat{p}/\sqrt{m\hbar\omega_m}$. Accordingly, the Heisenberg equations of the motion are:

$$\frac{d}{dt}\hat{Q} = \omega_m\hat{P} \quad (13)$$

$$\frac{d}{dt}\hat{P} = \omega_m\bar{\chi}\hat{a}^\dagger\hat{a} - \omega_m\hat{Q} + \sqrt{\lambda}w(t) \quad (14)$$

$$\frac{d}{dt}\hat{a} = -i(\omega_c - \omega_l - \omega_m\bar{\chi}\hat{Q})\hat{a} - \kappa\hat{a} + \varepsilon, \quad (15)$$

where:

$$\bar{\chi} = \frac{\chi}{\omega_m}\sqrt{\frac{\hbar}{m\omega_m}}; \quad \lambda = \frac{\hbar\eta}{m\omega_m} = \frac{\hbar}{m\omega_m}\gamma\iint d^3\mathbf{r}d^3\mathbf{r}'\frac{\exp\left[-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4r_C^2}\right]}{(2\sqrt{\pi}r_C)^3}\frac{\partial\varrho(\mathbf{r})}{\partial\mathbf{r}_1}\frac{\partial\varrho(\mathbf{r}')}{\partial\mathbf{r}'_1}. \quad (16)$$

One should remember that above stochastic differential equations should be understood in the Stratonovich sense. Using standard procedures [28], one can obtain the corresponding Itô form, which coincides with the Stratonovich form in this special case. Now we can add noise terms: the thermal Brownian motion induced by a bath, and the input noise of the laser field. Therefore the quantum Langevin equations are:

$$\frac{d}{dt}\hat{Q} = \omega_m\hat{P} \quad (17)$$

$$\frac{d}{dt}\hat{P} = \omega_m\bar{\chi}\hat{a}^\dagger\hat{a} - \omega_m\hat{Q} + \sqrt{\lambda}w(t) - \gamma_m\hat{P} + \hat{W}(t) \quad (18)$$

$$\frac{d}{dt}\hat{a} = -i(\omega_c - \omega_l - \omega_m\bar{\chi}\hat{Q})\hat{a} - \kappa\hat{a} + \varepsilon + \hat{a}_{\text{in}}\sqrt{2\kappa} \quad (19)$$

$$\frac{d}{dt}\hat{a}^\dagger = i(\omega_c - \omega_l - \omega_m\bar{\chi}\hat{Q})\hat{a}^\dagger - \kappa\hat{a}^\dagger + \varepsilon + \hat{a}_{\text{in}}^\dagger\sqrt{2\kappa} \quad (20)$$

where $\hat{a}_{\text{in}}(t)$ is the input noise with zero mean the correlation function¹

$$\langle \delta \hat{a}_{\text{in}}(t) \delta \hat{a}_{\text{in}}^\dagger(s) \rangle = \delta(t - s), \quad (21)$$

and $\hat{W}(t)$ is the quantum thermal Brownian motion operator with zero mean the correlation function:

$$\langle \hat{W}(t) \hat{W}(s) \rangle = \frac{\gamma_m}{2\pi\omega_m} \int_{-\infty}^{+\infty} d\omega \omega e^{-i\omega(t-s)} \left[1 + \coth\left(\frac{\hbar\omega}{2k_B T}\right) \right] \quad (22)$$

Since here the noise operators \hat{W} and \hat{a}_{in} are of quantum origin, we represent their stochastic average by $\langle \cdots \rangle$, meaning the trace over the corresponding quantum mechanical degrees of the freedom.

We now linearise the first term on the right-side of Eqs.(19,20). We write each operator as follows: $\hat{A} = A_s + \delta\hat{A}$, where A_s is the classical steady-state value of the operator \hat{A} , and $\delta\hat{A}$ is a small quantum fluctuation around this steady-state value. Doing so, and also assuming $a_s \gg 1$, the linearised quantum Langevin equations take the following form:

$$\frac{d}{dt} \delta\hat{Q} = \omega_m \delta\hat{P} \quad (23)$$

$$\frac{d}{dt} \delta\hat{P} = \omega_m \bar{\chi} (a_s \delta\hat{a}^\dagger + a_s^* \delta\hat{a}) - \omega_m \delta\hat{Q} + \sqrt{\lambda} w(t) - \gamma_m \delta\hat{P} + \hat{W}(t) \quad (24)$$

