Implications of the Lorentz signature of space-time: from bound states to clock synchronization, non-inertial frames, particle localization, relativistic entanglement, dark side of the universe

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Abstract.
In my contribution to IARD 2012 [1] I made a detailed review of my formulation of relativistic classical and quantum mechanics (RCM and RQM) of massive [2] and massless [3] particles in SR. See Refs.[4] for extensive reviews of the obtained results in special and general relativity (SR and GR). Here I will sketch the main conceptual points of it and I will speak about new developments in this framework.

The Lorentz signature of the space-times of SR and GR is a source of problems absent in non-relativistic theories. The classical mechanics (CM) of Newton gives a deterministic description of objects (particles, bodies) supposed to have a reality in an inertial frame of the Galilei space-time centered on an inertial mathematical observer playing no dynamical role beyond defining Cartesian coordinates. This space-time is assumed to be a given background container of the real objects, whose world-lines are described in terms of an absolute notion of time. At each instant there is an absolute Euclidean 3-space where the objects are localized. The inertial frames are connected by the transformations of the Galilei group. This description can be extended to non-inertial frames centered on mathematical accelerated observers.

This realistic description of the world-lines of particles is preserved in special relativity (SR). However, now they are described in the inertial frames of Minkowski space-time centered on inertial mathematical relativistic observers and the Poincaré group describes the transformations connecting the inertial frames. However, in SR there is no notion of absolute time and of absolute 3-space: only the whole Minkowski space-time is absolute and only the conformal structure (i.e. the light-cone describing the locus of incoming and outgoing radiation in every point) has an intrinsic meaning. As a consequence, we must introduce a *convention of clock synchronization* to define an instantaneous 3-space.

Usually RCM is formulated in inertial frames, whose Euclidean 3-spaces are defined by Einstein's convention [1]. Only with this convention does the 1-way velocity of light between two observers

[1] The inertial observer A sends a ray of light at x_i^o towards the (in general accelerated) observer B; the ray is

(it depends on how their clocks are synchronized) coincide with the 2-way velocity of light c of an inertial observer (it replaces the unit of length in relativistic metrology [5]).

However this description of RCM is still incomplete for interacting systems due to the following problems:

(i) There is not a unique notion of relativistic center of mass of a system of particles like in Newtonian mechanics;
(ii) There is the problem of the elimination of relative times in relativistic bound states (time-like excitations are not seen in spectroscopy);
(iii) It is highly non trivial to find the explicit form of the Poincaré generators (especially the Lorentz boosts) for interacting particles in the instant form of dynamics;
(iv) There is no accepted global formulation of non-inertial frames without the pathologies of the rotating disk and of Fermi coordinates.

The absence of time-like excitations in relative times in spectroscopy and the need of a sound formulation of the Cauchy problem in field theory require a metrological convention about the instantaneous 3-spaces, namely a clock synchronization convention. This can be done in a Wigner covariant way with a 3+1 splitting of space-time, the use of radar 4-coordinates centered on a time-like observer and the separation of the non-local (non-measurable) relativistic center of mass of the 3-universe, both in special relativistic inertial frames and in (either special or general) relativistic non-inertial ones of globally hyperbolic space-times asymptotically Minkowskian without super-translations.

A solution to all these problems has been given in Refs.[6, 7, 8] (see also the reviews in Ref.[4]). In these papers there is the first general theory of global non-inertial frames in Minkowski space-time. In the recent paper [9] there is the most updated construction of explicit non-inertial frames and a discussion on accelerated time-like observers avoiding the uniformly accelerated Rindler ones (they disappear in the non-relativistic limit).

The main differences from non-relativistic CM are the *non-local nature* of the relativistic collective variables proposed for the relativistic center of mass (implying their non-measurability with local measurements) and a *spatial non-separability* of the particles, which must be described by means of suitable Wigner-covariant relative 3-variables. Only relative variables are measurable due to this basic spatial non-separability already present at the classical level.

This formulation of RCM allows one to get a consistent definition of RQM of particles with an associated notion of relativistic entanglement as an extension of non-relativistic quantum mechanics (NRQM) [2] avoiding all the known relativistic pathologies. This was done in Ref.[2]. However in RQM the spatial non-separability is an obstruction to the identification of subsystems [2, 10] and leads to a theory of relativistic entanglement strongly different from the non-relativistic one, in which Alice and Bob cannot be decoupled observers.

reflected towards A at a point P of B world-line and then reabsorbed by A at x^o_f; by convention P is synchronous with the mid-point between emission and absorption on A's world-line, i.e. $x^o_P = x^o_i + \frac{1}{2}\,(x^o_f - x^o_i) = \frac{1}{2}\,(x^o_i + x^o_f)$. This convention selects the Euclidean instantaneous 3-spaces $x^o = ct = const.$ of the inertial frames centered on A. However, if the observer A is accelerated, the convention can breaks down due the possible appearance of coordinate singularities.

[2] Unlikely from the transition from Galilei space-time to the Minkowski one, the transition from CM to NRQM can be done only in an operational way due to the big unsolved foundational problems of NRQM (see for instance Ref.[11, 12])

This framework for RCM has been extended to classical field theory (CFT) in Refs.[7] [4] both for classical fields and fluids, but the extension of the approach to quantum field theory (QFT) has still to be done. In field theory the separation of the relativistic center of mass may help [7] in avoiding Haag theorem, but leads to extra problems for the definition of particles besides the known ones in non-inertial frames and in curved space-times.

