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Natural spectral-line broadening in atoms with unstable nuclei

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Abstract. Spontaneous emission from atoms with unstable nuclei is investigated. The decay law of an unstable nucleus is shown to be not exponential and to depend on a parameter characterizing the self-interaction of the unstable nucleus. This parameter, as well as the decay width, can be extracted from the line profile. It is demonstrated on the instance of Beryllium atom with the unstable neutron-rich ^{13}Be nucleus that for light atoms this parameter may have an essential effect on their spontaneous emission spectrum. Spontaneous emission from supercritical atoms is investigated as well. Their spectrum is continuous and depends on the decay width of the supercritical nucleus. The most sensitivity to the magnitude of the decay width is observed in the X-ray band.

1. Introduction

Atomic spectra are known to be a rich source of information about the structure of atomic nuclei. The interaction between the nucleus and atomic electrons is purely electromagnetic, its character is well known, that is why nuclear parameters can be extracted from atomic spectra without involving any model concepts. In Ref. [1] it was shown that spectroscopic methods may appear very promising for the investigation of unstable nuclei. The instability of the atomic nucleus has an unavoidable effect on the process of spontaneous emission, and hence, the spectrum carries information about the decay parameters of the unstable nucleus. The method of spectroscopic analysis developed in [1] is based on the approach of generalized quantum dynamics (GQD) [2]. Its advantage is that it allows one to regard an unstable system as an energy distribution from the very beginning and does not appeal to the eigenvalues of any Hamiltonian. Using this approach, it was shown [1] that the spectrum of a supercritical atom is continuous and depends not only on the decay width and the mass excess of the superheavy nucleus, but on a parameter characterizing the self-interaction of the nucleus. The developed approach is universal and may be applied to any unstable nucleus. In the present paper we proceed with the investigating the spontaneous emission spectrum from supercritical atoms and search for the spectral band especially sensitive to the magnitude of the decay width of the supercritical nucleus. Besides, we show how the above approach can be applied to the exploration of light unstable nuclei. We regard the spontaneous emission from the light Beryllium atom with the neutron-rich nucleus ^{13}Be and show that the parameter characterizing the nuclear self-interaction plays an essential role in this case.

2. The self-energy function of an unstable nucleus

The natural broadening of spectral line profiles $S(\omega)$ is defined by the probability of a photon being emitted with energy ω as the atomic system goes from the excited state $|i\rangle$ to a lower state $|f\rangle$:

$$S(\omega)=\frac{dW_i(\omega)}{d\omega}=A'\omega\sum_{\lambda}\int d\Omega_k\left|\langle f;\bar{k},\varepsilon_\lambda|U(t,0)|f\rangle\right|^2, \quad (1)$$

where ω, $\bar{k}$ and ε_λ are the energy, momentum and polarization of the photon respectively, A' is a normalization factor, $U(t,0)$ is the evolution operator in the representation picture connected with the evolution operator in the Schrödinger picture $U_S(t,0)$ by the following expression: $U(t,0)=e^{iH_0t}U_S(t,0)e^{-iH_0t}$. The operator $U_S(t,0)$ is just a Fourier transform of the Green operator:

$$U_S(t,0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dx\exp(-iz\tau)G(z). \quad (2)$$

In Ref. [2] it was shown that using the reduction procedure the Green operator can be redifined so that it describes the evolution of particles propagating freely or interacting only with vacuum. This is achieved by means of the insertion of the self-energy function $C(z)$ into the matrix elements of the free Green operator: $\langle m'|\tilde{G}_0(z)|m\rangle=\langle m'|m\rangle(z-E_m-C_m(z))^{-1}$, where E_m is the energy of the system in the state $|m\rangle$ and $C_m(z)$ is the energy of the interaction with vacuum in this state. The condition $z-E_m-C_m(z)=0$ determines the physical masses of particles. After such reduction the interaction in the system is described by the operator $M(z)$ which involves only the interaction of the particles with each other. The system of equations for the operators $C(z)$ and $M(z)$ was derived in [3] from the generalized dynamical equation (GDE), which in turn is a direct consequence of the first principles of quantum physics. Being equivalent to the Schrödinger equation in the case when the interaction in a quantum system is instantaneous, GDE allows one to extend quantum dynamics to the case of nonlocal-in-time interactions. This equation provides a new insight into many problems in nuclear physics [1,4], atomic physics [5-9] and quantum optics [10,11]. The approach to description of quantum dynamics constructed this way is more general and remains valid even in the case when no Hamiltonian can be constructed as an operator generating dynamics of the system. Moreover, it allows to take into account the instability of a state consistently and to regard it as an energy distribution from the very beginning, without appealing to the eigenvector system of any Hamiltonian.

