Ranking Schools' Academic Performance Using a Fuzzy VIKOR

Suhaina Musani[1,2] and Abdul Aziz Jemain[3]

[1]Faculty of Computing and Informatics, Universiti Malaysia Sabah, Labuan International Campus, Jalan Sg. Pagar, 87000 Wilayah Persekutuan Labuan, Malaysia.

[3]School of Mathematical Sciences, Universiti Kebangsaan Malaysia, 43600 Bangi, Selangor Darul Ehsan, Malaysia

Abstract. Determination rank is structuring alternatives in order of priority. It is based on the criteria determined for each alternative involved. Evaluation criteria are performed and then a composite index composed of each alternative for the purpose of arranging in order of preference alternatives. This practice is known as multiple criteria decision making (MCDM). There are several common approaches to MCDM, one of the practice is known as VIKOR (Multi-criteria Optimization and Compromise Solution). The objective of this study is to develop a rational method for school ranking based on linguistic information of a criterion. The school represents an alternative, while the results for a number of subjects as the criterion. The results of the examination for a course, is given according to the student percentage of each grade. Five grades of excellence, honours, average, pass and fail is used to indicate a level of achievement in linguistics. Linguistic variables are transformed to fuzzy numbers to form a composite index of school performance. Results showed that fuzzy set theory can solve the limitations of using MCDM when there is uncertainty problems exist in the data.

Keywords: Multi Criteria Decision Making; VIKOR; fuzzy numbers; composite index

1. Introduction

Since the last few years, the evaluation of the education system with excellent schools ranks has become increasingly popular as a benchmark of the education system. School ranking as an evaluating school performance will provide a direct influence on schools involved [1] [2]. According to Wu et al. [3], evaluating performance is important to the administration of educational institutions to determine the market which will influence the publics' perception to the institution. In addition, it also affects the expenses allocated for student recruitment and operations, as well as acting as a guide to strategic planning institution. On this basis, implementation of school ranking is not meant to punish but to identify schools in need of assistance in terms of infrastructure, financing, enhancement of teachers and also for the development of an environment conducive in teaching and learning process [4]. To achieve these goals, the study will focus on the standard school examination results. Assessment of

[2] Corresponding author. E-mail address: shonsue@yahoo.com.my (Suhaina Musani)

academic achievement on standardized examination results is the most practical way to rank the school in this study [5].

To choose an alternative by priority usually take into many factors to consider such as limited resources, organizational goals and requirements, risk and many others. Both qualitative and quantitative criteria may affect the assessment of each alternative will make the selection process more complex, challenging and unique. Then a more systematic method must be determined to tackle the problem of these criteria [6]. Often the criteria evaluated are determined by the subjective perception and personal considerations, then fuzzy MCDM approach can explain in more detail how decision makers can make an assessment of the alternatives, then choose the best solution. As stated in the assessment of different linguistic variables criteria, then the evaluation process should be carried out in a fuzzy environment. This study aims to develop an effective fuzzy MCDM approach to solve school ranking problems with application of fuzzy VIKOR method. The main idea of this method is to utilize compromise ranking method by comparing the measure of closeness to the ideal alternative to find the best solution. Recently, the usage of fuzzy VIKOR method has been increasing as a medium of decision making in fuzzy MCDM problem solving [7] [8] [9] [10] [11].

This article is organized as follows; the methodology fuzzy MCDM is presented briefly in the next section. An empirical study of a school ranking is presented in Section 3. Finally, conclusion is provided in the final section in this study.

2. Methodology

The Methodology of applying Fuzzy VIKOR for school ranking is presented in the following subsections.

2.1 Fuzzy MCDM

In the classic MCDM, evaluation of alternatives and weights measured in numbers or crisp and it depends on the consideration of the researcher. Usually the alternatives assessment and the important weights of the criteria cannot be measured reliably; in which case it may come from a variety of sources including the information cannot be quantified, imprecise, and uncertain with conflict of preferences involved in the selection process [12]. In this situation, fuzzy set theory is introduced into MCDM by Bellman and Zadeh [13] to model the uncertainty inherent in human judgment and is known as fuzzy MCDM.

