On solitons in the negative refracting medium

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Abstract. We consider the coupled electromagnetic waves propagating in a nonlinear medium, which is featured by a positive and negative refraction indexes. The backward waves can be propagating in this case. The example of the true soliton is discussed. In general case the coupled forward and backward solitary wave can be found. They are analogues to the optical solitons.

1. Introductionn

As a rule, the phase velocity (V_{ph}) and the Pointing vector (S) of the electromagnetic wave are collinear vectors. Frequently the Pointing vector is considered to be proportional to the group velocity vector. It is correct in the case of isotropic medium. Antiparallel orientation of the phase velocity and the Pointing vector (or group velocity vector) was first discussed in [1, 2]. In [3] it was indicated that antiparallel orientation of V_{ph} and S results in negative refraction. Subsequently, this idea was developed by Mandelstam in [4]. It has been predicted that when the real parts of the dielectric permittivity and magnetic permeability in the medium simultaneously take on negative values in some frequency range, antiparallel orientation of the V_{ph} and S occurs [5,6] and the property of negative refraction appears [7]. The existence of the media characterized by negative refractive index (NRI) was demonstrated experimentally first in the microwave and then in the near-infrared ranges. Reviews of the properties of the NRI materials and the nonlinear phenomena in NRI materials are presented in [8-13].

2. Sine-Gordon equation. Forward and backward solitons

The Sine-Gordon equation is the famous example of the evolution equation in the nonlinear wave theory [14, 15]. This equation is derived in many fields of physics and its properties are well known. In the one dimension case the Sine-Gordon equation is completely integrable one. Solitons and multi-solitons (in the strict sense) are the solutions of this equation. Thus the Sine-Gordon equation can be considered as an etalon equation in this context.

There are two forms of the Sine-Gordon equation

$$u_{,xx} - u_{,tt} = a\sin u \,, \tag{1}$$

and

$$u_{,xt} = a \sin u. \tag{2}$$

Linearization of these equations results in linear wave equations for description of the linear wave propagation

$$u_{,xx} - u_{,tt} - au = 0, \tag{3}$$

$$u_{,xt} - au = 0. \tag{4}$$

From equation (3) the dispersion relation follows in the form $\omega^2(k) = a + k^2$. The phase and group velocities are determined by the following expression

$$V_{ph} = \frac{\omega}{k} = \pm \frac{(a+k^2)^{1/2}}{k}, \quad V_g = \frac{d\omega}{dk} = \pm \frac{k}{(a+k^2)^{1/2}}.$$

It is necessary to emphasize that signs of the phase and group velocities are the same. Hence, the linear waves in this case are the forward waves.

Equation (4) results in the dispersion relation in the form $\omega(k) = ak^{-1}$. Thus, the phase and group velocities are determined by the following expression

$$V_{ph} = \frac{\omega}{k} = \frac{a}{k^2}, \quad V_g = \frac{d\omega}{dk} = -\frac{a}{k^2}.$$

In this case the signs of the phase and group velocities are opposite ones. Hence, the linear waves in this case are the backward waves.

If we are supposed that equation (1) is the nonlinear generalization of equation (3), we can say equation (1) describes the forward wave soliton. Oppositely, if the Sine Gordon equation (2) is the nonlinear generalization of equation (4), the solitons to equation (2) can be treated as the backward wave solitons.

Another example of the backward wave solitons has been represented in [16], where backward wave propagation was considered by using the Korteweg-de Vries equation. Propagation of the backward solitary waves (they are not solitons in the strict sense) was discussed in many papers. Review of the nonlinear phenomena in this field is presented in [12].

3. Coupled forward and backward waves

New phenomena in nonlinear optics were predicted in the case of the forward and backward wave interaction. Second and third harmonic generation in NRI metamaterials can be accompanied by quadratic and cubic backward wave soliton formation [17-20]. The forward wave and backward wave interaction is realized in a nonlinear coupler and in a nonlinear waveguide array, which consists of alternating waveguides of positive and negative refraction indexes [21-24]. Gap solitons and the bistability of continuous waves in an oppositely directed coupler represent new effects due to the positive-negative refraction phenomenon [21, 24].

The characteristic form of the evolution equations describing the forward and backward wave interaction is

$$i(e_{1,x} + v_1^{-1} e_{1,t}) = P_1[e_1, e_2], \quad i(-e_{2,x} + v_2^{-1} e_{2,t}) = P_2[e_1, e_2], \tag{5}$$

where e_1 (e_2) is a slowly varying envelope of forward (backward) wave, v_1 (v_2) is the group velocity of forward (backward) wave, and P_1 (P_2) is a functional describing the dispersion and coupling effects. For example, in the second harmonic generation case these functional can be written as

$$P_1[e_1, e_2] = -s_1 e_{1,tt} + e_2 e_1^*, \quad P_2[e_1, e_2] = -s_2 e_{2,tt} + e_1^2.$$

At this point the forward and backward wave propagating in cubic nonlinear medium will be considered. Let us assume that thephase velocities are collinear but the group velocities are oppositely directed. The normalized system of evolution equations can be written in following form

$$i(e_{1,x} + v_1^{-1}e_{1,t}) = s_1 e_{1,tt} + (|e_1|^2 + |e_2|^2)e_1 \,, \qquad (6)$$

$$i(-e_{2,x} + v_2^{-1}e_{2,t}) = s_2 e_{2,tt} + (|e_1|^2 + |e_2|^2)e_2 \,, \qquad (7)$$

where s_1 (s_2) is the coefficient of group velocity dispersion for the forward (backward) wave.

If the following ansatz

$$e_1(y) = a_1(y)e^{i\omega_1 t + i\kappa_1 x} \,, \quad e_2(y) = a_2(y)e^{i\omega_2 t + i\kappa_2 x} \,, \quad y = \tau_p^{-1}(t - x/V + t_0) \,,$$

is used, then the family of steady state solutions of the system of equations (6)-(7) can be found. Here $\omega_{1,2}$, $\kappa_{1,2}$, τ_p and t_0 are parameters of normalization. Parameter V is the solitary wave velocity.
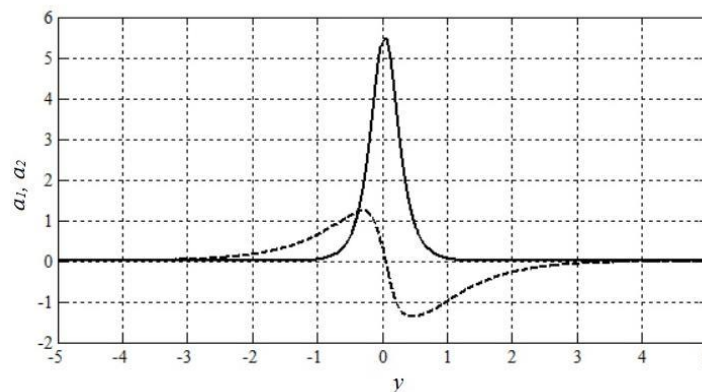

Figure 1. Real envelope of the forward soliton (full line) and backward soliton (dashed line) as function of characteristic variable y.

The kind of steady state solutions of system of equations (6)-(7) is defined by the boundary conditions at infinity ($y \to \pm\infty$). Thus, there are bright-bright solitary waves (i. e., a coupled bright forward wave and a bright backward wave) and dark-bright solitary waves (for example, a coupled bright forward wave and a dark backward wave). The first kind of the solitary wave is shown in figure 1. At some condition on the parameters $\omega_{1,2}$, $\kappa_{1,2}$ and $v_{1,2}$ the amplitudes a_1 and a_2 can be the same.

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