

# Modeling Of The Pores Form Influence On The Hydraulic Resistance Of Membranes And Their Permeability

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**Abstract.** Until the present time, modeling of the pores form influence on the hydraulic resistance of membranes and their permeability has not been analyzed. The present study suggests the determination of the optimum form of pores and the thorough consideration of the issue on the productivity of polymer membranes with the profile pores.

## 1. Introduction

The widespread notion about the structure of porous adsorption layers are based on the possibility of describing pores as formation, to which definite geometric sizes and form can be ascribed [1]. In correspondence with such notions, the classification of porous systems is usually based on likening the real systems to the artificial ones, constructed on the models with the organized structure by several rules [2].

The model with various types of spheres packs and other bodies of the same size is the most wide-spread. On the basis of this model, the model of capillaries of various form and length is constructed as well as the model of pores as conventional canals between the contacting spheres [3].

## 2. The main part

The work [4] introduces the experiments, where glass capillaries have been chosen as the model objects, correlations between geometric characteristics of which (diameter, length of the cylindrical section, length of the conic section and so on) are close to such ones for pores in the track-etched membranes. The configuration with sharp contractions ('zero' thickness of the wall) has been received by the following way. The metallic diaphragm with round hole 1,0 mm was attached to the end of the cylindrical capillary. The diaphragm thickness comprised 200 mcm, that is it was essentially less than the hole diameter.

The volume speed of the flow of the semi synthetic motor oil at the pressure differential is  $P = 353$  Pa. The corresponding calculation values show that the fault of measuring time and amount of flowing liquid introduced an error to the end result less than 1%. The coefficient of the dynamic viscosity of the oil was taken equal to 0,293 Pa·s (measured at 18,5<sup>o</sup>C). Under the given conditions, the liquid flow corresponded to Reynolds numbers of the order  $10^{-3} - 10^{-2}$ .



The wholly satisfactory fit of the measured and calculated values is being observed. The differences are partially stipulated by the errors of defining viscosity and pressure differential. Therefore, relations of flow speeds for various capillaries are more demonstrative. These relations, calculated separately from measured and predicted values, conform much better [7].

This is an advantage in the productivity, which has been achieved by the transition from the cylindrical form to the cone-cylindrical one. Let us note that in spite of the sufficiently gross simplifications, the experiment with capillaries doesn't show essential divergence with the calculation data. These results allow to conclude that the above mentioned method is sufficiently adequate and can be employed for the optimization of pores form in the membranes.

Using the derivative relations, we will calculate specific productivity of the membranes, having various configurations of pores. Within the same configurations, we will vary the correlation between  $R$  and  $r$ , as well as between  $L$  and  $l$ . Our task will be the finding of such combination of membrane parameters, at which the volume speed of the flow for the given fixed  $r$  will be maximum.

We calculate the specific productivity of the membrane, related to the ordinary porosity. In this case, the calculations results can be presented in two-dimensional system of coordinates, exactly in the form of functions  $Q_m / \varepsilon (R)$  at varied values of the parameters  $l$  and  $r$  and fixed  $L$ .

### 3. The experimental part

Let's consider the speed of the liquid flow through the membranes with pores of various configurations.

Let the pore be a twisting canal with the sharp contractions at the ends, which can be considered as the holes with zero thickness of the wall. The pressure fall in the internal site territory of the membrane pore by the width  $L$  can be found from Hagen-Poiseuille formula:

$$\Delta P_1 = \frac{8q\mu LU}{R^2} \quad (1)$$

where  $\Delta P_1$  is pressure differential,  $\mu$  – a coefficient of dynamic viscosity of the medium,  $U$  - speed of the flow inside the pore,  $R$  – pore radius,  $q$  - a coefficient of pores tortuousness.

Let's define the tortuousness coefficient by the formula

$$q = \frac{l}{L} \quad (2)$$

$l$  - average length of the pores.

