The manifestly covariant Aharonov-Bohm effect in terms of the 4D fields

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Abstract.
In this paper it is presented a manifestly covariant formulation of the Aharonov-Bohm (AB) phase difference for the magnetic AB effect . This covariant AB phase is written in terms of the Faraday 2-form F and using the decomposition of F in terms of the electric and magnetic fields as four-dimensional (4D) geometric quantities. It is shown that there is a static electric field outside a stationary solenoid with resistive conductor carrying steady current, which causes that the AB phase difference in the magnetic AB effect may be determined by the electric part of the covariant expression, i.e., by the local influence of the 4D electric field and not, as generally accepted, in terms of nonzero vector potential.

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1. Introduction

In a recent paper [1] the covariant generalizations of the Aharonov-Bohm (AB) effect [2] are considered. One of these generalizations, which will be investigated in this paper, is in terms of the space-time "area" integral of the electric and magnetic fields written in terms of the Faraday 2-form F, Eq. (6) in [1] or Eq. (1) here.

In this paper two important changes relative to [1] will be presented. The first change, which will be discussed in Sec. 2, refers to the mathematical formulation, whereas the second one refers to the physical interpretation of the AB phase shift and it will be discussed in Sec. 3, see also Sec. 10 in [3]. It is true that the expression for the AB phase difference, Eq. (6) in [1], is a covariant expression, but it is not the case with the decomposition of F in terms of the components of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, Eq. (7) in [1]. Instead of it a manifestly covariant decomposition of F, i.e., of $F_{\mu\nu}$, will be presented by Eq. (5). As can be seen from [4-9], in the four-dimensional (4D) spacetime, in contrast to the usual transformations (UT) of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, Eq. (2) here, or Eq. (11.148) in [10], according to which the transformed $\mathbf{E}'$ is expressed by the mixture of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, the mathematically correct Lorentz transformations (LT) always transform the 4D algebraic object representing the electric field only to the electric field; there is no mixing with the magnetic field, Eq. (4) here or Eqs. (42) and (43) in [3]. This is first shown by Minkowski in Sec. 11.6 in [11] and reinvented and generalized in terms of 4D geometric quantities in [4-9]. A brief discussion is given in [3]. Using such 4D electric and magnetic fields the manifestly covariant expression for the AB phase difference is given by Eq. (8), which replaces Eqs. (8) and (9) from [1].

In Sec. 3, we use the results from [3], particularly it refers to the discussion from Sec. 10 in [3]. There, it is mentioned that always there are external electric fields from stationary resistive conductors carrying constant currents, see, e.g., Sec. 4 in [12] and references therein. In Secs. 7-7.2 in [3] it is shown that in the 4D geometric approach to special relativity, the invariant special relativity (ISR), there is a static electric field outside a moving and a *stationary* solenoid with a steady current not only for resistive conductors but also for superconductors. Note that in the ISR an independent physical reality is attributed to the 4D geometric quantities and not, as usual, to the 3D quantities. Furthermore, in Sec. 8 in [3], it is discovered that there is such static 4D electric field not only outside a *moving* permanent magnet, as generally accepted in physical literature, but outside a *stationary* permanent magnet as well. As explained in [3] that result is based on the paper [13] in which the generalized Uhlenbeck-Goudsmit hypothesis is formulated, Eq. (9) in [13], i.e., Eq. (59) in [3]. The mentioned results for the existence of the 4D external electric fields may give the possibility to explain the experimentally observed fringe shift for the magnetic AB effect even in Tonomura's experiments [14], Sec. 10 in [3]: "in terms of forces, which so far have been overlooked." Here, in Sec. 3, these results from [3] are combined with the correct covariant formulation of the AB effect from Sec. 2, i.e., with Eqs. (10) and (11) for $\delta\alpha_E$, to explain the existence of the magnetic AB phase difference in terms of the overlooked 4D electric force and not, as usual, in terms of the vector potential.

The existence of the overlooked 4D external electric fields is one of the reasons why we do not consider the covariant AB phase in terms of the four-potentials, $\delta\alpha_{EB} = (e/\hbar)\oint A_\mu dx^\mu$, Eq. (5) in [1]. Another reason is that in [15] an axiomatic formulation of the electromagnetism is presented in which only the field equation for F is postulated, Eq. (4) in [15], i.e., Eq. (20) in [3]. It is shown in [15] that the electromagnetic field F can be taken as *the primary quantity* for the whole electromagnetism both in the theory and in experiments; F is a well-defined 4D *measurable* quantity. It yields the complete description of the electromagnetic field and there is no need to introduce either the potentials (thus dispensing with the need for the gauge conditions) or the field vectors. That formulation with the F field is a self-contained, complete and consistent formulation. The generalization of Eq. (4) in [15] to a moving medium is presented in [16]. There, [16], the field equations are written in terms of F and the generalized magnetization-polarization bivector $\mathcal{M}$ and not, as usual, in terms of F and the electromagnetic excitation tensor $\mathcal{H}$.

