

# Stochastic optimization of the scheduling of a radiotherapy center

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**Abstract.** Cancer treatment facilities can improve their efficiency for radiation therapy by optimizing the utilization of the linear accelerators (linacs). We propose a method to schedule patients on such machines taking into account their priority for treatment, the maximum waiting time before the first treatment, the treatment duration, and the preparation of this treatment (dosimetry). At each arrival of a patient, the future workloads of the linacs and the dosimetry are inferred. We propose a genetic algorithm, which schedules future tasks in dosimetry and a constraint programming formulation to verify the feasibility of a planning of dosimetry. This approach ensures the beginning of the treatment on time and thus avoids the cancellation of treatment sessions on linacs. Preliminary results indicate the improvements of this new procedure.

## 1. Introduction

Patients with cancer can wait significant amount of time in Quebec for their treatments. After the first consultation, a patient must take a series of exams before undergoing his treatment on a linear accelerator (linac), which irradiates the malignant tumor to kill the infected cells. The total waiting delay is measured as the number of waiting days between the first consultation and the beginning of the treatment. Managers need to reduce this delay and must report detailed statistics to state authorities on a regular basis. For example, a team of the McGill University Health Center in Montreal has decreased the delay from 76 days to 56 days in one year for patients suffering from cell lung cancers [1]. The team achieves such a goal by improving their processes and their patient flow. However, Quebec authorities have fixed an objective of 28 days as a maximum delay. There still need improvements for cancer care processes: this improvements are crucial knowing that cancer costs \$17.4 billion per year in Canada and that the number of patients diagnosed with cancer keeps increasing by 1.5% every year [2]. Legrain *et al* [3] have shown that an efficient utilization of the linacs can decrease this delay. Despite this improvement, the patient can face some unnecessary delays if the preparation of the cancer treatment (pretreatment) is not correctly planned and executed. The pretreatment consists



mainly in planning the shape, the intensity, and the direction of the beams of the linac: this part is called the dosimetry. If pretreatment planning is not completed, the beginning of the treatment can be postponed or even canceled. Indeed, the appointments on the linac are usually taken before the end of the pretreatment, and thus a delay can cancel the first ones. This paper presents tools to improve the efficiency of the pretreatment process. It aims at reducing pretreatment process times and at avoiding unnecessary cancellations.

This work has been realized in collaboration with the Centre Intégré de Cancérologie de Laval (CICL) and is based on previous work [3]. The main bottlenecks are first the linacs and second the dosimetry. We propose to deal both bottlenecks in an online fashion. When a patient arrives in the center, a clerk books his appointments on a linac according to the number of prescribed sessions. The challenge lies in evaluating the future resource needs to book the best possible appointments. A cancer treatment facility must treat two different kind of patients: palliatives and curatives. Palliative patients need a quick treatment to relieve their pain, they are considered as high priority patients and should start their treatment less than 3 days after their admission. The center delivers a more precise treatment to curative patients to maximize their recovery chances; they are considered to be lower priority patients and can wait until 14 or 28 days according to their type of cancer. The management should balance their resources between these two kinds of patients and must respect as much as possible these deadlines. The pretreatment is simplified and reduced to a series of fixed waiting times and two tasks in dosimetry. The first task is the preparation of the treatment and the second task is its verification. The two tasks must be performed by two different dosimetrists. The fixed delays allow to perform all the others tasks of the pretreatment. The scheduling of the pretreatment can thus be viewed as an hybrid flow-shop with recirculation.

Job-shop scheduling problems are known to be difficult to solve. Methods such as mixed-integer programming [4] or constraints programming [5] achieve poor performance on big and complex job-shops. Gélinas and Soumis [6] present a column generation approach which succeeds to solve problems with 250 jobs only if there is less than two operations by jobs. When the number of operations increases and the constraints become more complex, one prefers to use heuristics or meta-heuristics. Widmer and Hertz [7] propose a successful tabu search for a flow-shop problem; Cheng *et al* [8] give an important review of genetic algorithms applied to job-shop scheduling. Gröflin *et al* [9] also propose advanced tabu search to solve very complex job-shop scheduling problems. The flow-shop problem is a little easier, because operations in a job are always performed in the same order. Furthermore, when an operation can be realized on different or parallel machines, the problem is so-called the hybrid flow-shop problem. Ruiz and Vázquez-Rodríguez [10] present an extensive review of this problem. As the dosimetry process imposes that the preparation and the verification operations are performed by the same team of dosimetrists, this problem is modeled by an hybrid flow-shop with recirculation. Bertel and Billaut [11] provide a very efficient genetic algorithm with an interesting cyclic crossover mechanism. The genetic algorithm introduced here draws inspiration from this latter procedure.