In fuzzy MCDM, performance evaluation and weighting usually represented by fuzzy numbers. According to Liu et al. [14] triangular fuzzy numbers and trapezoidal fuzzy numbers (TzFN) are the most commonly used in the theory and practice of fuzzy number. In fact, the triangular fuzzy number is a special case of TzFN. When the two middle values are the same number, TzFN will become fuzzy triangular numbers. For the sake of simplicity and without loss of generality, TzFN prefer to represent linguistic variables in this study. For example, a positive TzFN $\tilde{A}$ marked as $(x_1 = 2, x_2 = 3, x_3 = 5, x_4 = 6)$ shown in Figure 1.

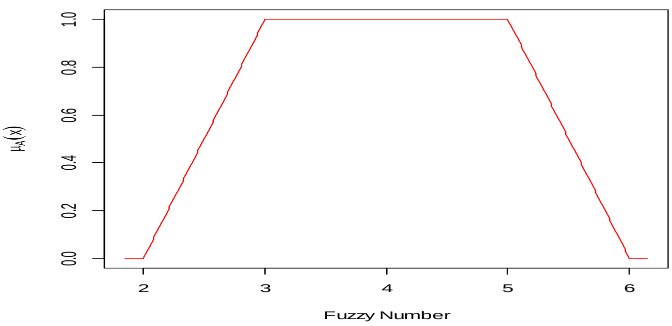

Figure 1: Trapezoidal fuzzy numbers $\tilde{A}$

Given any two positive TzFN $\tilde{A}=(a_1,a_2,a_3,a_4)$, $\tilde{B}=(b_1,b_2,b_3,b_4)$ and a positive real numbers *r*, the algebraic operations of the TzFN can be expressed as follows:

$$\tilde{A}\oplus\tilde{B}=[a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4],\quad \tilde{A}\Theta\tilde{B}=[a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1],$$
$$\tilde{A}\otimes\tilde{B}\cong[a_1b_1,a_2b_2,a_3b_3,a_4b_4],\qquad r\otimes\tilde{B}\cong[rb_1,rb_2,rb_3,rb_4]. \tag{1}$$

A linguistic variable is a variable whose value is specified in the form of non-numerical or in words [15]. The concept of a linguistic variable is very useful in dealing with situations which are too complex or too difficult to be described by the quantitative expression. The linguistic values represented by fuzzy numbers. Zadeh [16] provide a level of expertise that is more appropriate in fuzzy linguistic variables. For example, a student achieves academic level as excellence, honours, average, pass and fail depending on the subjective assessment by the assessor. Table 1 below gives the trapezoidal fuzzy numbers to five linguistic variables as previously stated, while Figure 2 shows the membership functions for the sake of visualization.

Table 1: Linguistic variable for each level of achievement

Linguistic Variable		Trapezoidal Fuzzy Numbers (TzFN)
Excellent	(g5)	(8,9,10,10)
Honours	(g4)	(6,7,8,9)
Average	(g3)	(3,4,5,7)
Pass	(g2)	(1,2,3,4)
Fail	(g1)	(0,0,0,2)

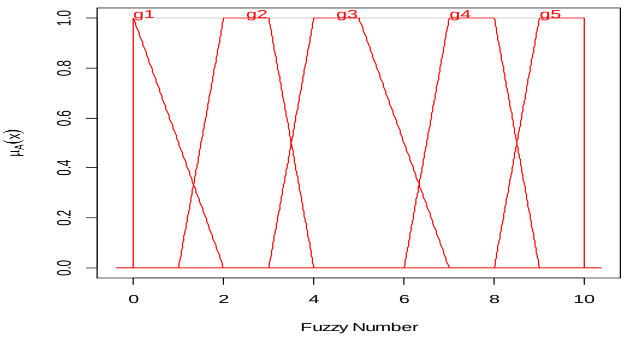

Figure 2: Membership functions for each level of academic achievement

2.2 The Fuzzy VIKOR Method

Multi Criteria Optimization and Compromise Solution methods (or VIKOR) has been developed for multi-criteria optimization in a complex system [17]. It determines the compromise solution and best solution from a set of alternatives. Compromise solution will be presented by comparing the degree of closeness to the ideal alternative and each alternative can be evaluated by each criterion function [18]. A systematic approach of a fuzzy VIKOR method for multi criteria in fuzziness environment is given in this section. According Tzeng et al. [19] this approach aims to find the best compromise solution between decision-makers to be consistent with the objectives of human cognition.