2. Covariant expression for the AB phase shift

The covariant expression for the AB phase difference in terms of the Faraday 2-form F is presented by Eq. (6) in [1], which is repeated here

$$\delta\alpha_{EB} = (-e/2\hbar)\int F_{\mu\nu}dx^\mu \wedge dx^\nu = (e/\hbar)\int F, \tag{1}$$

where $F = (-1/2)F_{\mu\nu}dx^\mu \wedge dx^\nu$. (The notation is the same as in [1]; dx^μ and dx^ν are differential four-vectors and throughout the paper we set $c = 1$.) In order to show that this covariant expression (1) reduces to the usual expressions with the 3-vectors, Eqs. (2) and (4) in [1], the Faraday 2-form F is decomposed using the components of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, Eq. (7) in [1]. It is worth mentioning that Eq. (7) in [1] is not mathematically correct; the expression $(-1/2)F_{\mu\nu}dx^\mu \wedge dx^\nu$ (it will be denoted as (F)) is covariant under the LT, but it is not the case with its decomposition $(E_x dx + E_y dy + E_z dz) \wedge dt + B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy$ (it will be denoted as (EB)). The expression (EB) is obtained from that one with $F_{\mu\nu}$, (F), using the usual identification of the components of $F_{\mu\nu}$ with the components of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, e.g., Eq. (11.138) in [10], see also Eq. (3) and the comment on it in [3]. In all traditional approaches it is supposed that the same identification holds in a relatively moving inertial frame of reference, see Eq. (7) in [3]. This means that it is considered that the components of $\mathbf{E}$ and $\mathbf{B}$ transform under the LT as the components of $F_{\mu\nu}$ transform, i.e., that the LT of the components of $\mathbf{E}$ and $\mathbf{B}$ (for the boost in the x direction) are

$$\begin{aligned} E'_x &= E_x,\ E'_y = \gamma(E_y - \beta B_z),\ E'_z = \gamma(E_z + \beta B_y), \\ B'_x &= B_x,\ B'_y = \gamma(B_y + \beta E_z),\ B'_z = \gamma(B_z - \beta E_y), \end{aligned} \tag{2}$$

see, e.g., Sec. 11.10 and Eq. (11.148) in [10], or the discussion and equations (9) and (10) in [3]. The essential feature of the transformations (2) is that *the transformed components* $E'_{x,y,z}$ *are expressed by the mixture of the components of the 3-vectors* $\mathbf{E}$ *and* $\mathbf{B}$, *and similarly for* $\mathbf{B}'$. The electric field $\mathbf{E}$ in one inertial frame is "seen" as slightly changed electric field $\mathbf{E}'$ and an *induced magnetic field* $\mathbf{B}'$ in a relatively moving inertial frame. From the time of Einstein's fundamental paper [17], the transformations (2) are always considered to be the relativistically correct LT (boosts) of $\mathbf{E}$ and $\mathbf{B}$, but we shall call them, as already said, the UT. As can be seen from Secs. 3.1 and 3.2 in [3], the above mentioned identification is synchronization dependent and it holds only if Einstein's synchronization [17] is used. There, it is also shown that the mentioned identifications are meaningless if only the Einstein synchronization is replaced by an asymmetric synchronization, the "radio" synchronization. That nonstandard synchronization is described in more detail in [18], see also [13]. This is also mentioned below, see Eq. (9) and the discussion with it. But, *different synchronizations are only different conventions and physics must not depend on conventions.*

Therefore, as first shown by Minkowski in Sec. 11.6 in [11] and independently reinvented and generalized in terms of the 4D geometric quantities in [4-9], $F_{\mu\nu}$ can be decomposed in a covariant manner

$$\begin{aligned} F_{\mu\nu} &= (v_\mu E_\nu - v_\nu E_\mu) + \varepsilon_{\mu\nu\alpha\beta} v^\alpha B^\beta, \\ E_\mu &= F_{\nu\mu} v^\nu, \quad B_\mu = (1/2)\varepsilon_{\mu\nu\alpha\beta} F^{\nu\alpha} v^\beta, \end{aligned} \tag{3}$$

where E_μ and B_μ are the components of the 4D electric and magnetic fields respectively, whereas v_μ are the components of the 4D velocity of a family of observers who measure electric and magnetic fields, see also Sec. 5 in [3]. Since $F_{\mu\nu}$ is antisymmetric it holds that $E_\mu v^\mu = B_\mu v^\mu = 0$, only three components of E_μ and B_μ are independent. In the 4D spacetime the mathematically correct decomposition of F into 4D electric and magnetic fields *and* the 4-velocity of the observer, Eq. (3), is already firmly theoretically founded and it is known to many physicists. The recent example is in [19]; it is only the electric part (the magnetic part is zero there). Similarly, in the component form as in (3), this decomposition is presented, e.g., in [20] and in the basis-free form with the abstract 4D quantities, e.g., in [21].

