

Monitoring Actuarial Present Values of Term Life Insurance By a Statistical Process Control Chart

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Abstract. Tracking performance of life insurance or similar insurance policy using standard statistical process control chart is complex because of many factors. In this work, we present the difficulty in doing so. However, with some modifications of the SPC charting framework, the difficulty can be manageable to the actuaries. So, we propose monitoring a simpler but natural actuarial quantity that is typically found in recursion formulas of reserves, profit testing, as well as present values. We shared some simulation results for the monitoring process. Additionally, some advantages of doing so is discussed.

1. Introduction

Tracking the performance of insurance policy for a group of policyholders is part and parcel of any insurance contract. Typically such work is done laboriously without much help from charting techniques such as a statistical process control (SPC) chart. In this paper, we explore the possibility of using SPC charts to simplify the meticulous work of monitoring performance of insurance policy.

One major challenge for applying SPC charts to insurance applications is the issue of what attribute or measure is of interest to track. Another is how to index this attribute or measure. Yet another is whether the attribute or measure is constant or variable and whether or not it is independent across time. All these are complex questions that complicate a straightforward use of an SPC charting technique.

To understand the challenges, we now review the literature and put these challenges in its context.

2. Literature

Methods of Statistical Process Control (SPC) have been used quite extensively in industrial settings to produce goods to certain quality specifications and to maintain product quality.

The first SPC chart was introduced by [1] which was used as in-stream control tool to guide the production process. Later, [2] popularized the SPC techniques within his Total Quality Management (TQM) idea. Japanese industries extensively espoused Deming's TQM idea where they found that technicians with little or no training in statistics can track product quality fairly well because all they

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had to do was to graphically check if the means randomly fall in a certain band of tolerance to signify acceptability of the product produced by the process [3]-[4].

SPC charts have been used not only in the industrial settings but also in the education setting [5] to track and monitor the consistency of ratings across time. It has also been applied in accounting by [6] to monitor financial reporting of public companies. The SPC technique was discussed by [7] for its applicability in finance to monitor stock market returns. To our knowledge, SPC techniques have yet to find its application in the field of insurance and actuarial science.

2.1. Rules When Tracking with SPC Chart

SPC charts involve tracking some statistical quantities from random samples across time. To provide a clear procedure to identify processes that are out of control, some rules were suggested by [8] as follows:

- 1) Any point is either beyond the upper or lower 3 sigma control limit.
- 2) At least eight consecutive points are on only one side of the control chart.
- 3) Two or three consecutive points are inside the control limit but outside the upper or lower 2 sigma warning limit.
- 4) Four or five consecutive points are outside the 1-sigma limits.
- 5) A non-random or unusual pattern in the data (such as a cyclic pattern).
- 6) One or more points near a warning or control limit.

Originally only the first rule was introduced but over the years more rules were introduced to increase the sensitivity of the SPC charts. Many researchers such as [9] have discussed the sensitivity of these rules.

2.2. Statistical Process Control in Actuarial Science?

Typically SPC charts involve tracking some statistical quantities from random samples across time. The inherent variation shown by the chart is known to be coming from random sampling of these statistical quantities and any departure there from is thought of as coming from some process shift.

Actuarial quantities to be monitored, however, pose some difficulties in applying the SPC charting technique. First, the quantities to be track such as actuarial present values of insurance portfolios are not necessarily random samples across time and most often are not. Second, for each time point, a sample loss observation such as death or accident may or may not be observed. Third, for longer insurance contracts, the actuarial present values can be perceived as a weighted average of the random loss event across several time points. Fourth, tracking a non-suitable quantity will result in futility rather than utility of the SPC technique. Thus, some ingenuity is needed to find a suitable quantity to track.

3. Procedure

3.1. Selection of Appropriate Actuarial Quantity

Two main quantities that are often at the heart of insurance policies are (1) interest rate risk and (2) probability of the loss event. One may track the first quantity fairly well with SPC charting techniques assuming that variable interest rates are fairly stationary around some risk-free target values. In this approach, the observed quantities are thought of as random samples across time. As for the second quantity, the probability of a loss event is typically not independent of tracking time. As such, if such quantity were to be tracked, it cannot be summed or averaged across time nor can the same tracking limits be used.