VIKOR algorithm based on modified fuzzy numbers stated as follows:

Step 1: Expressed multi criteria decision making problem in the matrix format.

There are m alternatives can defined as A_i (i = 1, 2, …, m) which will be evaluated based on the criteria selected that is C_j (j = 1, 2, …, n). Each criteria has five grade achievement g = 1, 2, …, 5. Subjective evaluation is done to determine the decision matrix $X = \{x_{ijg}, i = 1,2,...,m; j = 1,2,...,n; g = 1,2,...,5\}$ using linguistic variable as shown in Table 1.

Decision matrix can be expressed as follows:

$$X = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array}, \; i = 1, 2, \ldots, m \; ; j = 1, 2, \ldots, n$$

$$W = [w_1, w_2, ..., w_n]$$

Where $A_1, A_2, ..., A_m$ are the alternatives to be chosen, $C_1, C_2, ..., C_m$ are the evaluation criteria, x_{ij} is the rating of alternative A_i with respect to C_j, w_j is the importance weight of the j th criterion holds.

Step 2: Construct a fuzzy decision matrix.
The aggregated fuzzy rating x_{ijg} of alternatives with respect to trapezoidal fuzzy numbers is modification from the method of arithmetic weighted average [20] and calculated using the following equation:

$$\tilde{X} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{g=1}^{5} x_{ijg} \otimes TzFN = \left[\tilde{X}_{ij}\right]_{m\times n} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \tag{2}$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, ..., \tilde{w}_n]$$

This method is most often used an aggregation process because of simple and flexible operations and fits well with the goals of the study. $\tilde{x}_{ij}$ and $\tilde{w}_j$ are linguistic variables denoted by trapezoidal fuzzy number where $\tilde{x}_{ij}$ is the rating of alternative A_i with respect to C_j, $\tilde{w}_j$ is the importance weight of the j th criterion. A trapezoidal fuzzy number can be defined as $\tilde{x}_{ij} = (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij})$.

Step 3: Evaluate the fuzzy importance weight of criteria.
The fuzzy weighted values for each criterion will be determined based on the importance of each criterion. Degree of importance of each criterion depends on the burden borne by each school. Relative value is directly proportional to the number of candidates sitting for specific subjects. This clearly shows the value of a higher weight should be given to the criteria that have more number of candidates because it brings an additional burden to ensure that each candidate can understand the subject well [21]. Therefore, if the number of candidates taking the subject j, then the fuzzy importance of subjects is given as specified by Diakoulaki et al. [22]:

$$\tilde{w}_j = \tilde{s}_j \Big/ \sum_{j=1}^{n} \tilde{s}_j \tag{3}$$

$\tilde{s}_j$ is the standard deviation value for the criterion C_n. Standard deviation $\tilde{s}_j$ is given as follows:

$$\tilde{s}_j = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(\tilde{x}_{mn} - \tilde{\tilde{x}}_n\right)^2} \tag{4}$$

with $\tilde{\tilde{x}}_n = \frac{1}{M}\sum_{m=1}^{M}\tilde{x}_{mn}$, $0 \le \tilde{w}_j \le 1$ and M = Total number of alternative.

Step 4: Determine the fuzzy best value ($\tilde{x}_j^*$) and fuzzy worst value ($\tilde{x}_j^-$)

$$\tilde{x}_j^* = \max_i \tilde{x}_{ij},$$
$$\tilde{x}_j^- = \min_i \tilde{x}_{ij}. \tag{5}$$

Step 5: Compute the normalized fuzzy decision matrix. The normalized fuzzy decision matrix is calculated to ensure that each criterion value between 0 and 1, so that all the criteria are the standard and are comparable with each other. In this situation, VIKOR method using linear normalization to stabilize [17]. Linear normalization formula indicated by the score $\tilde{S}_i$ and $\tilde{R}_i$ as follows:

$$\tilde{S}_i = \sum_{j=1}^{n}\tilde{w}_j\left(\frac{\tilde{x}_j^* - \tilde{x}_{ij}}{\tilde{x}_j^* - \tilde{x}_j^-}\right) \quad \text{and} \quad \tilde{R}_i = \underset{j}{Max}\left[\tilde{w}_j\left(\frac{\tilde{x}_j^* - \tilde{x}_{ij}}{\tilde{x}_j^* - \tilde{x}_j^-}\right)\right] \tag{6}$$