Pretreatment scheduling does not have an extensive literature. Castro and Petrovic [12] present a mathematical programming formulation and some heuristics to solve the whole pretreatment scheduling problem. They assume that the appointments on linacs have already been booked: the pretreatment planning must thus be built considering these appointments as hard due dates. The contribution of this paper is thus to propose a matheuristic solving the pretreatment scheduling and the radiotherapy booking problems at the same time in an online fashion: it is a flow-shop problem with two kind of machines.

Section 2 presents the whole formulation and algorithms. Section 3 shows preliminary results on a real instance from the CICL. Section 4 closes the article with final remarks.

## 2. Methodology

The model proposed by Legrain *et al* [3] is here modified to take into account the dosimetry scheduling. The formulation (1) presents a stochastic optimization model. The problem is modeled as a two-machines flow shop: the dosimetry and the linacs. The planning of the dosimetry and appointments on linacs are represented by columns. At the arrival of patient  $j$ , the model infers the average cost of linacs' planning on a set  $\Omega_j$  of future patients scenarios. The variables  $x_{ij}$ ,  $y_{il}^\omega$ , and  $v_i^\omega$  are equal to 1 if respectively the linac's appointments  $i \in S_j$  is chosen for patient  $j$ , the linac's appointments  $i \in S_l$  is chosen for patient  $l$  in the scenario  $\omega \in \Omega_j$ , and the dosimetry planning  $i \in S^D$  is chosen for all patients of the scenario  $\omega \in \Omega_j$ . These variables are equal to 0 otherwise. The linacs appointments are represented by the parameters  $a_{ilk}^m$  ( $= 1$  if the patient  $l$  is booked on linac  $m$  the day  $k$  for the appointments  $i$ , 0 otherwise),  $b_{ij}$  (the day of the first appointment for the appointments  $i$ ) and the cost  $c_{ij}$ . The price  $c_{ij}$  of a linac appointments is a combination of waiting times and deadlines violation costs. The end of the pretreatment for patient  $l$  in the dosimetry planning  $i$  is the parameter  $r_{ij}$ . Finally, the variables  $z_{mk}^\omega$  count the number of slots booked in overtime the day  $k$  on linac  $m$ . They are bound for the linac  $m$  by  $O_{day}$  on a day and by  $O_{week}$  on a week.

$$\min \sum_{i \in S_j} c_{ij} x_{ij} + \mathbb{E}_{\omega \in \Omega_j} \left[ \sum_{l \in \mathcal{P}^\omega} \sum_{i \in S_l} c_{il} y_{il}^\omega + \sum_{k \in H} \sum_{m \in M} c^o z_{mk}^\omega \right] \quad (1a)$$

subject to:

$$\sum_{i \in S_j} x_{ij} + \sum_{i \in S_l} y_{il}^\omega \geq 1 \quad (1b)$$

$$\sum_{i \in S_l} y_{il}^\omega \geq 1, \quad \forall \omega \in \Omega_j, \forall l \in \mathcal{P}^\omega \quad (1c)$$

$$\sum_{i \in S^D} v_i^\omega \geq 1, \quad \forall \omega \in \Omega_j \quad (1d)$$

$$\sum_{i \in S_j} b_{ij} x_{ij} + \sum_{i \in S_l} b_{il} y_{il}^\omega - \sum_{i \in S^D} r_{ij} v_i^\omega \geq 0, \quad \forall \omega \in \Omega_j \quad (1e)$$

$$\sum_{i \in S_l} b_{il} y_{il}^\omega - \sum_{i \in S^D} r_{il} v_i^\omega \geq 0, \quad \forall \omega \in \Omega_j, \forall l \in \mathcal{P}^\omega \quad (1f)$$