As shown in Ref.[10] the non-separability and non-locality due to the relativistic center of mass (i.e. the Lorentz signature of the space-time) both at the classical and quantum levels reduce the relevance of the still debated quantum non-locality of NRQM. As shown in Ref.[3] these properties are present also in CFT and their extension to QFT is a difficult open problem, which adds to the existing problems with the notion of particle in QFT [13, 14] (instability under either unitary or non-unitary Bogoliubov transformations). The relativistic non-separability and non-locality point towards non-local QFT and rise the problem of the validity of *micro-causality* (quantum operators at space-like distances commute) at the relativistic level. This issue has already been raised by Busch [15] with the notion of *unsharp observables* (a local operator not measurable with local actions in a given 3-region). According to him sharp spatial localization is an *operationally meaningless idealization* (it requires an infinite amount of energy with unavoidable pair production; the quantum nature of the constituents of the detectors should be taken into account; and so on).

We have shown [10] the status of the particle localization problem both at the relativistic and non-relativistic levels showing the existing theoretical problems and some of the experimental limitations. One would need the identification of some relevant (well mathematically defined) position basis of wave functions with compact support (or like the over-complete coherent state basis for the harmonic oscillator) for the position operator to be used for every type of localization problem.

By taking into account the quantum nature of both the preparing apparatus and of the detectors we have shown that also at the non-relativistic level the real quasi-isolated system with a decoupled center of mass is the full set of "apparatus + detectors + observed system" and that the prepared and detected particles, moving along mean classical Newton trajectories, are described by relative variables [10].

Since, in any case, electromagnetism is always present, this implies that the relativistic picture is valid at every level in experiments.

We have shown the *non-factual* nature of the mathematical time-like observers and of their mathematical synchronization conventions for building reference frames (see also Ref.[16] in the framework of quantum information). It seems quite difficult to develop a theory in which Alice and Bob are dynamical observers [3] and exchange information by using a dynamical electromagnetic field! Instead Alice and Bob are present in every protocol for relativistic quantum information (see Ref.[18] for an old review emphasizing the problems with special relativity), a quickly developing theory, in which the study of the spatio-temporal trajectories of the investigated quantum systems is nearly always lacking.

The main open problem is that if one describes the free massive Klein-Gordon field in non-

[3] First steps in this direction are done with *quantum metrology* (see Ref.[17], ch.7), in which a quantum system is described by means of the relative variables with respect to another quantum system (the observer) without using variables defined with respect to an external classical reference frame (this relational approach is consistent with relativistic non-separability). It is under investigation what is the change in the information and in the entanglement if one goes from the description with respect to a quantum observer to the one with respect to another quantum observer.

inertial frames (like it is done in the Tomonaga-Schwinger formulation on arbitrary space-like hyper-surfaces of Minkowski space-time) then generically the time evolution is not unitarily implementable (the implied Bogoliubov transformation is not of the Hilbert-Schmidt type). How to avoid this no-go theorem, already present before going to GR where only non-inertial frames are allowed by the equivalence principle?

The described state of affairs [10] is in accord with Bohr's point of view according to which we need a classical description of the experimental apparatus. It seems that all the realizable experiments must admit a quasi-classical description not only of the apparatus but also of the quantum particles: they are present in the experimental area as classical effective particles with a mean trajectory and a mean value of 4-momentum (measured with time-of-flight methods).

As shown in Ref.[21], if one assumes that the wave function describes the given quantum system (no ensemble interpretation), the statement of Bohr can be justified by noting that the wave functions used in the preparation of particle beams (semi-classical objects with a mean classical trajectory and a classical mean momentum determined with time-of-flight methods) are a special subset of the wave functions solutions of the Schroedinger equation for the given particles. Their associated density matrix, pervading the whole 3-space, admits a *multi-polar expansion around a classical trajectory* having *zero dipole*. This implies that in this case the equations of the Ehrenfest theorem give rise to the Newton equations for the Newton trajectory (the monopole; it is not a Bohmian trajectory) with a classical force augmented by forces of a quantum nature coming from the quadrupole and the higher multipoles (they are proportional to powers of the Planck constant). As shown in Ref.[21] the mean trajectories of the prepared beams of particles and of the particles revealed by the detectors are just these classically emerging Newton trajectories impled by the Ehrenfest theorem for wave functions with zero dipole. Also all the intuitive descriptions of experiments in atomic physics are compatible with this emergence of classicality. In these descriptions an atom is represented as a classical particle delocalized in a small sphere, whose origin can be traced to the effect of the higher-multipole forces in the emerging Newton equations for the atom trajectory. The wave functions without zero dipole do not seem to be implementable in feasible experiments. No explanation is given of the probabilistic Born rule, but it is suggested that the random unique outcomes have a quasi-classical localization given by these Newton trajectories.

An extremely important (till now unnoticed) point is that the fixation of the gauge freedom of GR (and of every generally covariant theory of gravity), i.e. the choice of the non-inertial frame and of its 4-coordinates, is nothing else that *the establishment of conventions for relativistic metrology* [5], an operation done from atomic physicists, NASA engineers and astronomers. In Ref.[22, 23] there is the Hamiltonian formulation of ADM tetrad gravity in asymptotically Minkowskian space-times admitting global non-inertial frames. It turns out that the trace of the extrinsic curvature (the York time) of the non-Euclidean 3-spaces is a gauge variable to be fixed with a metrological convention (it is the remnant of the gauge freedom in clock synchronization in GR). As shown in the third paper of Refs.[20], the inertial gauge freedom in the York time is connected with the existence of dark matter at the cosmological level. It is possible that at least part of dark matter is a relativistic inertial effect eliminable with a suitable convention for the ICRS celestial reference system. Moreover the York time is also connected with dark energy.

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