Step 6: Compute the index VIKOR $\tilde{Q}_i$

$$\tilde{Q}_i = v\left(\frac{\tilde{S}_i - \tilde{S}^-}{\tilde{S}^+ - \tilde{S}^-}\right) + (1-v)\left(\frac{\tilde{R}_i - \tilde{R}^-}{\tilde{R}^+ - \tilde{R}^-}\right) \tag{7}$$

where

$$\tilde{S}^+ = \max_i \tilde{S}_i, \tilde{S}^- = \min_i \tilde{S}_i$$
$$R^+ = \max_i \tilde{R}_i, \tilde{R}^- = \min_i \tilde{R}_i$$

v is introduced as the weight in strategy of the maximum group utility. From the literature, it has been inferred that the VIKOR index value is mostly taken as $v = 0.5$.

Step 7:
Sorting the value $\tilde{S}$, $\tilde{R}$ and $\tilde{Q}$ in descending order. The best alternative in order of $\tilde{Q}$ is the maximum possible value of $\tilde{Q}$ based on merit points that was done in this study and symbolized $A^{(1)}$. With the second largest alternative referred to $A^{(2)}$ and so on until an alternative with the smallest value of $\tilde{Q}$ is expressed as $A^{(m)}$.

Step 8:
The alternatives $A^{(1)}$ that are in the best position with the maximum value of $\tilde{Q}$ will be proposed as the best alternatives in providing a compromise solution if and only if satisfy two conditions:
C 1: Acceptable advantage.
The alternative $A^{(1)}$ accepted as the best advantages when the difference index VIKOR $\tilde{Q}$ between alternative $A^{(2)}$ and $A^{(1)}$ must be greater than or equal to the value of *DQ*, or in other words

$$\tilde{Q}_{(A^{(2)})} - \tilde{Q}_{(A^{(1)})} \ge DQ \text{ with } DQ = \frac{1}{M-1}.$$

C 2: Acceptable stability in decision making.
Alternative $A^{(1)}$ must also be in the best ranked by $\tilde{S}$ or $\tilde{R}$.

When one of the conditions is not satisfied, a set of compromise solution will be proposed as follows:
i. If ***C 1*** is not satisfied:
Then the alternative set $A^{(1)}, A^{(2)},..., A^{(m)}$ considered together with its best $A^{(m)}$ determined by the relationship $\tilde{Q}_{\left(A^{(m)}\right)} - \tilde{Q}_{\left(A^{(1)}\right)} < DQ$.
ii. If ***C 2*** is not satisfied:
Thus, both alternative $A^{(1)}$ dan $A^{(2)}$ are recommended as the best option position [7] or in other words the two alternative can be described as the best alternative.
R software 2:15 was used to analyse data using fuzzy VIKOR method.

3 Empirical Study

This study used a sample data of academic achievement for the Sijil Pelajaran Malaysia (SPM) examination results from one of the state in Malaysia. It is aim to evaluate and rank the schools containing multiple conflicting criteria subject. The selected ten schools (School 1, School 2, …, School 10) are to be evaluated by four major subjects (Subject 1, Subject 2, Subject 3, Subject 4) in five grades for each subject which are excellent (g5), honours (g4), average (g3), pass (g2) and fail (g1) as shown in Table 2:

Table 2: Data on the percentage of academic achievement, x_{ijg}

	Subject																			
	Subject 1					Subject 2					Subject 3					Subject 4				
School	g1	g2	g3	g4	g5	g1	g2	g3	g4	g5	g1	g2	g3	g4	g5	g1	g2	g3	g4	g5
1	0.017	0.238	0.213	0.336	0.196	0.174	0.234	0.191	0.204	0.196	0.106	0.264	0.140	0.162	0.328	0.120	0.295	0.115	0.150	0.321
2	0.110	0.294	0.298	0.212	0.086	0.209	0.402	0.176	0.148	0.066	0.275	0.275	0.199	0.144	0.106	0.206	0.324	0.139	0.080	0.252
3	0.099	0.167	0.167	0.257	0.311	0.186	0.190	0.072	0.176	0.376	0.323	0.173	0.123	0.114	0.268	0.241	0.159	0.027	0.100	0.473
4	0.000	0.000	0.025	0.364	0.610	0.000	0.076	0.254	0.415	0.254	0.000	0.085	0.186	0.356	0.373	0.025	0.136	0.110	0.178	0.551
5	0.052	0.278	0.268	0.309	0.093	0.224	0.316	0.235	0.143	0.082	0.469	0.250	0.146	0.094	0.042	0.306	0.337	0.082	0.092	0.184
6	0.061	0.287	0.243	0.291	0.117	0.143	0.313	0.165	0.187	0.191	0.367	0.218	0.162	0.144	0.109	0.223	0.266	0.135	0.135	0.240
7	0.031	0.336	0.288	0.205	0.140	0.276	0.443	0.110	0.123	0.048	0.333	0.228	0.259	0.114	0.066	0.368	0.342	0.075	0.066	0.149
8	0.070	0.140	0.193	0.333	0.263	0.14	0.333	0.123	0.246	0.158	0.228	0.228	0.333	0.140	0.070	0.246	0.351	0.105	0.123	0.175
9	0.207	0.414	0.207	0.135	0.036	0.366	0.429	0.071	0.054	0.080	0.509	0.287	0.130	0.065	0.009	0.291	0.400	0.100	0.073	0.136
10	0.143	0.388	0.240	0.158	0.071	0.235	0.281	0.143	0.189	0.153	0.51	0.201	0.139	0.093	0.057	0.297	0.25	0.073	0.078	0.302

The modified fuzzy VIKOR method is used to solve this multi criteria decision making problem and the computational procedures are stated as follow:

Step 1: The observations in decision matrix described the percentage of students who obtained the results of each subjects are shown in Table 2. For School 1, the percentage of students who obtained a fail grade for Subject 1 is 1.7 percent, percentage of students earned a pass grade is 23.8 percent, average grade is 21.3 percent, 19.6 percent for honours and 17.4 percent for excellent grade.

Step 2: Five grades for each subject use the linguistic variables. The corresponding fuzzy numbers of five linguistic variables are shown in Table 1 and membership functions for each linguistic variable are shown in Figure 2. According to equation (1) and (2), convert the linguistic variables into TzFN

$(\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij})$ as well aggregate the skor (percentage x_{ijg}) with TzFN. Fuzzy decision matrix can be referred in Table 3. To be clearer, aggregate the skor and TzFN of School 1 with respect to Subject 1 is computed as:

$$\tilde{x}_{School1,Subject1,\tilde{a}} = (0.017\times 0)+(0.238\times 1)+(0.213\times 3)+(0.336\times 6)+(0.196\times 8)=4.460$$

$$\tilde{x}_{School1,Subject1,\tilde{b}} = (0.017\times 0)+(0.238\times 2)+(0.213\times 4)+(0.336\times 7)+(0.196\times 9)=5.443$$

$$\tilde{x}_{School1,Subject1,\tilde{c}} = (0.017\times 0)+(0.238\times 3)+(0.213\times 5)+(0.336\times 8)+(0.196\times 10)=6.426$$

$$\tilde{x}_{School1,Subject1,\tilde{d}} = (0.017\times 2)+(0.238\times 4)+(0.213\times 7)+(0.336\times 9)+(0.196\times 10)=7.460$$

Table 3: Aggregated trapezoid fuzzy number decision matrix

	Subject															
	Subject 1				Subject 2				Subject 3				Subject 4			
School	$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{d}$	$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{d}$	$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{d}$	$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{d}$
1	**4.460**	**5.443**	**6.426**	**7.460**	3.600	4.426	5.251	6.421	4.277	5.170	6.064	6.983	4.103	4.983	5.863	6.778
2	3.147	4.037	4.927	6.249	2.340	3.131	3.922	5.242	2.585	3.309	4.034	5.403	3.235	4.029	4.824	5.916
3	4.694	5.595	6.495	7.450	4.471	5.285	6.100	6.982	3.368	4.045	4.723	5.900	4.623	5.382	6.141	6.936
4	7.144	8.144	9.144	9.559	5.364	6.364	7.364	8.364	5.763	6.763	7.763	8.576	5.941	6.915	7.890	8.475
5	3.680	4.629	5.577	6.804	2.531	3.306	4.082	5.459	1.583	2.115	2.646	4.219	2.602	3.296	3.990	5.194
6	3.704	4.643	5.583	6.770	3.461	4.317	5.174	6.291	2.441	3.074	3.707	5.127	3.406	4.183	4.961	6.079
7	3.550	4.520	5.489	6.668	1.895	2.618	3.342	4.680	2.215	2.882	3.548	5.075	2.154	2.785	3.417	4.711
8	4.825	5.754	6.684	7.684	3.439	4.298	5.158	6.263	2.632	3.404	4.175	5.667	2.807	3.561	4.316	5.491
9	2.135	2.928	3.721	5.099	1.607	2.241	2.875	4.232	1.139	1.630	2.120	3.750	2.227	2.936	3.645	4.900
10	2.628	3.485	4.342	5.653	3.066	3.832	4.597	5.821	1.629	2.119	2.608	4.201	3.354	4.057	4.760	5.828