To avoid the difficulties discussed above and to be relevant to insurance applications, we propose tracking the actuarial present value of a 1-year insurance policy across time. Embedded in this singular quantity is both the interest rate risk and loss probability mentioned earlier. That is, over successive time periods h , we will monitor the observed quantity

$$\hat{A}_{x+h:\overline{1}|} = (1 + i_h)^{-1} \hat{q}_{x+h} \quad (1)$$

against the target

$$A_{x+h:\overline{1}|} = v q_{x+h} \quad (2)$$

where $v = (1 + i)^{-1}$. As can be seen in the above equations, not only does the observed 1-year present values change with time, the target in equation (2) can also change.

To track the performance of 1-year insurance policies, we need to set up 2-sigma warning and 3-sigma control limits (see [8]). The following variance of the 1-year insurance policy is instrumental in setting this up.

$$\text{Var}(v^{K(x)+1}) = {}^2A_{x+h:\overline{1}|} - \left(A_{x+h:\overline{1}|}\right)^2 = v^2 p_{x+h} q_{x+h} \quad (3)$$

The 2-sigma warning and 3-sigma control limits are then defined as follows:

$$\begin{aligned} A_{x+h:\overline{1}|} \pm k \sqrt{\text{Var}(v^{K(x)+1})/n_h} &= v q_{x+h} \pm k v \sqrt{p_{x+h} q_{x+h} / ({}_h p_x \times n)} \\ &= v \left(q_{x+h} \pm k \sqrt{p_{x+h} q_{x+h} / (n \prod_{j=0}^{h-1} p_{x+j})} \right) \end{aligned} \quad (4)$$

where $k = 2$ or 3 for the two types of limits respectively and n_h represents the size of the group expected to survive at time h .

3.2. Data Sources

For this paper, we use the 1989-91 US life Table for female lives as available in [10]. In particular and for illustration, we concentrate on a group of adult females of initial working age of 25 and consistently monitor the 1-year term life insurance for them. A short excerpt of this single-decrement US life Table is given below:

Table 1. 1989-91 U.S. Life Table (for female lives)

x	l_x	d_x	p_x	q_x
25	98325	58	0.99941	0.00059
26	98267	59	0.99940	0.00060
27	98208	61	0.99938	0.00062

For simplicity, we consider the case where benefits are payable only at the end of the year of death. The target and 2- and 3-sigma limits come from the table as described in equations (2) and (4) earlier. The simulated random data of size n_h 25+ h year old adult females come from the following population:

- 1) The random samples of loss events of size $n_h = 1000 \prod_{j=0}^{h-1} p_{25+j}$ (at ages 25+ h , $h = 0, 1, 2, \dots, 25$) comes from Bernoulli $b(1, q_{x+h})$. Note that $n_0 = n = 1000$.
- 2) The random sample of 25 interest spot-rate risk i_h comes from a Normal distribution with mean $i = 0.06$, and standard deviation $\sigma = 0.015$. That is, i_h comes from a $N(0.06, 0.015)$ distribution.

We will refer to this data set as scenario 1.

The simulated observations of loss events above are then used to define the new probability of loss (such a death) events q_{x+h} . The interest spot-rate risk i_h provide the discounting function specific to period h . The loss probability and the discounting function together define the sample observations in period h as described in (1) above.

To provide comparison when the portfolio is beyond expectation, we provide also the following scenarios when the simulated observation data come from the following population:

- 1) Scenario 2 (process shift with disturbances in life Table probabilities): In addition to the random samples from scenario 1, the random samples of loss events of size $n_h = 1000 \prod_{j=0}^{h-1} p_{25+j}$ (at ages $25+h$, $h=16, 17, \dots, 25$) comes from Bernoulli $b(1, q_{x+h} + W)$ where W comes from $N(0.007, 0.003)$.
- 2) Scenario 3 (process shift with larger disturbances in the spot-rate interest risk): The random sample of 25 interest spot-rate risk i_h as in scenario 1 before except when $h=16$ to 25 , the spot-rates come from a Normal distribution with mean $i = 0.09$, and standard deviation $\sigma = 0.03$.
- 3) Scenario 4 (combined disturbances in loss probability and spot-rates): a combination of scenarios 1 and 2 above.

We now track the simulated observation across the h time periods.

4. Results and Discussions

Here we share the results of the simulation study to illustrate the potential use of the SPC charts in Actuarial applications.

Figure 1 below provides a simulated result for a 1-year term life insurance portfolio for 1000 participants with end-of-year benefit of 1 unit. As can be seen in figure 1, the observations are all within the warning and control limits. This signifies that there are no unexpected violations in the statistical process for the insurance portfolio.

