

# Group velocity equalisation in multimode waveguides using inverse scattering designs

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**Abstract.** In this paper, using an inverse scattering approach, we describe how the selection of mode effective indices and thus phase velocities can be used to control group velocity in a waveguide. As such it is shown that differential group delay can be equalised or minimised over a wavelength of choice. A particular feature of the new designs is the development of rings and a peaked core which may split depending upon the number of guided modes. These designs show characteristics comparable with commercially available fibres but with refractive index profiles that differ from typical graded-index designs.

## 1. Introduction

There is currently great interest in spatial division multiplexing [1] in order to overcome the impending “capacity crunch” [2] of single-core, single-mode optical fibre transmission systems. In particular, mode-division multiplexing is of particular interest because of the large number of modes that can be used in a single fibre. However, in order to use such methods it is important to control the deleterious effect of differential group delay (DGD) as well as mode-coupling. As a result work has been done to investigate the tailoring of optical fibre refractive index (RI) profiles to meet these needs [3–5]. One particular design by Gruner-Nielsen et al. consists of a graded-index core and an outer trench RI. While the performance of this design in terms of minimisation of DGD and low mode coupling between the  $LP_{01}$  and  $LP_{11}$  modes is very good, it relies upon a parametric design process. We were interested to know whether direct control of mode effective indices could be used to control both of these characteristics.

In this paper we extend our work [6] on the inverse scattering (IS) design of single-mode planar waveguides to the case of multimode planar waveguides through the use of Darboux transformations as have been used previously by Mills[7] in the design of optical interconnects. We show that through manipulation of the phase velocities of the waveguide it is possible to manipulate the group velocities to the extent of controlling the wavelength of equalisation and/or minimising the DGD over large wavelengths. It is clear that we have performed this investigation in planar waveguides rather than optical fibres, but we believe that the qualitative features of the designs will carry over.

## 2. The Darboux transformations

As has previously been described and utilised by other authors [8,9], there exists a correspondence between a bound state in quantum mechanics and a discrete propagating mode in a waveguide. The IS theory allows for the construction of a potential with a given set of bound states a priori through the



use of the Darboux transformations [10]. In this paper we will not describe in detail the process involved other than to say that associated with the set of  $N$  propagation constants  $\beta_m$  are a set of bound state eigenvalues  $k_m$  defined through the relationship

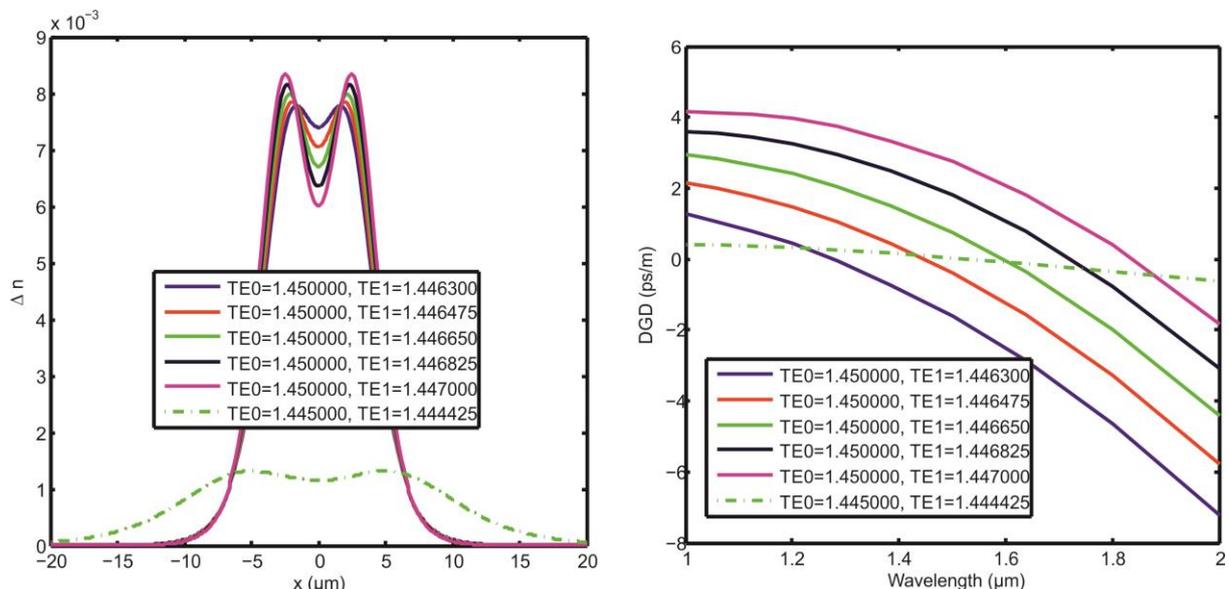
$$k_m^2 = k_0^2 n_2^2 - \beta_m^2 \quad (1)$$

Where  $k_0 = 2\pi/\lambda_d$  is the free-space wavenumber at the design wavelength  $\lambda_d$ ,  $n_2$  is the refractive index of the cladding of the waveguide and  $\beta_m$  is the  $m$ 'th propagation constant.