$$\sum_{i \in S_j} a_{ijk}^m x_{ij} + \sum_{l \in \mathcal{P}^\omega} \sum_{i \in S_l} a_{ilk}^m y_{il}^\omega \leq F_k^m + z_{mk}^\omega, \quad \forall m \in M, \forall k \in H, \forall \omega \in \Omega_j \quad (1g)$$

$$\mathbb{1}_{\mathcal{P}_p}(j) \sum_{i \in S_j} a_{ijk}^m x_{ij} + \sum_{l \in \mathcal{P}_p^\omega} \sum_{i \in S_l} a_{ilk}^m y_{il}^\omega \geq z_{mk}^\omega, \quad \forall m \in M, \forall k \in H, \forall \omega \in \Omega_j \quad (1h)$$

$$\sum_{k=b}^{b+4} z_{mk}^\omega \leq O_{week}, \quad \forall m \in M, \forall b \in \mathcal{B}, \forall \omega \in \Omega_j \quad (1i)$$

$$z_{mk}^\omega \in [0, O_{day}], \quad \forall m \in M, \forall k \in H, \forall \omega \in \Omega_j \quad (1j)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in S_j \quad (1k)$$

$$y_{il}^\omega \in \{0, 1\}, \quad \forall l \in \mathcal{P}^\omega, \forall i \in S_l, \forall \omega \in \Omega_j \quad (1l)$$

$$v_i^\omega \in \{0, 1\}, \quad \forall i \in S^D, \forall \omega \in \Omega_j \quad (1m)$$

Constraints (1b) and (1c) ensure respectively that patient  $j$  and all future patients are scheduled. Constraints (1d) choose a dosimetry schedule for each scenario  $\omega$ . Constraints (1e) and (1f) ensure respectively that patient  $j$  and all future patients have their pretreatment ready on time for their first treatment on a linac: they are precedence constraints from the flow-shop problem. Constraints (1g) verify that the capacity (including the overtime) of each linac is not exceeded. Constraints (1h) force palliative patients to be scheduled in available overtime slots. Constraints (1i) and (1j) bound respectively the weekly and daily overtime on each linac. Constraints (1k), (1l), and (1m) are integrality constraints. Finally, the objective (1a) is divided in two parts: the cost of a planning for the patient  $j$  and the average future cost of linacs' planning.

The columns  $x_{ij}$ ,  $y_{il}^\omega$  representing patients appointments on linacs are all inserted once at the beginning. The others columns  $v_i^\omega$  are generated during the process with the genetic algorithm 1 inspired by Bertel and Billaut [11].

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**Algorithm 1 Genetic Algorithm**


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**Population:** list of chromosomes:  $P := [ ]$

**Initialization:** add  $N/4$  chromosomes built with dispatching rules, fill the rest with random chromosomes

**while** solving time  $< T$  AND not enough different chromosomes with a negative reduced cost  
**do**

**for all** chromosome  $c$  in population  $P$  **do**

**Cyclic Crossover:** with probability  $p_c$ , cross  $c$  with a random different chromosome

**Mutation:** with probability  $p_m$ , swap randomly two positions of  $c$

**Intensification:** with probability  $p_l$ , make a small local search with insertions on  $c$

**end for**

**Add** all new chromosomes to  $P$

**Selection:** keep the best  $N$  chromosomes

**end while**

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The dosimetry scheduling problem has two tasks and a time window per job. The time windows correspond to some fixed delays: they ensure that all the others pretreatment tasks than dosimetry's ones are performed within this waiting times. The two tasks represent the preparation and the verification of the treatment by a dosimetrist. As the formulation (1) follows a Dantzig-Wolfe scheme [6], this flow-shop problem minimizes the weighted completion time (due to the dual variables linked to constraints (1e) and (1f)). Algorithm 1 tries thus to build in a certain amount of time  $T$  a population  $P$  of  $N$  chromosomes with a negative reduced cost. A chromosome is a sequence of jobs and represents a dosimetry planning. A job can be found two times in a chromosome: the first appearance corresponds to the preparation of the dosimetry, while the second corresponds to the verification. A planning is then made from a chromosome by simply scheduling in the chromosome order each task as early as possible. Finally, the intensification phase aims at improving some chromosomes: several insertions are tested to decrease the reduced cost.