The pressure differential on the hole in the partition of zero thickness can be expressed in the following way:

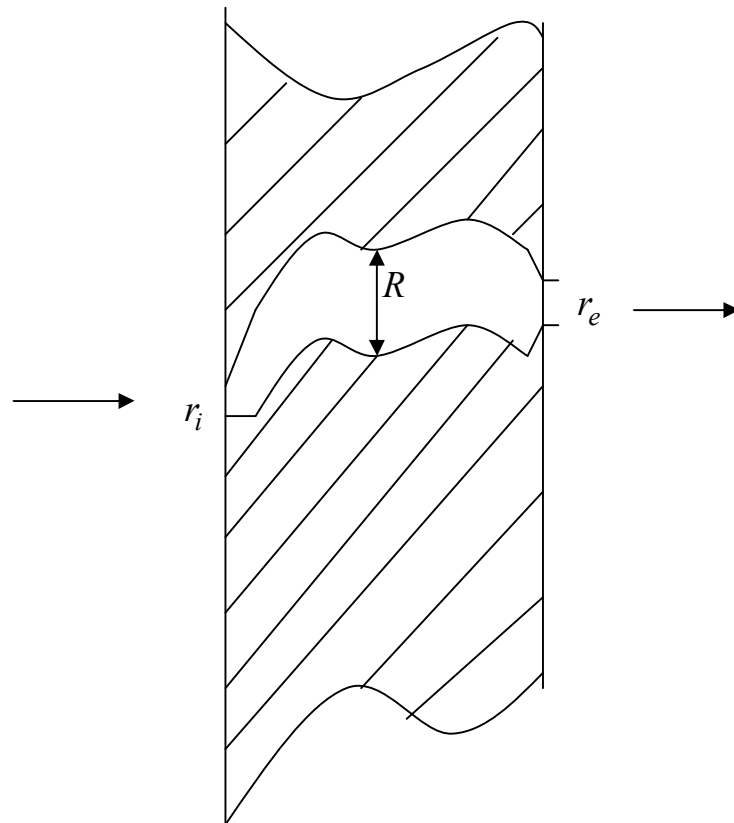
$$\Delta P_2 = \frac{3\alpha\mu\pi R^2 U}{r^3} \quad (3)$$

Here  $r$  – a hole radius at the output of the pore on the membrane surface,  $\alpha$  - a coefficient, taking into account the asymmetric arrangement of the exit hole of the pore relating to its axis.

In the first approximation, we will accept that the pressure differential in all pores will be equal to the total of the pressure differentials in the cylindrical section and at the end contractions:

$$\Delta P = \Delta P_1 + \Delta P_2^i + \Delta P_2^e \quad (4)$$

Where  $\Delta P_2^i$  - differential at the internal edge;  $\Delta P_2^e$  - differential at the external edge.



**Figure 1. Flow diagram through the pore.**

Let  $N$  – a number at the area unit of the membrane surface.

Then the coefficient of the free cross-section of the membrane or surface porosity is defined by the formula:

$$p = \pi^2 N \quad (5)$$

The connection between volume porosity  $\varepsilon$  of the membrane with pores of the given configuration and surface porosity is calculated in the following way:

$$\varepsilon = pq \quad (6)$$

From here, we get the expressions for the differential at the membrane edge:

$$\Delta P_2 = \frac{3\alpha\mu\pi\varepsilon U}{pr} \quad (7)$$

$$\Delta P_2 = \frac{3\alpha\mu\pi q U}{r} \quad (8)$$

Then the interconnection between the flow, filtered through the liquid membrane  $Q$  and its geometric characteristics will look like:

$$\Delta P = \frac{\mu q^2 Q}{\varepsilon F} \left[ \frac{8L}{R^2} + 3\pi \left( \frac{\alpha_i}{r_i} + \frac{\alpha_e}{r_e} \right) \right] \quad (9)$$

where  $F$  - membrane area.

2. Let the pore have conic contraction at the ends that is close to the real form of the pores, which can be received in the practice [6].

We assume that the cylindrical section by the length  $L$  at the ends transits to the conic contraction by the length  $l_k$ , ended by the holes of the radius  $r$ .