Given that there exists Brown's identity [11] connecting the phase velocity and group velocity to a weighed integral of power densities in waveguide regions we proposed that control of dispersive properties would be possible through the above method. With the propagation constants supplied a priori the potential is derived and its dispersive properties obtained through direct scattering. We applied this method to the design of planar waveguides with two, three and four modes. In all cases the design wavelength was chosen to be  $\lambda_d = 1.55\mu\text{m}$  and a cladding index of  $n_2 = 1.444$ . In addition, it was assumed in calculating the DGD that material dispersion is the same for all modes and profile dispersion is negligible.

### 3. Dual-mode designs

We first select the effective index of the  $\text{TE}_0$  mode and vary the spacing of the  $\text{TE}_1$  mode with respect to this and the cladding. We find that through the variation of the  $\text{TE}_1$  mode effective index we may change the wavelength for which group velocity equalisation is achieved. We illustrate this procedure in figure 1 while demonstrating how the refractive index of the design is also altered. It can be seen that as the effective index of the  $\text{TE}_1$  mode approaches that of the  $\text{TE}_0$  mode we have a deepening of the central dip and a movement towards longer wavelengths of the dispersion equalisation wavelength. This behaviour was originally discussed by Stolen [12] where he considered optical fibres with ring index profiles.

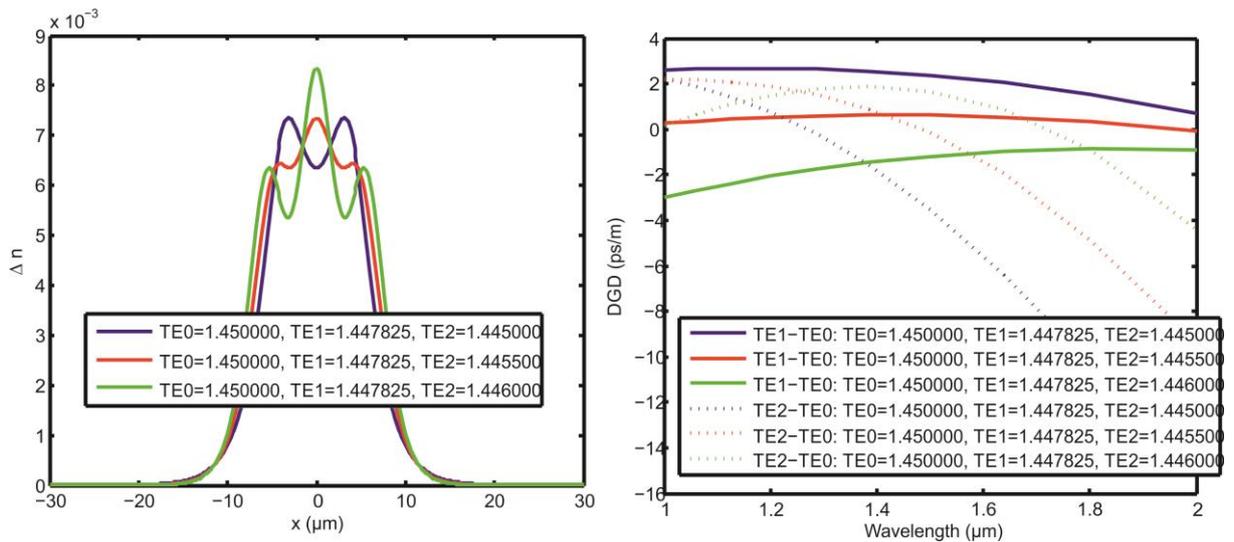


**Figure 1.** Dual mode waveguide designs with varying group velocity equalisation wavelength specified by changing  $\text{TE}_1$  mode effective index for fixed  $\text{TE}_0$

It is also interesting to note that reducing the effective index of  $\text{TE}_0$  results in a scaling down and across of the refractive index profile as well as a reduction in the slope of the DGD as can be seen in figure 1. As such the DGD can be set to zero at a design wavelength and minimised over a large wavelength about this point.