The Green operator of the atomic system with an unstable nucleus, which undergoes a spontaneous atomic transition with a photon emission, has the following structure:

$$G(z)=\tilde{G}_{DP}(z)M(z)\tilde{G}_f(z)M(z)\tilde{G}_i(z), \quad (3)$$

where $\langle i|\tilde{G}_i(z)|i\rangle=(z-E_i-C(z-E_i))^{-1}$ is the Green operator of the atomic system in the excited state $|i\rangle$ with the energy E_i, $\langle f;\bar{k},\varepsilon_\lambda|\tilde{G}_f(z)|f;\bar{k},\varepsilon_\lambda\rangle=(z-E_f-\omega-C(z-E_f-\omega))^{-1}$ is the Green operator of the atomic system in a lower state $|f\rangle$ with the energy E_f and a photon in the state $|\bar{k},\varepsilon_\lambda\rangle$, $\langle DP|\tilde{G}_{DP}(z)|DP\rangle=(z-E_{DP}-\omega)^{-1}$ is the Green operator of the daughter particles of the nuclear decay with the total energy E_{DP}. All the above operators are taken in the Furry picture and the Coulomb interaction between the central nucleus and atomic electrons is already accounted for.

The self energy function of the atom with an unstable nucleus is split up to the self-energy of its electrons $C_{e^-}(z)$ and the self-energy of the nucleus $C_N(z)$: $C(z) = C_N(z) + C_{e^-}(z)$. In general, it has a real and an imaginary part: $C(z) = \Delta E + i\Gamma(z)/2$, where $\Gamma(z)/2 \equiv \mathrm{Im}C(z)$. The real part determines the shift of an energy level, while the imaginary part describes the decay law of the system and in the case of exponential decay coincides with the decay width up to the factor 1/2. Further we will regard the energies of atomic levels with account of the Lamb shift and we will deal with the renormalized masses of the particles. Thus, the real part of the self-energy function is already taken into account and only the imaginary part remains: $C(z) = i\,\mathrm{Im}C(z)$.

The decay law of the superheavy nucleus is defined by the behaviour of its self-energy function $C_N(z)$ near the point $z = E_N$, E_N being the energy of the supercritical nucleus: $E_N = E_{DP} + \Delta M$, ΔM being the mass excess. The variation of the self-energy function is generally weak and we can restrict ourselves to the first term of its Taylor expansion: $C_N(z) = C_N(E_N) + dC_N(z)/dz\big|_{z=E_N}(z - E_N) + \ldots$. Obviously, $2\,\mathrm{Im}\,C_N(E_N)$ is just the decay width Γ_0. The parameter $T_1 = dC_N(z)/dz\big|_{z=E_N}$ was discussed in [1] and looks as follows: $T_1 = \lambda R$, where $\lambda = \dfrac{\mu}{\Delta M}$ is the ratio of the reduced mass of the nucleus μ to its mass excess ΔM. The factor R characterizes the self-interaction of the unstable nucleus. It should be emphasized that this parameter is model independent and provides a fundamental characteristic of the nuclear decay process. Along with the decay width it determines the energy distribution of the unstable state. The value of R determines a nonexponential correction to the decay law. From the solution of the generalized dynamical equation for the nuclear self-energy function [1] it was obtained that $R \sim 1$. With all this, we arrive at the following form of the nuclear self-energy function:

$$2\,\mathrm{Im}\,C_N(E_{DP} + \omega) = \Gamma_0 + \lambda \cdot R \cdot \omega. \tag{4}$$

As for the electronic self-function, its dependence on z is not essential in comparison with the nuclear one, so it can be just equated to the decay width W of the corresponding atomic level: $C_{e^-}(z) = iW/2$.

3. Spontaneous emission from supercritical atoms

In the case of supercritical atoms $C_{e^-}(z)$ is negligibly small and the total self-energy function is reduced to that of the nucleus: $C(z) \equiv i\,\mathrm{Im}C_N(z)$. We will investigate the supercritical nucleus on the instance of the double-Uranium giant nuleus. The heavy Uranium nuclei undergo strong Coulomb repulsion and fly apart in a very short time. The daughter particles are two Uranium nuclei. The lifetime of the superheavy nucleus τ_{SH} plays the key role for the principal probability of the observation of vacuum decay [12] in heavy-ion collision experiments. It is estimated by nuclear physics to be of the order of $\tau_{SH} \sim 10^{-21}$ s [13] (the corresponding decay width is $\Gamma_V \sim 10^5$ eV). At the same time, an assumption was made in [14] that a mechanism can exist which holds the nuclei together for a longer time $\tau_{SH} \sim 10^{-19}$ (the corresponding decay width is smaller: $\Gamma_D \sim 10^3$ eV). The spectroscopic approach described above could help to determine the decay time more precisely.