Step 3: Fuzzy important weight of criteria are addressed in Table 4. To make it explicit, the fuzzy weight of Subject 1 with respect to $\tilde{a}$ ($\tilde{w}_{Subjek1,\tilde{a}}$) is calculated as:

$$\bar{\tilde{x}}_{\tilde{a}} = \frac{1}{10}\sum_{m=1}^{10}\tilde{x}_{m\tilde{a}} = \frac{1}{10}(39.967) = 3.997,$$

$$\tilde{s}_{Subject1,\tilde{a}} = \sqrt{\frac{1}{10}\sum_{m=1}^{10}\left(\tilde{x}_{m\tilde{a}} - \bar{\tilde{x}}_{\tilde{a}}\right)^2} = \sqrt{\frac{1}{10}\begin{pmatrix}(4.460-3.997)^2+(3.147-3.997)^2+(4.694-3.997)^2+\\(7.144-3.997)^2+(3.680-3.997)^2+(3.704-3.997)^2+\\(3.550-3.997)^2+(4.825-3.997)^2+(2.135-3.997)^2+\\(2.628-3.997)^2\end{pmatrix}}$$

$$=1.404$$

$\tilde{s}_{Subject1,\tilde{b}} = 1.451$, $\tilde{s}_{Subject1,\tilde{c}} = 1.499$, $\tilde{s}_{Subject1,\tilde{d}} = 1.229$,

$\tilde{s}_{Subject2,\tilde{a}} = 1.156$, $\tilde{s}_{Subject2,\tilde{b}} = 1.242$, $\tilde{s}_{Subject2,\tilde{c}} = 1.329$, $\tilde{s}_{Subject2,\tilde{d}} = 1.186$,

$\tilde{s}_{Subject3,\tilde{a}} = 1.391$, $\tilde{s}_{Subject3,\tilde{b}} = 1.551$, $\tilde{s}_{Subject3,\tilde{c}} = 1.712$, $\tilde{s}_{Subject3,\tilde{d}} = 1.436$,
$\tilde{s}_{Subject4,\tilde{a}} = 1.172$, $\tilde{s}_{Subject4,\tilde{b}} = 1.258$, $\tilde{s}_{Subject4,\tilde{c}} = 1.346$, $\tilde{s}_{Subject4,\tilde{d}} = 1.126$.

$$\tilde{w}_{Subject1,\tilde{a}} = \frac{1.404}{\begin{pmatrix}1.404+1.451+1.499+1.229+1.156+1.242+1.329+1.186+ \\ 1.391+1.551+1.712+1.436+1.172+1.258+1.346+1.126\end{pmatrix}} = \frac{1.404}{21.489} = 0.065$$

Table 4: Fuzzy important weight of the criteria

Subject 1				Subject 2				Subject 3				Subject 4			
$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{d}$	$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{d}$	$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{d}$	$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{d}$
(**0.065**,	0.068,	0.070,	0.057)	(0.054,	0.058,	0.062,	0.055)	(0.065,	0.072,	0.080,	0.067)	(0.055,	0.059,	0.063,	0.052)

Step 4: The fuzzy best value ($\tilde{x}_j^*$) and fuzzy worst value ($\tilde{x}_j^-$) are listed in Table 5.