¿From the mathematical viewpoint it is trivially to see how, e.g., E_μ from (3) is transformed under the LT; in the mathematically correct LT the transformed components E'_μ are not determined only by $F'_{\mu\nu}$, as in all usual approaches, e.g., Eqs. (11.147) and (11.148) in [10], but also by v'^μ. This is first shown by Minkowski in Sec. 11.6 in [11]. Let v, E and B are 1×4 matrices and F is a 4×4 matrix; their components are implicitly determined in the standard basis. Minkowski first described how v and F separately transform under the LT A (the matrix of the LT is denoted as A). The LT of the 4-velocity v is $v' = vA$ and the LT of the field-strength tensor F is $F' = A^{-1}FA$, then, as shown by Minkowski, the mathematically correct LT of $E = vF$ is $E = vF \longrightarrow E' = (vA)(A^{-1}FA) = (vF)A = EA$. This means that under the LT *both* quantities, the field-strength tensor F (4×4 matrix) *and the 4-velocity* v (1×4 matrix) are transformed and their product transforms as any 1×4 matrix transforms. As already stated that mathematically correct procedure is reinvented and generalized using the 4D geometric quantities both in the tensor formalism and in the geometric algebra formalism in [4-9]. Particularly, the comparison with Minkowski's results, Sec. 11.6 in [11], is presented in [9]. The essential point is that *the 4D electric field E transforms by the LT again to the 4D electric field E'; there is no mixing with the 4D magnetic field B*, i.e., *the components E_μ transform by the LT again to the components E'_μ of the same 4D electric field and there is no mixing with* B_μ,

$$E'_0 = \gamma(E_0 + \beta E_1),\ E'_1 = \gamma(E_1 + \beta E_0),\ E'_{2,3} = E_{2,3}, \tag{4}$$

for a boost along the x^1 axis. It is easily seen that the UT, Eq. (11.148) in [10], i.e., Eq. (2) here, will be simply obtained in this 4D geometric approach if *only* the components $F_{\mu\nu}$ are transformed *but not* the components v^μ. Such procedure corresponds to the usual identifications of the components of $F_{\mu\nu}$ with the components of the 3-vectors **E** and **B** in both relatively moving inertial frames of reference. A short derivation of these results can be seen in [7]. In this case there is no need to write the transformations for the components B_μ since they transform as in (4). This means that it is proved in [4-9] that, contrary to the generally accepted opinion, *the UT of the 3-vectors* **E** *and* **B**, Eq. (2), *are not the LT*, but that the mathematically correct LT are given by Eq. (4). For a brief review see Sec. 5 in [3] or Sec. 3 in [22]. It is interesting that although Eq. (3) is known to many physicists, e.g., [20, 21], it is not noticed that the mathematically correct LT of, e.g., $E_\mu = F_{\nu\mu}v^\nu$, necessarily require that both $F_{\nu\mu}$ and v^ν have to be transformed and not only $F_{\nu\mu}$. In the 4D spacetime, from the mathematical viewpoint, the 4D electric and magnetic fields are correctly defined and they transform as any other 4-vector transforms, i.e., according to Eq. (4).

Hence, instead of Eq. (7) in [1] we have

$$\begin{aligned} F &= (-1/2)F_{\mu\nu}dx^\mu \wedge dx^\nu = \\ &\quad (-1/2)[(v_\mu E_\nu - v_\nu E_\mu) + \varepsilon_{\mu\nu\alpha\beta}v^\alpha B^\beta]dx^\mu \wedge dx^\nu. \end{aligned} \tag{5}$$

In Eq. (5) both expressions for F are manifestly covariant under the LT, which does not hold, as already stated, for Eq. (7) in [1]. In contrast to the usual treatment from [1], $\delta\alpha_{EB}$ that is given by Eq. (8) below *is the same for all relatively moving inertial observers and for all coordinate bases used by them; the principle of relativity is naturally satisfied.* This proves a mathematical and relativistic correctness of this manifestly covariant approach.

For the reader's convenience and for easier comparison with [1] we have written, e.g., Eq. (3), only with components, but as F is a 4D geometric quantity, a 2-form ($F = (-1/2)F_{\mu\nu}dx^\mu \wedge dx^\nu$), so is, e.g., the electric field E, a 4D geometric quantity, an 1-form ($E = E_\mu dx^\mu$). Both, F and E in these relations are written in a specific coordinate basis, the standard basis, with the Einstein synchronization of distant clocks and Cartesian space coordinates. In [1], as in all usual covariant approaches, the standard basis is exclusively used, but, as pointed out above, different systems of coordinates are allowed in an inertial frame and they are all equivalent in the description of physical phenomena. Thus, for example, one can use the above mentioned asymmetric synchronization, the "radio" synchronization. The important difference relative to the usual formulation with 3-vectors is that in the 4D spacetime a 4D geometric quantity is *the same 4D quantity* for all inertial observers and for all coordinate bases used by them, $E = E_\mu dx^\mu = E'_\mu dx'^\mu = E_{\mu,r}dx^{\mu,r} = \ldots$, where the primed quantities are the Lorentz transforms of the unprimed ones and the quantities with the index "r" are in the coordinate basis with the "radio" synchronization. Observe that in [18] and in the second and third papers in [23] the "radio" synchronization is used throughout the papers. Moreover, in Eq. (4) in [18] it is presented the transformation matrix that connects Einstein's system of coordinates with another system of coordinates in the same reference frame. Also, Eq. (1) in [18], it is derived such form of the LT, which is independent of the chosen system of coordinates, including different synchronizations. Since in the ISR every 4D geometric quantity is invariant under the LT the principle of relativity is automatically satisfied and there is no need to postulate it outside the mathematical formulation of the theory as in Einstein's formulation of SR, [17].