The spectral line profile for superheavy atoms was derived in [1] and is as follows:

$$S(\omega) = \frac{A}{\Delta M^2 + \Gamma_0^2} \frac{\omega}{\left(\omega - \Delta M\right)^2 + \left(\Gamma_0 + \lambda R \omega\right)^2 / 4}. \tag{5}$$

The photon emission probability corresponding to Γ_V (Γ_D) will be denoted as $S_V(\omega)$ ($S_D(\omega)$). Here the magnitude of R is not essential in comparison with other parameters ($\Delta M, \Gamma_0$) and its variation will not have a notable effect on the spectrum. Let us explore the influence of the decay width. The numerical values of the parameters in equation (5) are as follows: $\Delta M \approx 9 \cdot 10^8 eV$ [14], $\lambda \approx 1.2 \cdot 10^2$ for the double-Uranium system [1]. In the optical range ($\omega \sim 1.6 - 3.2$ eV) the photon energy in expression (5) can be neglected in comparison with ΔM and Γ_0:

$$P(\omega) = A\omega\left(\Delta M^2 + \Gamma_0^2\right)^{-2}. \tag{6}$$

The photon emission probability is then linearly dependent on the photon energy and the derivative is constant. The numerical value of the derivative differs for the two assumed values of the decay width. This difference manifests itself in the 10th decimal index if $\Gamma_0 \equiv \Gamma_V \sim 10^5$ eV and in the 14th decimal index if $\Gamma_0 \equiv \Gamma_D \sim 10^3$ eV. Measurements of such accuracy are possible owing to the modern optical technologies [15]. In the ultraviolet range the difference manifests itself in the 8th decimal index.

In the soft X-ray band ($\omega \sim 124 \div 5000$ eV) the term $\lambda R\omega$ becomes comparable to Γ_D and in the hard X-ray region ($\omega > 5000$ eV) it reaches the value of Γ_V. The difference between the spectral line profiles manifests itself in the 6th decimal index, the shape of the profile being different as well (figure 1). Thus, the X-ray region is most sensitive from the experimental point of view.

By further increasing the photon energy overcomes the decay width and for gamma photons ($\omega > 124000$) the latter becomes negligible in formula (5). Thus, for high energy photons formula (5) looks as follows:

$$P(\omega) = \frac{A}{\Delta M^2} \frac{\omega}{(\omega - \Delta M)^2} \tag{7}$$

and becomes independent on the decay width. In this part of the spectrum the emission probability is primarily defined by ΔM and hence can be used for the experimental tests of this parameter.

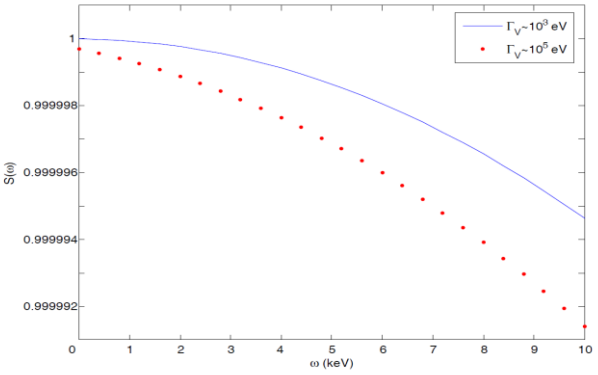

Figure 1. The emission probability for the X-ray band.

4. Spectrum of the light nucleus ^{13}Be

Let us consider the electron transition $2p_{1/2} \to 1s_{1/2}$ in a hydrogen-like atom of Beryllium with the neutron-rich nucleus ^{13}Be. The nucleus ^{13}Be decays via neutron emission:

$$^{13}_{4}\mathrm{Be} \to {}^{12}_{4}\mathrm{Be} + {}^{1}_{0}\mathrm{n},$$

with the half-life time $\tau_N = 0.51 \cdot 10^{-9}$ s ($\Gamma_0 = 0.13 \cdot 10^{-7}$ eV). The lifetime of the excited atomic state $2p_{1/2}$ is $\tau_A = 0.62 \cdot 10^{-13}$ s (accordingly, $W = 0.11 \cdot 10^{-5}$ eV). And hence the atomic transition occurs before the nucleus decays. But the nuclear instability will influence the atomic spectrum.