$$\frac{d}{dt} \delta\hat{a} = -i(\omega_c - \omega_l - \omega_m \bar{\chi} Q_s) \delta\hat{a} + i\omega_m \bar{\chi} a_s \delta\hat{Q} - \kappa \delta\hat{a} + \delta\hat{a}_{\text{in}} \sqrt{2\kappa} \quad (25)$$

$$\frac{d}{dt} \delta\hat{a}^\dagger = i(\omega_c - \omega_l - \omega_m \bar{\chi} Q_s) \delta\hat{a}^\dagger - i\omega_m \bar{\chi} a_s^* \delta\hat{Q} - \kappa \delta\hat{a}^\dagger + \delta\hat{a}_{\text{in}}^\dagger \sqrt{2\kappa}. \quad (26)$$

For the dimensionless position operator $\delta\hat{Q}$, the solution of the Fourier transform of above equations is given by:

$$\delta\hat{Q}(\omega) = -\frac{\omega_m}{d(\omega)} \left[\left(\hat{W}(\omega) + \sqrt{\lambda} w(\omega) \right) (\Delta^2 + (\kappa + i\omega)^2) - i\omega_m \sqrt{2\kappa} \bar{\chi} \left(a_s^* (\kappa + i(\omega - \Delta)) \delta\hat{a}_{\text{in}}(\omega) + a_s (\kappa + i(\omega + \Delta)) \delta\hat{a}_{\text{in}}^\dagger(-\omega) \right) \right], \quad (27)$$

where $d(\omega) = (\Delta^2 + (\kappa + i\omega)^2)(\omega(\omega - i\gamma_m) - \omega_m^2) + 2|a_s|^2 \bar{\chi}^2 \omega_m^3 \Delta$, and $\Delta = \omega_c - \omega_l - \omega_m \bar{\chi} Q_s$. Taking the Fourier transforms, the CSL noise and also the thermal Brownian noise operator yield:

$$\mathbb{E}(w^*(\omega) w(\omega')) = \frac{1}{2\pi} \delta(\omega - \omega'); \quad \langle \delta\hat{a}_{\text{in}}(\omega) \delta\hat{a}_{\text{in}}^\dagger(\omega') \rangle = \frac{1}{2\pi} \delta(\omega - \omega') \quad (28)$$

$$\langle \hat{W}^\dagger(\omega) \hat{W}(\omega') \rangle = \frac{\gamma_m \omega}{2\pi\omega_m} \left[1 + \coth\left(\frac{\hbar\omega}{2k_B T}\right) \right] \delta(\omega - \omega') \quad (29)$$

From the Wiener-Khinchin theorem [29], we also have:

$$\mathbb{E}(\langle \delta\hat{Q}^\dagger(\omega) \delta\hat{Q}(\omega') \rangle) = \mathcal{S}(\omega) \delta(\omega - \omega'), \quad (30)$$

where $\mathcal{S}(\omega)$ is the corresponding spectral density of the dimensionless position operator $\delta\hat{Q}$. Thus, one finds:

$$\mathcal{S}(\omega) = \frac{1}{2} \int_{-\infty}^{+\infty} d\omega' \left(\mathbb{E}(\langle \delta\hat{Q}^\dagger(\omega) \delta\hat{Q}(\omega') \rangle) + \mathbb{E}(\langle \delta\hat{Q}^\dagger(\omega') \delta\hat{Q}(\omega) \rangle) \right). \quad (31)$$

¹ See the following paragraphs for the precise definition of $\delta\hat{a}_{\text{in}}(t)$.

Introducing Eq.(27) into Eq.(30) yields:

$$\mathbb{E}(\langle \delta \hat{Q}^\dagger(\omega) \delta \hat{Q}(\omega') \rangle) = \delta(\omega - \omega') \frac{\omega_m^2}{2\pi |d(\omega)|^2} \times \left\{ 4\kappa \bar{\chi}^2 |a_s|^2 \omega_m^2 (\kappa^2 + \omega^2 + \Delta^2) + ((\Delta^2 + \kappa^2 - \omega^2)^2 + 4\kappa^2 \omega^2) \left(\frac{\gamma_m \omega}{\omega_m} \left[1 + \coth \left(\frac{\hbar \omega}{2k_B T} \right) \right] + \lambda \right) \right\} \quad (32)$$