For simplicity and for easier comparison with [1] we shall introduce the inertial frame of "fiducial" observers ($v^\mu = (1, 0, 0, 0)$) with the standard basis (Einstein's synchronization) in it, which will be called the "f"-frame. In that frame it holds that $E_0 = B_0 = 0$ and only the spatial components of E_μ and B_μ remain. From (3) it follows that these components are

$$E_i = F_{0i}v^0 = F_{0i}, \quad B_i = (1/2)\varepsilon_{0ijk}F^{kj}; \tag{6}$$

the same components as in, e.g., Eq. (11.138) in [10]. Observe that the "f"-frame is not any kind of a preferred frame, because any inertial frame can be chosen to be that frame and it is usually taken that the laboratory frame is the "f"-frame. However, in any other relatively moving inertial frame, the S' frame, the "fiducial" observers are moving, and the components v^μ transform as in (4), $v'^\mu = (\gamma, -\beta\gamma, 0, 0)$. Hence, as already shown by Minkowski in Sec. 11.6 in [11], for the transformations from the "f"-frame, see [7], $(E_\mu)' = [F_{\nu\mu}v^\nu]' = [F_{0i}v^0]' = F'_{\nu\mu}v'^\nu = E'_\mu$, and Eq. (4) is obtained; *the components E_μ transform by the LT again to the components E'_μ*. Let us take in (5) that $E_1 = E_x$, ... , $B_1 = B_x$, ... , $\varepsilon_{0123} = 1$, $dx^0 = dt$, ... , $dx^3 = dz$, then in the "f"-frame the second covariant expression in (5) corresponds to the expression (EB) that is used in [1]. In a relatively moving inertial frame S' the LT (4) will give that E'_0 and B'_0 will be different from zero and these terms cannot exist in the approach from [1], which deals with the expression (EB), i.e., with the fields as the 3-vectors.

In the usual formulation the physical meaning of 3-vectors $\mathbf{E}$ and $\mathbf{B}$ is determined by the the Lorentz force as a 3-vector $\mathbf{F} = q\mathbf{E} + q\mathbf{u} \times \mathbf{B}$ and by Newton's second law $\mathbf{F} = d\mathbf{p}/dt$, $\mathbf{p} = m\gamma_u\mathbf{u}$.

However, in the 4D spacetime, the Lorentz force K is not a 3-vector, but it is a 4D geometric quantity. K is the contraction of the electromagnetic 2-form F with particle's 4-velocity u (it is defined to be the tangent to its world line). The components of K in the standard basis are $K_\mu = qF_{\mu\nu}u^\nu$, where u^μ is the 4-velocity (components) of a charge q, or with E_μ and B_μ, using the decomposition of $F_{\mu\nu}$, (3), they become

$$K_\mu = q[(v_\mu E_\nu - v_\nu E_\mu) + \varepsilon_{\mu\nu\alpha\beta}v^\alpha B^\beta]u^\nu. \tag{7}$$

In the 4D spacetime, the physical meaning of E_μ and B_μ is determined by the Lorentz force K_μ and by the 4D expression for Newton's second law $K_\mu = dp_\mu/d\tau$, $p_\mu = mu_\mu$, where p_μ is the proper momentum (components) and τ is the proper time. All components E_μ and B_μ, thus E_0 and B_0 as well, are equally well physical and measurable quantities by means of the mentioned K_μ and the equation of motion, i.e., the 4D expression for Newton's second law. Obviously, regardless of the fact that majority of physicists believe that only the 3-vectors $\mathbf{E}$ and $\mathbf{B}$ are physical and measurable quantities, *in the 4D spacetime, the 4D geometric quantities are properly defined both theoretically and experimentally.* In view of this discussion it is obvious that the question what physically are E_0 and B_0 is equivalent to the question - what is the temporal component x_0 of the position 4-vector. This is particularly visible if the Einstein synchronization is replaced by the "radio" synchronization in which the space and time are not separated, see Eq. (9) below. Then, the usual 3-vector $\mathbf{r}$, and similarly the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, are meaningless. This fundamental difference between the usual formulation with the 3D quantities and the formulation with the 4D geometric quantities is exposed in much more detail, e.g., in [24].

It is also shown in, e.g., [5, 6, 15] that *the LT of the 4D* E_μ *and* B_μ, (4), *are in a true agreement (independent of the chosen inertial reference frame and of the chosen system of coordinates in it) with all experiments in the electromagnetism, whereas it is not the case with the UT of the 3D* $\mathbf{E}$ *and* $\mathbf{B}$, (2). Thus, for example, it is shown in [5] that the conventional theory with the 3D $\mathbf{E}$ and $\mathbf{B}$ and their UT yields different values for the motional emf ε for relatively moving inertial observers, $\varepsilon = UBl$ and $\varepsilon = \gamma UBl$, whereas the approach with 4D geometric quantities and their LT yields always the same value for ε, which is defined as a Lorentz scalar, $\varepsilon = \gamma UBl$. This result is very strong evidence that the usual approach is not relativistically correct, i.e., it is not in agreement with the principle of relativity. It is for the experimentalists to find the way to precisely measure the emf ε for the considered problem of a conductor moving in a static magnetic field and to see that in the laboratory frame $\varepsilon = \gamma UBl$ and not simply $\varepsilon = UBl$. That problem is of a considerable importance in practice. The similar discussion with the same conclusions was presented for the Faraday disk in [6]. In the already mentioned [15] and in [25] the Trouton-Noble paradox is considered. It is shown that in the geometric approach with 4D quantities *the 4D torques* will not appear for the moving capacitor if they do not exist for the stationary capacitor, which means that with 4D geometric quantities the principle of relativity is naturally satisfied and *there is not the Trouton-Noble paradox.* The same conclusion holds in the low-velocity approximation $\beta \ll 1$, or $\gamma \simeq 1$. It is also shown in the same geometric approach with 4D torques that there is no Jackson's paradox [24] and the "charge-magnet paradox" [22].