The distinction from the previous case is that the electron self-energy is not negligible now and should be taken into account as well, therefore, $C(z) = iW/2 + iC_N(z)$, where $C_N(z)$ is defined by formula (4) as before. The full Green operator of the system is now

$$G(z) = \tilde{G}_f(z) M(z) \tilde{G}_i(z), \tag{8}$$

where $\langle i | \tilde{G}_i(z) | i \rangle = \left(z - E_i - W/2 - C_N(z - E_i) \right)^{-1}$ is the Green operator of the atomic system in the excited state $|i\rangle$ with the energy E_i and the decay width W, $\left\langle f; \bar{k}, \varepsilon_\lambda \middle| \tilde{G}_f(z) \middle| f; \bar{k}, \varepsilon_\lambda \right\rangle = \left(z - E_f - \omega - C_N(z - E_f - \omega) \right)^{-1}$ is the Green operator of the atomic system in a lower state $|f\rangle$ with the energy E_f and a photon in the state $|\bar{k}, \varepsilon_\lambda\rangle$. The daughter particles are now a neutron ${}^1_0\mathrm{n}$ and a Beryllium atom with the nucleus ${}^{12}_4\mathrm{Be}$. Their parameters are as follows: $\mu = 0.87 \cdot 10^9$ eV, $\Delta M = 33247823$ eV and hence $\lambda = 26.1$.

Substituting this into (4) and then into (5) we obtain the following form of the spectral line profile:

$$S(\omega) = A \frac{\omega}{\left(\omega - \omega_{fi} \right)^2 + \left(W + \Gamma_0 + \lambda R \omega \right)^2 / 4}, \tag{9}$$

where $\omega_{fi} = 30.2$ eV is the atomic transition energy. Thanks to the fact, that all the intrinsic characteristics for the ^{13}Be are well known, the only parameter which can be varied here is the factor R. The spectral line profiles obtained for different values of this factor are displayed in figure (2).

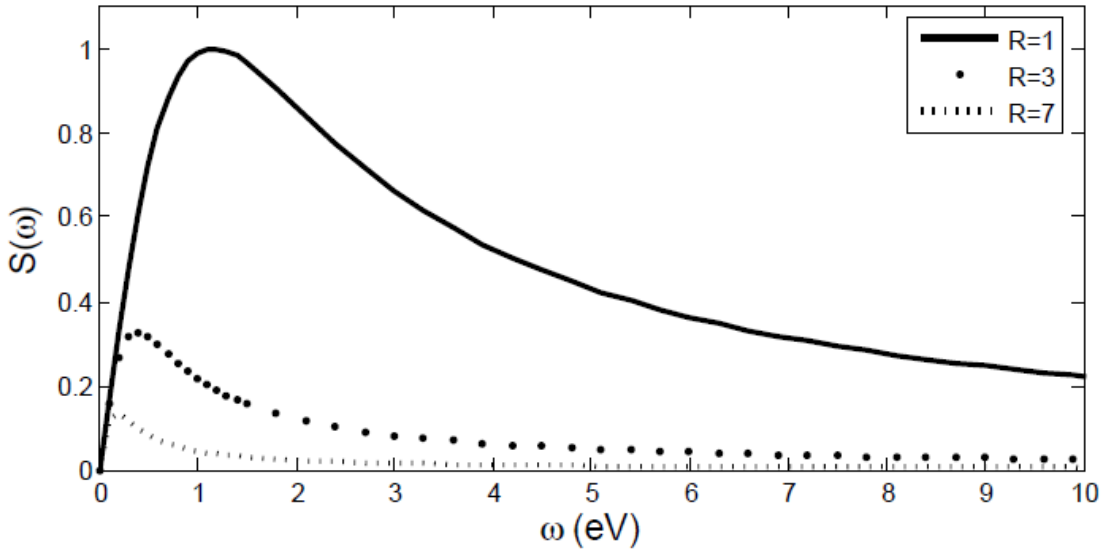

Figure 2. The radiation spectrum of the ^{13}Be atom for the photon energies $\omega \sim 0 \div 10$ eV depicted for different values of the factor R.

As it is obvious from the figure, the shape of the spectrum depends dramatically on the value of the factor R. This dependence is mostly expressed in the optical and ultraviolet parts of the spectrum, where the highest experimental accuracy is available.

5. Summary

The spectra of unstable atoms have been investigated in the framework of generalized quantum dynamics. For very heavy supercritical atoms the spectrum appears to be continuous and to depend mostly on the intrinsic parameters of the nucleus, such as its mass excess and decay width. The X-ray range has been shown to be most sensitive to the magnitude of the decay width of the superheavy nucleus and can be exploited for its experimental determination. The power of the developed approach was demonstrated for light unstable nuclei on the instance of the Beryllium atom with the unstable nucleus ^{13}Be. It was shown that its spectrum is strongly influenced by the factor depending on the nuclear interaction model and hence the spectrum can be used for the test of theoretical nuclear interaction models.

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