Accordingly, for the spectral density, we find:

$$\mathcal{S}(\omega) = \frac{\omega_m^2 \left\{ 2\kappa \bar{\chi}^2 |a_s|^2 \omega_m^2 (\kappa^2 + \omega^2 + \Delta^2) + [(\Delta^2 + \kappa^2 - \omega^2)^2 + 4\kappa^2 \omega^2] \left(\frac{\gamma_m \omega \coth(\hbar \beta \omega)}{\omega_m} + \lambda \right) \right\}}{\pi |d(\omega)|^2} \quad (33)$$

with $\beta = 1/2k_B T$. Here the term $1/|d(\omega)|^2$ determines the shape of the spectrum, while other terms in the nominator contribute to the intensity of the spectrum. Notice that the spectral density of the intracavity field $\mathcal{S}_{\text{cav}}(\omega)$ is related to $\mathcal{S}(\omega)$ via the simple relation: $\mathcal{S}_{\text{cav}}(\omega) = |f(\Delta)|^2 \mathcal{S}(\omega) + \text{additional terms}$ where $f(\Delta) = 2\bar{\chi} a_s \frac{\kappa - i\omega}{\Delta^2 + (\kappa - i\omega)^2}$ (see Eq.(15) in [30]). So the following arguments for $\mathcal{S}(\omega)$ can be directly also applied for $\mathcal{S}_{\text{cav}}(\omega)$. The contribution of the CSL noise is shown in Fig.1, which manifests in the intensity of the spectrum by increasing the area under $\mathcal{S}(\omega)$. The relative increase of the area under the spectral density can be considered as a measure for the CSL effect on the spectrum [19]:

$$\mathcal{A} = \frac{\int_{-\infty}^{+\infty} d\omega \mathcal{S}(\omega) - \lim_{\lambda \rightarrow 0} \int_{-\infty}^{+\infty} d\omega \mathcal{S}(\omega)}{\lim_{\lambda \rightarrow 0} \int_{-\infty}^{+\infty} d\omega \mathcal{S}(\omega)}. \quad (34)$$

Here \mathcal{A} gives us a quantitative estimate of the relative increase of the area under the spectral density when the CSL effects are included. As one can see in Fig.1, the spectrum is well-localized around $\omega \sim \omega_m$. Accordingly, with a very good approximation the spectral density can be written as follows:

$$\mathcal{S}(\omega) \approx \frac{\omega_m^2 \left\{ 2\kappa \bar{\chi}^2 |a_s|^2 \omega_m^2 (\kappa^2 + \omega_m^2 + \Delta^2) + [(\Delta^2 + \kappa^2 - \omega_m^2)^2 + 4\kappa^2 \omega_m^2] (\gamma_m \coth(\hbar \beta \omega_m) + \lambda) \right\}}{\pi |d(\omega)|^2}. \quad (35)$$

Introducing Eq.(35) into Eq.(34), one finds:

$$\mathcal{A} \approx \lambda \frac{(\Delta^2 + \kappa^2 - \omega_m^2)^2 + 4\kappa^2 \omega_m^2}{2\kappa \bar{\chi}^2 |a_s|^2 \omega_m^2 (\kappa^2 + \omega_m^2 + \Delta^2) + [(\Delta^2 + \kappa^2 - \omega_m^2)^2 + 4\kappa^2 \omega_m^2] (\gamma_m \coth(\hbar \beta \omega_m))} \quad (36)$$

As one can see, \mathcal{A} scales linearly with the collapse frequency λ . Accordingly, the larger λ , the stronger the CSL effects and thus the better for experimental searches. For example, considering the numerical values introduced in the caption of Fig.1, one finds: $\mathcal{A} \approx 10^{-10} (\lambda/1\text{Hz})$. A very important question is that how the collapse frequency λ depends on parameters characterizing the mechanical oscillator (e.g., mass, size, density and etc.). As given in Eq.(16), the collapse frequency is $\lambda = \eta x_0^2$. It is similar to the famous Joos/Zeh decoherence rate (see Eq.(3.59) in [31]), however here $x_0 = \sqrt{\hbar/m\omega_m}$ and depends on the mass. The term η is usually called as the localization strength (or the localization rate), and it depends on the mass and the geometry of the oscillator (see Eq.(8)). Accordingly, λ depends on the interplay between x_0 and η ; for example the delocalization distance x_0 decreases by increasing the mass while the localization strength η increases by increasing the mass. The detailed behaviour of λ with the mass and geometry of the mechanical oscillator is the topic of our future research.