Table 5: Fuzzy best value ($\tilde{x}_j^*$) and fuzzy worst value ($\tilde{x}_j^-$)

	Subject 1	Subject 2	Subject 3	Subject 4
$\tilde{x}_j^*$	(7.144, 8.144, 9.144, 9.559)	(5.364, 6.364, 7.364, 8.364)	(5.763, 6.763, 7.763 8.576)	(5.941, 6.915, 7.890, 8.475)
$\tilde{x}_j^-$	(2.135, 2.928, 3.721, 5.099)	(1.607, 2.241, 2.875, 4.232)	(1.139, 1.630, 2.120, 3.750)	(2.154, 2.785, 3.417, 4.711)

Step 5: Skor $\tilde{S}_i$ and $\tilde{R}_i$ are computed respectively in Table 6 as states in equation (6).

Step 6: According to equation (7) the score $\tilde{S}^+$, $\tilde{S}^-$, $\tilde{R}^+$ and $\tilde{R}^+$ are listed below:

$$\tilde{S}^+ = 0.902, \tilde{S}^- = 0.101$$

$$R^+ = 0.120, \tilde{R}^- = 0.050$$

By applying equation (7), the index VIKOR $\tilde{Q}_{School1}$ can be calculated as:

$$\tilde{Q}_{School1} = 0.5 \times \left(\frac{0.288-0.101}{0.902-0.101}\right) + (1-0.5) \times \left(\frac{0.050-0.050}{0.120-0.050}\right) = 0.117 \ .$$

Therefor the values $\tilde{Q}_i$ are shown in Table 6.

Table 6: Ranking of schools for $v = 0.5$

Schools	$\tilde{S}_i$	Ranking	$\tilde{R}_i$	Ranking	$\tilde{Q}_i$ (index VIKOR)	Ranking
1	0.288	9	0.050	10	0.117	9
2	0.821	3	0.083	6	0.685	4
3	0.322	8	0.069	8	0.271	8
4	0.101	10	0.055	9	0.035	10
5	0.704	6	0.093	4	0.682	5
6	0.776	4	0.104	2	0.801	3
7	0.686	7	0.074	7	0.536	7
8	0.715	5	0.090	5	0.665	6
9	0.867	2	0.100	3	0.833	2
10	0.902	1	0.120	1	1.000	1

Step 7: The values of $\tilde{S}_i$, $\tilde{R}_i$ dan $\tilde{Q}_i$ are calculated for all schools by selecting v = 0.5 (consensus) as shown in Table 6. As stated in Table 6, the larger $\tilde{Q}_i$ implies the better performance of the schools. Hence $A^{(1)} = 1.000$, $A^{(2)} = 0.833$, …, $A^{(m)} = 0.035$.

Step 8:
Once the value $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$ are obtained by descending order, ***C1*** tested whether filled by the following equation:

$$\tilde{Q}_{(A^{(2)})} - \tilde{Q}_{(A^{(1)})} \geq DQ,$$

The difference value Q for alternative $A^{(1)}$ and $A^{(2)}$ is -0.167. This means $\tilde{Q}_{(A^{(2)})} - \tilde{Q}_{(A^{(1)})}$ is less than threshold DQ = 0.111. Therefore, first condition ***C1*** is not satisfied then the ***C2*** is tested. The result shows that the second condition ***C2*** is satisfied. Since ***C2*** is satisfied, this research propose the school 10, 9, 6, 2, 5, 8, 7, 3, 1 and 4 as a set of compromise solutions by order preference such an analysis are presented in Table 6 or can be stated as follows:

$$School10 > School9 > School6 > School2 > School5 >$$
$$School8 > School7 > School3 > School1 > School4.$$

4 Conclusion

VIKOR method is an effective technique for analysing various types of criteria, and it has been widely used in the rank of a compromise in the field of management. The provided case study has demonstrated the capability of the proposed fuzzy MCDM model to effectively solve school ranking problem under a fuzzy environment. In this method, the ranking of schools are assessed in linguistic variable by trapezoidal fuzzy numbers and the importance weights of criteria are also in fuzzy number. Several studies have applied the method of performance analysis in educational institutions such as Chen and Tzeng [1], Wu et al. [2] and Wu et al. [3]. This study used fuzzy VIKOR method to determine the priority ranks of the performance for ten schools. By using the suggested approach, the ambiguities involved in the assessment of academic achievement on examination results data could be effectively represented and processed to assure a more effective evaluation process. Based on the result, this study can give management implication for the school administrators and Ministry of Education who wish to take countermeasures. In addition, the process and results of this study could provide a reference point for other schools and related educational institutions in their efforts to improve their performances, conduct academic evaluations and to legislate educational policies. To accurately reflect the real performance situations of schools in Malaysia, future research is recommended to take daily schools, boarding schools, religious schools and vocational/technical schools into consideration.