At this point it is worth noting that in the mathematically correct approach, in general, there is no room for the 3-vectors in the 4D spacetime. Let us better explain that statement. It is written in [1] after Eq. (7) that: "$F = B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy = \mathbf{B} \cdot d\mathbf{S}$ where the differential forms expression has been converted back to three-vector notation and $d\mathbf{S}$ is the differential area." However, such an equality is mathematically impossible and incorrect. Namely, in the mathematically correct formulation dx, dy, dz have to be understood as differential 4-vectors dx^1, dx^2, dx^3, respectively, the 4D geometric quantities that are properly defined on the 4D spacetime; the wedge product refers to such 4D quantities and not to the usual scalar differentials. On the other hand, $\mathbf{B}$ and $d\mathbf{S}$, as *geometric quantities in the 3D space*, are constructed from the components and *the unit 3-vectors* $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, e.g., $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$. The unit 3-vectors have nothing to do with the basis in the 4D spacetime. The LT are properly defined on the 4D spacetime and they cannot transform the 3-vectors. Hence, in the 4D spacetime it is not mathematically correct to state as in [1]: ".. the expression in (6) reduces to $\delta\alpha_{EB} = (e/\hbar)\int F = (e/\hbar)\int \mathbf{B} \cdot d\mathbf{S}$ which is equivalent to the 3-vector expression (2)." In the 4D spacetime the covariant expression $((e/\hbar)\int F)$ is the correct one, but it is not the case with the usual expression for the magnetic flux with the 3-vectors $((e/\hbar)\int \mathbf{B} \cdot d\mathbf{S})$; *they cannot be equal.* The same objection refers to all other relations with the 3-vectors in [1]. Hence, in this geometric approach, using (1) and (5), the manifestly covariant expression for the AB phase difference becomes

$$\delta\alpha_{EB} = (-e/2\hbar)\int [(v_\mu E_\nu(x) - v_\nu E_\mu(x)) + \varepsilon_{\mu\nu\alpha\beta} v^\alpha B^\beta(x)] dx^\mu \wedge dx^\nu. \tag{8}$$

In Sec. 3 in [1] it is investigated "the usual magnetic AB set-up of an infinite solenoid but with a time dependent magnetic field and vector potential, i.e., $\mathbf{B}(t)$ and $\mathbf{A}(t)$." As noted in [1] for that situation the scalar potential is still zero, $\phi = 0$. At first, it is worth mentioning that, as explained in Sec. 3 in [3], in a correct covariant formulation there is no static case. The 1-form A ($A = A_\mu dx^\mu$) and the Faraday 2-form F are both, always function of the position four-vector x; $A(x)$ and $F(x)$. If, for example, the usual 3-vector fields $\mathbf{A}(\mathbf{r})$, $\mathbf{B}(\mathbf{r})$ do not explicitly depend on the coordinate time t in one frame, then the LT will mix the time and space coordinates; they cannot transform the spatial coordinates from one frame only to spatial coordinates in a relatively moving inertial frame of reference. What is static case for one inertial observer is not more static case for relatively moving inertial observer, but a time dependent case. Furthermore, if an observer uses the "radio" synchronization and not Einstein's synchronization, then the space and time are not separated and the usual 3-vector $\mathbf{r}$ is meaningless. As can be seen from Eq. (13) in [3] the components of the position 4-vector x in the commonly used coordinate basis with Einstein's synchronization and that one with the "radio" synchronization are connected as

$$x_r^0 = x^0 - x^1 - x^2 - x^3, \quad x_r^i = x^i, \tag{9}$$

and the same relation holds, e.g., for (A_r^0, A_r^i), or (E_r^0, E_r^i).

This consideration suggests that the results from Sec. 3 in [1] for the time dependent, infinite solenoid, have to be reexamined using the correct covariant formulation (8). We shall only discuss the AB phase difference determined by Eqs. (8) and (9) in [1]. It is calculated using Eq. (7) from [1]. This will be compared with (8). As already mentioned above, in the 4D spacetime, Eq. (7) from [1] is not mathematically correct and the same holds for Eqs. (8) and (9) from [1], which deal with the 3-vectors. The part of the AB phase difference with B_μ from (8) is $\delta\alpha_B = (-e/2\hbar)\int \varepsilon_{\mu\nu\alpha\beta} v^\alpha B^\beta(x) dx^\mu \wedge dx^\nu$ and it replaces Eq. (8) from [1]. Only in the "f"-frame that part becomes $\delta\alpha_B = (-e/2\hbar)\int \varepsilon_{0ijk} v^0 B^k(x) dx^i \wedge dx^j$ and, as can be seen by the use of $B_1 = B_x$, etc. that mathematically correct expression corresponds to Eq. (8) from [1], i.e., to $\delta\alpha_B = (e/\hbar)\int \mathbf{B}\cdot d\mathbf{S}$. The essential difference is that all quantities in this covariantly defined $\delta\alpha_B$ are properly defined in the 4D spacetime and they correctly transform under the LT, like (4), which is not the case with the 3D quantities from Eq. (8) in [1].