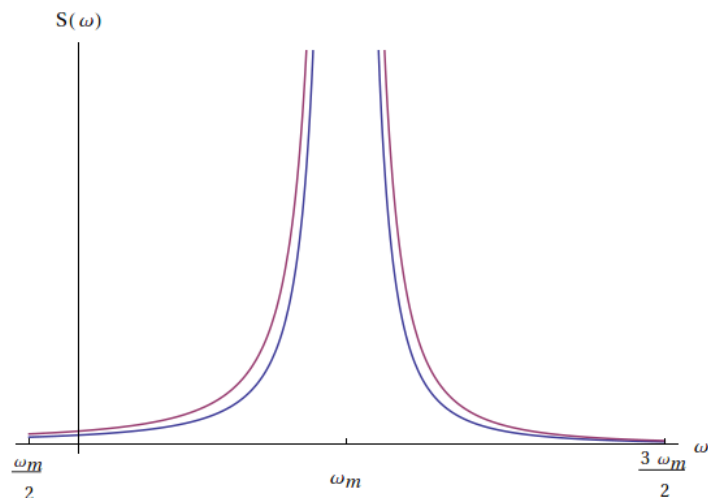


Figure 1. The spectral density as given in Eq.(33). For the blue line, we set $\lambda = 0$ (no CSL effect) and for the red line, we set: $\lambda = 10^{10}$ Hz (with CSL effect), which is a quite unrealistic value that we consider only for the sake of illustration. The parameters are set as follows: $m = 150$ ng, $\omega_m/2\pi = 2.75 \times 10^5$ Hz, $\gamma_m/2\pi = 10^{-5}\omega_m$, $\Delta = 4\kappa$, $\mathcal{P} = 4$ mW, $\kappa = 5 \times 10^7$ Hz, $T = 1$ mK, $\omega_l/2\pi = 2.78 \times 10^{15}$ Hz, $L = 25$ mm.

4. Concluding remarks

Collapse models provide a quantitative explanation on how macroscopic classical world emerges from the microscopic quantum world. They assume a universal noise field inducing appropriate Brownian motion corrections to standard Schrödinger dynamics. Accordingly, one *derives* the random localization of the wave function at the end of a measurement, of course with the correct probabilities given by the Born rule. The strength of the collapse process grows with the size of the system. Therefore, microscopic systems behave quantum mechanically, while macroscopic objects are classical. At the intermediate mesoscopic scale, collapse models predict deviations from standard quantum predictions. These deviations depend on the statistical properties of the collapse noise and also on parameters characterizing the system (e.g., mass, size, density and etc.). In the CSL model, statistical properties of the collapse noise (i.e., its mean value and its autocorrelation) are fixed by two phenomenological parameters.

All experiments to date have been at scales where the collapse effects are negligible for all practical purposes. However, recent experimental progress in revealing quantum features of larger objects, opens new path to search for testing at unprecedented scales where collapse models can be falsified. The most serious challenge in this regard is creating macroscopic superpositions, because all experimental schemes proposed so far are based on the natural idea of creating a macroscopic quantum superposition in space.

Quite recently, an alternative approach has been proposed, based on measuring the fluctuating properties of light (e.g., the spectral density) which interacts with a quantum system. The collapse manifests as a change (e.g., an extra broadening) in lineshapes of the light spectrum. The great advantageous of this new approach is that here there is no need for the preparation of a quantum superposed state. It has been discussed that promising results can be revealed in the spectrum of light interacting with a radiation pressure-driven mechanical oscillator in a cavity optomechanics setting. Proper measures have been introduced to quantify the collapse effect on the spectrum. These measures depend on the collapse frequency. Therefore, it is very important to quantify the collapse frequency, e.g., to explain how it depends on the parameters of the mechanical oscillator (e.g., its size, mass, density and etc.). Here, the collapse frequency depends linearly on the localization strength η and quadratically on the delocalization distance x_0 (i.e. $\lambda = \eta x_0^2$). The localization strength increases with the mass, as expected; while the delocalization distance decreases with the mass. This is the very particular thing that distinguishes our example from other cases studied so far in the literature. In previous cases, only the behaviour of the localization strength with the mass has been investigated, because the delocalization distance was independent from the mass; while in our case it depends also on

the mass, and thus one needs to carefully study the interplay of the mass on the localization strength and on the delocalization distance. Accordingly, the precise behaviour of the collapse frequency depends on this dual role of the mass, and for different geometries it may produce different behaviours.