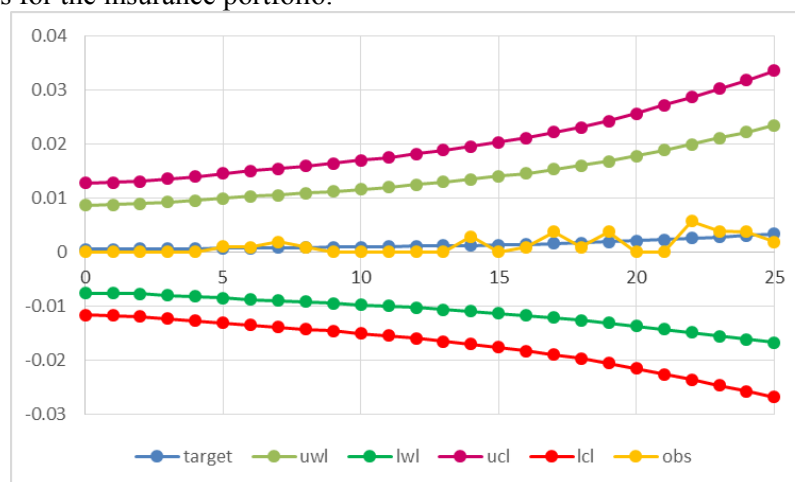


Figure 1. Statistical Process Control Chart for a 1-year term Life Insurance

Figure 2 below provides the result when there is a simulated upward shift in the loss probability.

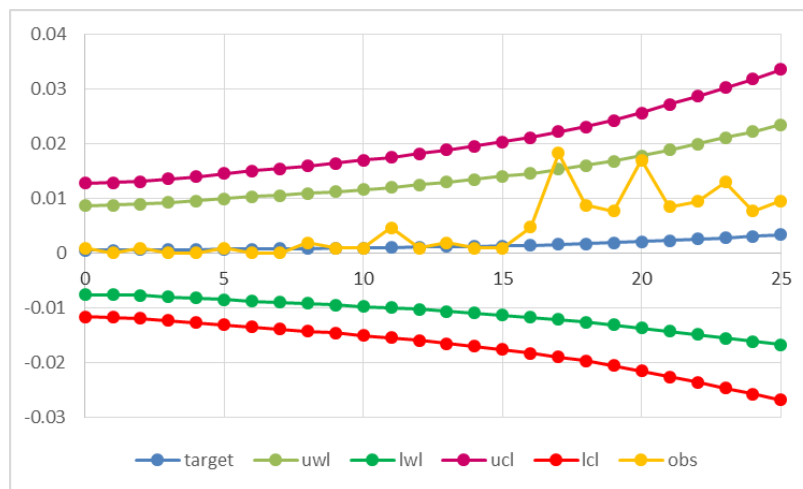


Figure 2. SPC Chart for a 1-year term Life Insurance with some disturbance in loss probabilities

As shown by figure 2, we observe a violation of the upper warning limit at time $h=17$. We also have an observation nearing the upper warning limit at $h=20$. It is also apparent that the process has shifted from the target values since all observations after time $h=16$ were oscillating around a higher mean than the target values. Figure 2 shows four of the six characteristics of an “out of control” process described by [8] earlier. That is, although no points are outside the 3 sigma limits and less than 2 consecutive points are outside the 2 sigma warning limits, the other four rules described by [8] earlier were violated.

Figure 3 below provides the results when there is a simulated shift in the interest spot-rates. As shown by figure 3, although there are some disturbances in spot-rates, there does not seem to be any noticeable disturbance in the process as described by [8].

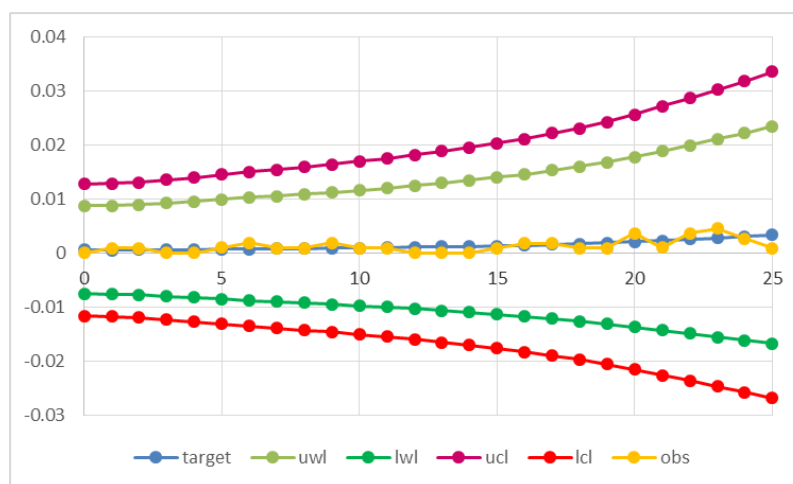


Figure 3. SPC Chart for a 1-year term Life Insurance with some disturbance in spot-rates