References

[1] Chen C H & Tzeng G H 2011 Creating the aspired intelligent assessment systems for teaching materials *Expert Systems with Applications* 38(10): 12168-12179
[2] Bradley S, Johnes G & Millington J 2001 The effect of competition on the efficiency of secondary school in England *European Journal of Operational Research* 135:545-568
[3] Wu H Y, Chen J K, Chen I S & Zhuo H H 2012 Ranking universities based on performance evaluation by a hybrid MCDM model *Measurement* 45(5): 856-880
[4] Wu H Y, Lin Y K & Chang C H 2011 Performance evaluation of extension education centres in universities based on the balanced scorecard *Evaluation and Program Planning* 34(1): 37-50

[5] Borhan M & Jemain A A 2011 Assessing schools' academic performance using a belief structure *Social Indicators Research* 106:187–197

[6] Tzeng G H, Teng M H, Chen J J & Opricovic S 2002 Multicriteria selection for a restaurant location in Taipei *International Journal of Hospitality Management* 21:171-187

[7] Büyüközkan G & Ruan D 2008 Evaluation of software development projects using a fuzzy multi-criteria decision approach *Mathematics and Computers in Simulation* 77(5–6): 464-475

[8] Jeya Girubha R & Vinodh S 2012 Application of fuzzy VIKOR and environmental impact analysis for material selection of an automotive component *Materials & Design* 37(0): 478-486

[9] Sanayei A, Farid M S & Yazdankhah A 2010 Group decision making process for supplier selection with VIKOR under fuzzy environment *Expert Systems with Applications* 37(1): 24-30

[10] Shemshadi A, Shirazi H, Toreihi M & Tarokh M J 2011 A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting *Expert Systems with Applications* 38(10): 12160-12167

[11] Yücenur G N & Demirel N Ç 2012 Group decision making process for insurance company selection problem with extended VIKOR method under fuzzy environment *Expert Systems with Applications* 39(3): 3702-3707

[12] Chen L Y & Wang T C 2009 Optimizing partners' choice in IS/IT outsourcing projects: The strategic decision of fuzzy VIKOR *International Journal of Production Economics* 120(1): 233-242

[13] Bellman R E, Zadeh L A 1970 Decision making in a fuzzy environment. *Management Science* 17(4):141–164

[14] Liu H C, Liu L L, Nan & Mao L X 2012 Risk evaluation in failure mode and effects analysis with extended VIKOR method under fuzzy environment *Expert Systems with Applications* 39(17): 12926-12934

[15] Herrera F, Herrera-Viedma, E & verdegay J L 1996 A model of consensus in group decision making under linguistic assessments *Fuzzy Sets and Systems* 78(1): 73-87

[16] Zadeh L A 1975 The concept of a linguistic variable and its applications to approximate reasoning-Part I *Information Sciences* 8: 199–249

[17] Opricovic S & Tzeng G H 2004 Compromise Solution by MCDM Methods: A Comparative Analysis of VIKOR and TOPSIS *European Journal of Operational Research* 156(2): 445-455

[18] Opricovic S & Tzeng G H 2007 Extended VIKOR method in comparison with outranking methods *European Journal of Operational Research* 178(2): 514-529

[19] Tzeng G H, Lin C W & Opricovic S 2005 Multi-Criteria Analysis of Alternative-Fuel Buses for Public Transportation *Energy Policy* 33(11): 1373-1383

[20] Zarghami M & Szidarovszky F 2009 Revising the OWA operator for multi criteria decision making problems under uncertainty *European Journal of Operational Research* 198(1): 259-265

[21] Musani S & Jemain A A 2013 A fuzzy MCDM approach for evaluating school performance based on linguistic information *AIP Conference Proceeding: The 2013 UKM FST Postgraduate Colloquium* 1571: 1006:1012

[22] Diakoulaki D, Mavrotas G & Papayannakis L 1995 Determining objective weights in multiple criteria problems: The CRITIC method *Computers & Amp; Operations Research* 22(7): 763-770