The part of the AB phase difference with E_μ from (8) is

$$\delta\alpha_E = (-e/2\hbar)\int (v_\mu E_\nu(x) - v_\nu E_\mu(x)) dx^\mu \wedge dx^\nu \tag{10}$$

and, the same as for $\delta\alpha_B$, it is the same quantity for all relatively moving inertial observers and for all bases used by them. Only in the "f"-frame $\delta\alpha_E$ from (10) becomes

$$\delta\alpha_E = (e/\hbar)\int v_0 E_i(x) dx^i \wedge dx^0 \tag{11}$$

and it can be compared with Eq. (9) from [1]. For that comparison $F_{\mu\nu}$ is written in terms of A_μ as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. In that expression it is considered that A_μ are the primary quantities whereas $F_{\mu\nu}$ are derived from them. But, as clearly shown in [15], the F field is *the primary quantity* for the whole electromagnetism and not the four potential, which is gauge dependent. However, here, for the comparison with [1], we use the above relation with A_μ. Then, (3) is used to get E_μ in terms of A_μ, $E_\mu = F_{\alpha\mu} v^\alpha = (\partial_\alpha A_\mu - \partial_\mu A_\alpha) v^\alpha$. In the "f"-frame $E_0 = 0$ and $E_i = (\partial_0 A_i - \partial_i A_0) v^0$, what corresponds to the components of the usual three-vector $\mathbf{E}$, e.g., E_1 corresponds to $E_x = -\partial A_x/\partial t - \partial_x \phi$; remember that if A_μ is written in the usual notation it is $A_\mu = (\phi, -A_x, -A_y, -A_z)$ and in the "f"-frame $v^\mu = (1, 0, 0, 0)$. Hence, in the "f"-frame, $\delta\alpha_E = (e/\hbar)\int (\partial_0 A_i - \partial_i A_0) dx^i \wedge dx^0$, which for $A_0 = 0$ becomes $= (e/\hbar)\int \partial_0 A_i dx^i \wedge dx^0$ and, by the procedure from [1], it corresponds to Eq. (9) in [1], i.e., to $\delta\alpha_E = (-e/\hbar)\int \mathbf{B}\cdot d\mathbf{S} = -\delta\alpha_B$. Thus, *only* in the "f"-frame and for $A_0 = 0$ "the two parts cancel exactly." Observe that the condition $A_0 = 0$ is not a Lorentz covariant condition; in a relatively moving inertial frame A_0 will be $\neq 0$. Furthermore, as seen from (9), in the basis with the "radio" synchronization the temporal and spatial components of A_μ cannot be separated, which means that in the 4D spacetime the condition $A_0 = 0$ has not a well-defined meaning. The similar objections hold for the whole discussion presented in [26].

3. The Aharonov-Bohm effect in terms of fields

It is really surprising that both in all numerous theoretical discussions, e.g., [1, 2, 27, 28, 26], in the experiments with microscopic solenoids [29] and also in the recent experiment with macroscopic solenoid [30], it is never noticed that in the rest frame of the solenoid there are *always* external *static* electric fields for *stationary, resistive* conductor carrying *constant current.* In an ohmic conductor there are quasistatic surface charges, which generate not only the electric field inside the wire driving the current, *but also a time independent electric field outside it.* That electric field is proportional to the current, see, e.g., Sec. 4 in [12] and references therein. As mentioned in [12] the existence of such quasistatic surface charges was first pointed out by Kirchhoff, Refs. [18-20] in [12]. There are no analytic solutions for these surface charges and the external electric fields for the case of finite solenoids; for an infinite solenoid see [31]. The distribution of the surface charges and the magnitude of the induced electric fields depend not only on the geometry of the circuit but even of its surroundings. *These external electric fields from steady currents are firmly experimentally confirmed*, see, e.g., [12], *and they are well-known in electrical engineering.* In [12], two other contributions to the external electric field are discussed, but, as explained in Sec. 10 in [3], they are of no concern here. It is worth mentioning that the expression from Sec. 4 in [12] is for a cylindrical wire of length l carrying a constant current I and that wire is a part of a square circuit. That expression is not appropriate for a finite solenoid with steady current. In [31] an infinite solenoid with steady current is considered and it is appropriate for the case considered in [1]. There, in [31], a uniform cylindrical resistive sheet of the radius a with a "line" battery with terminals at potentials $\pm V_0/2$ driving current *azimuthally* in it is considered. In Sec. IV, [31], it is presented (i) the magnitude of the electric field outside the solenoid, Eq. (11),

$$E = (V_0/\pi)(a/r\rho), \tag{12}$$

where r and ρ are the polar radii measured from the center (axis) and from the baterry (respectively), and (ii) the electric lines of force, Fig. 3. It is visible from Fig. 3. in [31] that the electric field has radial and poloidal components, where the latter ones follow the direction of the current just outside the solenoid in the same way as the magnetic vector potential.