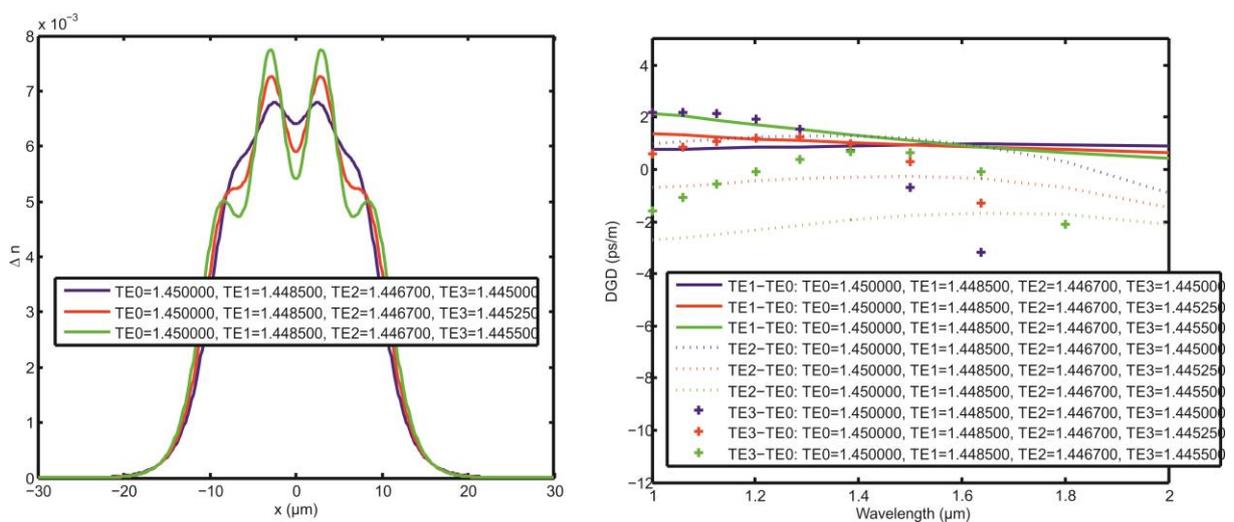
#### 4. Three and four-mode designs

In this section we show that designs can be obtained with minimisation of DGD with respect to  $TE_0$  of all modes. This is achieved by varying the effective indices of the propagating modes. In particular, in figure 2 we show a design for a three mode waveguide, coloured red, for which DGD of all modes is brought to within  $\sim 1$  ps/m. This DGD is close in range to that associated with graded-index few-mode fibres produced by OFS ( $\pm 0.4$  ps/m) [13] but with a different design.



**Figure 2.** Three mode waveguide designs showing variation with wavelength of DGD with the red curves showing DGD of all modes to within  $\sim 1$  ps/m

It is shown that equi-distant effective indices (blue curve), which approximate the standard parabolic RI profiles, result in waveguides with the highest DGD. Varying the inter-modal effective-index differences results in a pronounced central peak followed by the development of a ring. Increasing the contrast of the central peak and emphasising the development of the outer ring leads to minimisation of DGD at an ever larger wavelength as shown by the green design.



**Figure 3.** Four mode waveguide designs showing variation with wavelength of DGD with the red curve corresponding to a DGD of all modes of  $\sim 1$ ps/m

In figure 3 we illustrate a four mode waveguide design, once again coloured red, for which DGD of all modes is once again  $\sim 1$  ps/m. From figure 2 and 3 it can be seen that the overall qualitative shape of the three and four mode designs is the same with the slight development of a ring but in the four mode case an additional splitting of the central peak region. As in the two mode and three mode cases this is to be expected from coupler theory where approximately degenerate modes are associated with separation of the waveguide cores.

## 5. Conclusions

In this paper we have shown that control of effective indices of modes through the use of the Darboux transformation can be used parametrically to equalise or minimise differential group delay over a range of wavelengths. In addition we have identified that these waveguides can all be described qualitatively as consisting of an outer ring and a peaked core which may in turn split as the number of guided modes increases.

Our designs have shown characteristics comparable with those of commercially available few-mode fibres. While our investigation has been limited to planar waveguides there are similarities in intensity profiles between TE modes and LP modes and it is expected from Brown's identity that characteristics of these planar designs can be adapted for use in future optical fibres.

## 6. References

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