The algorithm 2 presents the global booking procedure for each patient  $j$ . We solve the slave problems obtained by the Benders' decomposition [13] applied on (1) instead of solving the whole formulation. A dispatching rule schedules all the pending dosimetry tasks to build an operational planning. Algorithm 2 uses specifically the earliest late start time rule which gives a good solution for a parallel machine problem [14]. Yet the feasibility of a set of pretreatment tasks is checked with a constraints programming model. Finally, the computation of the average reduced costs is explained in [3].

**Algorithm 2 Online Scheduling Algorithm**


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Initialize waiting list of dosimetry operations:  $Q := []$ 
for all patient  $j$  arrivals do
  schedule dosimetry tasks until current patient arrival day with a dispatching rule
  remove all scheduled dosimetry's operations from  $Q$ 
  for all scenarios  $\omega$  do
    solve slave problems with value  $x = 0$  for event  $\omega$ 
    while genetic algorithm adds new columns do
      insert new columns
      solve slave problems for event  $\omega$ 
    end while
  end for
  select a schedule  $i$  which is:
  • minimum for the average reduced cost
  • feasible for all dosimetry's operations in  $Q$  (checked with constraints programming)
  update remaining resources  $F_k^m$ 
  insert the two dosimetry tasks of patient  $j$  in  $Q$ 
end for

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**3. Results**

The instance used for the computational test is a real data set of the CICL with 181 patients on 77 working days. There are two linacs with 29 slots plus 3 in overtime and 4 dosimetrists. The scenarios used for the online algorithms have been drawn from the empirical patients distribution of the CICL: 5 scenarios are sampled for each patient. However, a proportion (70%) of curative patients are known in advance as they have undergone a surgery and/or a chemotherapy treatment in the center.

Table 1 compares four algorithms according to different criteria: the number of violated deadlines, the average waiting time, the number of used overtime slots, the number of patients which have respectively violated a deadline or seen their first appointment canceled because of the pretreatment. The CICL algorithm is the one currently used by the facility. The online clairvoyant algorithm is presented in [3]. These two procedures, unlike the two following, do not take into account the dosimetry planning in their decision process, but just a fixed delay. The online scheduling clairvoyant algorithm follows the procedure 2. Finally, the offline procedure proposes the optimal solution of the problem by solving it once at the end when all patients are known.

**Table 1.** Comparison of different algorithms on a real instance.

Algorithm	Deadline			Average delay			Overtime slots	Appointment	
	>3 days	>14 days	>28 days	3 days	14 days	28 days		delayed	canceled
CICL	15	4	0	1.54	9.20	8.63	17	0	2
Online Clairvoyant	5	0	0	0.80	8.02	10.37	3	1	4
Online Scheduling Clairvoyant	1	0	0	0.26	7.70	11.44	0	0	0
Offline	1	0	0	0.61	7.28	11.57	0	0	0

Table 1 shows that the offline and online scheduling clairvoyant algorithms outperform the two others procedures. They are nearly perfect with just 1 deadline violation, 0 overtime slots and 0 appointment delayed or canceled. The online clairvoyant algorithm has just 5 deadline violations and 3 overtime slots. However, one patient has been delayed because of the pretreatment and 4 patients pretreatments have not been ready on time to start the treatments. This last point means that the fixed delay was not enough to finish the pretreatment on schedule. The CICL

algorithm cannot achieve to respect the deadlines as 19 deadlines are violated. Finally, the procedure 2 needs in average 15 seconds per patient to book his appointments.

#### 4. Conclusion

We propose to model and solve a complex radiotherapy stochastic scheduling problem in an online fashion. A stochastic and complex column generation model is used for each patient. The relaxations made in our previous work decrease the solving time. The genetic algorithm allows also to reach this reasonable solving time by generating good columns even if the underlying flow-shop problem is extremely difficult. These preliminary results show that our online scheduling clairvoyant algorithm achieves results comparable to the optimal solution. Our procedure also outperforms the others proposed online algorithms. For our future works, we aim at providing a deep sensitivity analysis and more extensive computational tests.

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