In the recent experiment [30] the absence of electromagnetic forces outside the solenoid that are predicted by Boyer's force picture [32] has been experimentally investigated by means of a time-of-flight experiment for a macroscopic solenoid. It is looked for a time delay for electrons passing on opposite sides of the solenoid. As discussed above in the generally accepted theory the electron wave packets are influenced by nonzero vector potential, i.e., by the quantum action of the magnetic flux even when electrons pass through the field-free regions of space. On the other hand in Boyer's semiclassical theory [32] there is a back-action force of the solenoid on the electron, which gives rise to a time delay and to a phase shift that exactly matches the AB-phase shift. It is shown in [30] that there is no time delay and it is concluded that there are no fields predicted by Boyer's force picture [32]. In his comment on the results obtained in [30] Boyer [33] stated: "the Aharonov-Bohm phase shift has never been observed for such a macroscopic solenoid, ..." In [33], it is also argued that if the solenoid resistance is large, as in [30], then the back forces will be small and there is no time lag, but for the microscopic solenoids it is the opposite case. It has to be pointed out that neither the authors of [30] nor Boyer [32, 33] knew anything about the electric fields caused by the quasistatic surface charges that exist outside the resistive conductors carrying *constant* currents. This means that it is not true that the paper [30] shows *experimentally* that forces cannot be responsible for the magnetic AB phase shift. The electric forces caused by the mentioned quasistatic surface charges have nothing to do with Boyer's force picture, [32, 33]. Thus, the main result from [30] about the absence of the time delay does not imply that the electrons travel in a field-free region. Obviously, the electric fields from quasistatic surface charges have to be taken into account for the explanation of the AB phase difference in the magnetic AB effect as well, i.e., in the usual magnetic AB set-up of an infinite solenoid, which is considered in [1] and also in the case of finite macroscopic [30] and microscopic [29] solenoids. From the viewpoint presented here the AB phase difference in the magnetic AB effect *is not* due to the vector potential, i.e., according to Eq. (2) from [1] due to the quantum action of the magnetic flux, but *it is due to the mentioned external electric field from stationary solenoids with steady currents.* In that case, contrary to the generally accepted opinion, the electron does not travel in the field-free region, but the electron wave packets are *locally* influenced by the electric field. A similar expression as (11) is obtained in [34], Eq. (28), but their procedure is not relativistically correct and the 3D electric field that enters into their Eq. (28) is proportional to the square of the current.

In order to clarify the situation from the experimental viewpoint we consider that some new experiments are required: the measurement in a *single* experiment of the AB phase shift and the time delay, as suggested in [33], and the measurement of the mentioned external electric fields *separately* from AB-studies.

The above consideration implies that in the expression for $\delta\alpha_{EB}$ (8) there is no need to take into account the magnetic part $\delta\alpha_B$, i.e., the non-local effect of the magnetic field. In the 4D spacetime only the local effects are important and physically justified. This means that from the viewpoint of this approach with 4D geometric quantities *the AB phase difference is even for the magnetic AB effect exclusively determined by the covariant expression* $\delta\alpha_E$ from (10), i.e., by the local influence of the 4D electric field. If the rest frame of the solenoid, the laboratory frame, is taken to be the "f"-frame then $\delta\alpha_E$ is given by Eq. (11). In our opinion the magnetic part $\delta\alpha_B$ of $\delta\alpha_{EB}$ (8) could be taken into account only in the case that the solenoid's magnetic field is not entirely restricted to the coil's interior but exists in the coil's exterior as well, i.e., along the electron's trajectory. The same conclusion that only the local effect of the 4D electric field, i.e., $\delta\alpha_E$ (10) ((11)) is important and physically meaningful holds in the same measure for the time dependent set-up that is considered in Sec. 3 in [1]. Thus, in that case there is no cancellation of the non-local effect of the magnetic field, $\delta\alpha_B$, with the local effect of the electric field, $\delta\alpha_E$ (10) ((11)), because, as explained above, only the electric field from the solenoid with current exists in the region outside the solenoid and consequently it can locally influence the electron travelling through that region. It is interesting that, as can be seen from Sec. 4 in [35], if the current in the solenoid varies linearly with time then it creates a time independent external electric field, see Eq. (8) and Fig. 1 in [35]. Hence, for the solenoid with such a time-dependent current there will be no time-dependent AB phase shift although only $\delta\alpha_E$ (10) ((11)), the electric part of $\delta\alpha_{EB}$ (8), is considered to be physically correct and justified.

Note that in this approach with the 4D geometric quantities the 3D quantities from the usual approaches, e.g., from [1, 26, 12, 34, 35], etc. have to be interpreted in a different way. Thus, for example, the *components* of the electric field 3-vector in [1] have to be understood as the *spatial components* in the standard basis of the 4D electric field; the rest frame of the solenoid is taken to be the "f"-frame and therefore the temporal component $E_0 = 0$ (also $B_0 = 0$). Also, in this geometric approach the components K_μ of the Lorentz force are given by (7). As discussed above, in the case considered in [1] only the electric part of K from (7) is physically important.