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References

- [1] Leggett A J 2002 *J. Phys.: Condens. Matter* **14** R415
- [2] Adler S L and Bassi A 2009 *Science* **325** 275
- [3] Adler S L 2004 *Quantum Theory as an Emergent Phenomenon* (Cambridge: Cambridge Univ. Press)
- [4] Weinberg S 2012 *Phys. Rev. A* **85** 062116
- [5] Joos E *et al* 2003 *Decoherence and the Appearance of a Classical World in Quantum Theory* 2nd edition (New York: Springer)
- [6] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford Univ. Press)
- [7] Adler S L 2003 *Stud. Hist. Philos. Mod. Phys.* **34** 135; Vacchini B 2007 *J. Phys. A: Math. Theor.* **40** 2463; Adler S L 2014 Gravitation and the noise needed in objective reduction models *Preprint* arXiv:1401.0353v4; Arndt M and Hornberger K 2014 *Nat. Phys.* **10** 271
- [8] Bassi A and Ghirardi G C 2003 *Phys. Rep.* **379** 257
- [9] Bassi A, Lochan K, Satin S, Singh T P and Ulbricht H 2013 *Rev. Mod. Phys.* **85** 471
- [10] Penrose P 1996 *Gen. Rel. Grav.* **28** 581; *do.* 2014 *Found. Phys.* **44** 557
- [11] Pearle P 1976 *Phys. Rev. D* **13** 857; *do.* 1989 *Phys. Rev. A* **39** 2277
- [12] Ghirardi G C, Rimini A and Weber T 1986 *Phys. Rev. D* **34** 470
- [13] Diósi L 1987 *Phys. Lett. A* **120** 377; *do.* 1989 *Phys. Rev. A* **40** 1165
- [14] Ghirardi G C, Pearle P and Rimini A 1990 *Phys. Rev. A* **42** 78
- [15] Ghirardi G C, Grassi R and Benatti F 1995 *Found. Phys.* **25** 5
- [16] Adler S L 2007 *J. Phys. A* **40** 2935
- [17] Marquardt F and Girvin S M 1993 *Physics* **2** 40; Aspelmeyer M *et al* 2010 *J. Opt. Soc. Am. B* **27** A189; Aspelmeyer M, Kippenberg T J and Marquardt F 2014 *Rev. Mod. Phys.* **86** 1391; Romero-Isart O 2011 *Phys. Rev. A* **84** 052121; Hornberger K, Gerlich S, Haslinger P, Nimmrichter S and Arndt M 2012 *Rev. Mod. Phys.* **84** 157; Juffmann T, Ulbricht H and Arndt M 2013 *Rep. Prog. Phys.* **76** 086402
- [18] Bahrami M, Bassi A and Ulbricht H 2014 *Phys. Rev. A* **89** 032127
- [19] Bahrami M, Paternostro M, Bassi A and Ulbricht H 2014 *Phys. Rev. Lett.* **112** 210404
- [20] Nimmrichter S, Hornberger K and Hammerer K 2014 *Phys. Rev. Lett.* **113** 020405
- [21] For a more detailed calculation, see Section II of Bedingham D J 2013 *Phys. Rev. D* **88** 045032
- [22] Gisin N 1989 *Hel. Phys. Acta* **62** 363; *do.* 1990 *Phys. Lett. A* **143** 1
- [23] Gisin N and Rigo M 1995 *J. Phys. A* **28** 7375
- [24] Polchinski J 1991 *Phys. Rev. Lett.* **66** 397
- [25] Adler S L 2000 *J. Math. Phys.* **41** 2485; *do.* Statistical Dynamics of Global Unitary Invariant Matrix Models as Pre-Quantum Mechanics 2002 *Preprint* arXiv:hep-th/0206120, Section 5F; *do.* 2003 *Phys. Rev. D* **67** 025007, added note
- [26] Diósi L 1988 *Phys. Lett. A* **129** 419
- [27] Agarwal G S 2012 *Quantum Optics* (NY: Cambridge Univ. Press) Chapter 20
- [28] Arnold L 1974 *Stochastic differential equations: Theory and applications* (New York: Wiley) Section 10.2.
- [29] Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge Univ. Press) Section 2.4.
- [30] Paternostro M, Gigan S, Kim M S, Blaser F, Böhm H R and Aspelmeyer M 2006 *New J. Phys.* **8** 107
- [31] Joos E and Zeh H D 1985 *Z. Phys. B* **59** 223