Some explanation of the results can be offered here. Since we are monitoring 1-year term life actuarial present values, the discounting function does not show much effect over a short period of one year. If the contracts we monitor are more than 1-year, the compound effects of the spot-rates on the observed actuarial present values may have been more noticeable.

Figure 4 below provides the results when there is a simulated shift in the interest spot-rates as well as a shift in the loss probabilities.

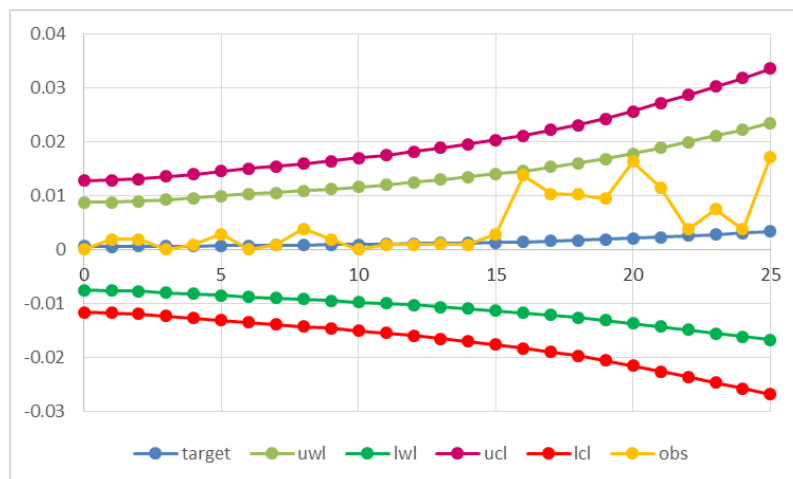


Figure 4. SPC Chart for a 1-year term Life Insurance with some disturbance in spot-rates and some disturbances in loss probabilities

As shown by figure 4, we have observations nearing the upper warning limit at $h=16$ and $h=20$. It is also apparent that the process has shifted since all observations after time $h=16$ were oscillating around a higher mean than the target values. Figure 4 shows four of the six characteristics of an “out of control” process described by [8]. That is, although no points are outside the 3 sigma control limits and no points are outside the 2 sigma warning limits, the other four rules described by [8] earlier were violated.

5. Conclusion

The use of SPC charts for actuarial applications have been explored in this study. It is found that 1-year term life insurance contracts are suitable to be monitored across time. It is beneficial to monitor such short-term contracts for several reasons. One, profit or loss on insurance portfolios are measured at least annually and could be measured as frequent as monthly or quarterly. Two, although longer-term contracts are not monitored here, the recursive relationships in actuarial present values can be used on the one-year contracts to derive the effects on the longer-term contracts.

Our simulation results suggest that the effect of changes in loss probabilities affect the observed actuarial present values more than the same effects in spot-rates. This could be partly due to our focus on short-term 1-year contracts where spot-rates for one-year may not provide much of a difference in short term actuarial present values.

6. Study Limitations

Our results indicate that the effect of changes in the process in terms of loss probabilities may affect the observed actuarial present values more than the same effects on spot-rates. If we were able to monitor longer-term contracts rather than 1-year terms, our results may have been different. In particular, if we were able to monitor more than 1-year contract terms, we would have been able to see the compounding effects of investments based on the interest spot-rates and the long-term compounding effects of the mortality rates. However, this is not only difficult as described earlier but also since profit determinations for insurance companies must be done at least once a year, tracking over longer periods practically does not help companies determine their level of annual profits.

In this paper, we have provided some simulated results to demonstrate the use of SPC in actuarial science where such methods have not been used before. More research is however needed to know more about the promise of such method.

Moreover, more extensive simulation work than the one done here is needed if the focus is to tease out which of the two components (loss probability or spot-rates) of the observed actuarial present values is more susceptible to disturbances.

7. References

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Acknowledgments

The author appreciates KFUPM for providing research facilities for this study.