The pressure fall  $\Delta P_k$  in the conic site territory of the pore at the narrow angles of conicity  $\gamma$  can be found from the following correlation [5]:

$$\Delta P_k = \frac{8q\mu Q R^2 l_k}{3r^4 \varepsilon F} (z + z^2 + z^3) \quad (10)$$

where

$$z = 1 - \frac{r_l}{R} \quad (11)$$

Summing the pressure falls in the internal and conic site territories; we will receive an expression for the resulting pressure differential through the asymmetric membrane:

$$\Delta P = \frac{8q\mu Q}{\varepsilon F} \left[ \frac{Lq}{R^2} + \frac{R^2}{3} \left( \frac{l_{ki}}{r_i^4} (z_i + z_i^2 + z_i^3) + \frac{l_{ke}}{r_e^4} (z_e + z_e^2 + z_e^3) \right) \right] \quad (12)$$

If the pore has an asymmetric configuration, there is the contraction only on one side of the membrane and we will receive:

$$\Delta P = \frac{8q\mu Q}{\varepsilon F} \left[ \frac{Lq}{R^2} + \frac{R^2}{3} \left( \frac{l_{ki}}{r_i^4} (z_i + z_i^2 + z_i^3) \right) + \frac{3\pi q \alpha_e}{r_e} \right] \quad (13)$$

#### 4. Results

Analysis of the results leads to the following conclusions:

1. Even at zero thickness of the selective layer, the membrane productivity growth is constrained by the factor 4-5.
2. With the growth of the internal radius of pores related to the ordinary porosity, the specific productivity of membranes passes through maximum. The state of the maximum depends on the thickness of the selective layer.
3. At the thickness of the selective layer 3 mcm, the advantage in the productivity is insignificant. At the thickness of the selective layer 1 mcm, the growth of the productivity is essential, whereby the optimum correlation between internal radius of pores and pores radius in the selective layer comprises 3- 3,5.

#### 5. Conclusion

In our calculations, we assume that the volume porosity should be constrained by the size 0,15. The choice of this boundary is somewhat conventional; it is mainly imposed by the requirement to the mechanic strength of the membrane. As it is known from the experiment, track-etched membranes based on the films are sufficiently durable at the porosity not more than 0,15-0,25. More thorough procedure of optimization should take into consideration that the strength depends not only on the volume porosity but also on film thickness as well as on the pores density [8]. At equal values of the volume porosity, the membranes with higher density of pores have slightly worst mechanic characteristics than the membranes with less density of pores [9]. Therefore the final selection of optimum parameters of membranes can be realized only experimentally.

The carried out theoretical consideration allows calculating the specific productivity of membranes with thin selective layers. In spite of the approximate character of model calculations, they

are essential for the optimization of the membranes structure and the further processing of the technology of their production. The following step in our research will be the production of membranes with pores of various profiles and comparison of their properties with the calculated ones.

## References

- [1] M. Nakagaki 1991 *Physical Chemistry membranes* (Moscow) p.256.
- [2] N.I. Nikolaev 1980 *Diffusion in membranes* (Moscow) pp.134-167
- [3] A.V. Smirnov, A.S. Bartov 2005 *Probabilistic approach in assessing the selectivity of the membrane to separate ions* Membranes (№ 2 (26)) pp. 23-30
- [4] P.Yu.Apel and S.N.Dmitriyev 2004 *Optimization of track membranes formes* Critical technologies, Membranes, №3 (23), pp. 32-37
- [5] Zh.Umarova, Sh.Eskendirov 2012 *Mechanism of Gases Transfer through Polymer Membranes and Membrane Module Calculation Model to Separate Gases* Procedia Technology Volume 1 pp. 356–361
- [6] Zh.Umarova 2011 *Algorithm for Solving the Model of Multistage Hollow Fiber Membrane Systems and the Identification of Model Parameters* Informatics Engineering and Information Science, Volume 251 pp 466-476
- [7] Zh.Umarova, S.Botayeva, A.Yegenova, A.Usenova 2014 *Mathematical Model and Calculation Algorithm of Micro- and Meso Levels of Separation Process of Gaseous Mixtures in Molecular Sieves* International Conference on Mathematics, Engineering & Industrial Applications (ICoMEIA 2014) proceedings book (Malaysia) p.74
- [8] A.V. Smirnov, A. Bartov 2005 *Probabilistic approach in assessing the selectivity of the membrane to separate ions* Membranes № 2 (26) pp. 23-30
- [9] E.R.Volkova, E.A.Denysiuk 2003 *New method for estimating the diffusion coefficient of the penetrant in polymeric materials* Aerospace and high technology (Perm) p.45