4. Conclusions

As seen from the preceding discussion the correct covariant formulation of the AB phase shift (8) deals with the 4D geometric quantities that properly transform under the mathematically correct LT (4). In the 4D spacetime Eq. (8) replaces Eqs. (8) and (9) from [1], which deal with the 3D quantities that transform under the UT (2). Both, the 3D quantities and their UT (2) are ill-defined in the 4D spacetime. As *proved* in [4-9], contrary to the generally accepted opinion, the UT (2) are not the mathematically correct LT. The main result that is obtained in this paper is that *even for the magnetic AB effect* (a stationary solenoid with *resistive* conductor carrying either steady current or the current that varies linearly with time) *the AB phase difference is exclusively determined by the covariant expression* $\delta\alpha_E$ from (10), i.e., by the *local* influence of the 4D electric field. Thus, here, it is shown that in the 4D spacetime only the electric part of $\delta\alpha_{EB}$ (8), i.e., $\delta\alpha_E$ (10) ((11)) is physically correct and meaningful. The reason for it is that there are *static* electric fields outside a *stationary, resistive* conductor carrying *steady current*, which means that it is not true that, e.g., in experiments [29, 30], the electron travels in the field-free region of space. The existence of the mentioned electric fields is firmly experimentally confirmed; for some experiments see, e.g., [12] and references therein. All this together shows that the magnetic AB phase shift considered in [1] is not a topological phase shift.

In Sec. 7.1 in [3] it is shown that the external static electric fields, the "relativistic" second-order electric fields, would need to exist not only for resistive conductors with steady currents but even for superconducting solenoids with steady currents. In Sec. 7.2 in [3] different experiments for the detection of the second-order electric fields outside a stationary superconductor with steady current are discussed. Furthermore, what is very important for the explanation of the AB effect, in Sec. 8 in [3] such second-order electric fields are predicted to exist outside a *stationary* permanent magnet as well. As discussed in Sec. 10 in [3], these results could explain the experimentally observed fringe shift for the magnetic AB effect even in Tonomura's experiments [14] *in terms of previously overlooked electric forces and not, as generally accepted, in terms of nonzero vector potentials.*

Similarly, the qualitative theoretical explanations of the quantum phase shifts in terms of the classical forces as the 4D vectors in the Aharonov-Casher and the Röntgen effects are presented in [7, 36]. Furthermore, in [37], the dipole moments are quantized and it is shown that the expectation value for the quantum force 4D vector is not zero in the case of the Aharonov-Casher and the Röntgen effects and in the neutron interferometry. Hence, in these experiments too the phase shifts are not due to force-free interaction of the dipole, i.e., they also are not the topological phase shifts.

The covariant AB effect in terms of F and not in terms of a vector potential is also investigated in [38].

It is interesting to note that recently another local explanation of the AB effect is proposed in [39]. There, it is argued that if the solenoid in the AB effect is treated in the framework of quantum theory then the effect can be explained by the local action of the field of the electron on the solenoid. In some respects there is a similarity between Boyer's calculation [32, 33] and Vaidman's determination [39] of the AB phase shift. Boyer in [32, 33] calculates the force exerted by the electron on a solenoid (represented by a line of magnetic dipoles) and then relies on Newton's third law to obtain a back-action force of the solenoid on the electron. The same Boyer's force approach is investigated in [30] but a solenoid is considered as a stack of current loops. However, Newton's third law is violated for the electromagnetic interaction and to overcome this difficulty a hidden momentum is often introduced, particularly in the case with current loops, see, e.g. [30, 40] and references therein. But, as shown, e.g., in [3] and [22], if an independent physical reality is attributed to the 4D geometric quantities and not, as usual, to the 3D quantities, then there is no need for the introduction of some "hidden" 3D quantities and there are no electromagnetic paradoxes. Vaidman, in [39], see Fig. 4 in [39], considers that the electron produces change in the magnetic flux of the solenoid, which causes an electromotive force on charged solenoids (in his example). This leads to the change in their velocity and to the shift of the wave packet of the cylinders and finally to the correct expression for the AB phase, Eq. (5) in [39] (arXiv: 1301.6153). *Observe that this phase shift is for the source (solenoids) and not for the passing electron.* Then, Vaidman states: "Since in quantum mechanics the wave function is for all parts of the system together, the change of the wave function of the source leads to observable effect in the interference experiment with the electron." (See Eqs. (8) and (9) in the first paper in [39] for the change in the total wave function of the electron and the solenoid.) It is worth noting that in Boyer's picture [32, 33] it is impossible to detect the predicted force on the solenoid since it requires the detection of the force of a single electron on a macroscopic object. For the same reason, in Vaidman's picture [39], it is impossible to detect the mentioned electromotive force and the change in the angular velocity of the solenoids. Thus, both Boyer's force and Vaidman's electromotive force cannot be experimentally verified, which means that neither of these approaches have some physical, experimental, foundation.

On the other hand, the theory presented here is based on the existence of the static *electric fields* outside a *stationary, resistive* conductor carrying *steady current*, and these fields are already firmly experimentally verified.

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