3+1 dimensional viscous hydrodynamics at high baryon densities

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Abstract. A 3+1 dimensional event-by-event viscous hydrodynamic + cascade model is applied for the simulation of heavy ion collision reactions at $\sqrt{s_{NN}} = 6.3 \dots 200$ GeV. UrQMD cascade is used for the pre-thermal (pre-hydro) and final (post-hydro) stages of the reaction. The baryon, as well as electric charge densities are consistently taken into account in the model. For this aim the equation of state based on a Chiral model coupled to the Polyakov loop is used in hydrodynamic phase of evolution.

As a result of the model adjustment to the experimental data, the effective values of the shear viscosity over entropy density η/s are obtained for different collision energies in the BES region. A decrease of the effective values of η/s from 0.2 to 0.08 is observed as collision energy increases from $\sqrt{s} \approx 7$ to 39 GeV.

1. Introduction

During the last years an extensive analysis of the experimental data from Au-Au collisions with maximal energy at BNL RHIC collider and Pb-Pb collisions at CERN LHC has been done in the various models based on the viscous hydrodynamic approach, with the aim to extract the values of transport coefficients of the medium created, e.g. shear viscosity to entropy density ratio η/s. There are still uncertainties coming from the different models of the pre-thermal state used, however a combined analysis of top RHIC/LHC data within the same model points to the temperature-dependent η/s of QGP medium which has to increase with temperature [1].

The aim of the present study is to conduct a similar analysis for heavy ion collisions at lower collision energies corresponding to RHIC Beam Energy Scan program and highest collision energies achievable at the future FAIR/NICA experiments, and estimate the effective value of η/s of hydrodynamic medium created. For this aim, a state-of-the-art event-by-event viscous hydrodynamic + cascade model is used.

2. Model description

The first key ingredient for the applications of the hydro-based model to the BES collision energy range is a full 3-dimensional hydrodynamic evolution. This requires non-trivial space-time rapidity dependence of initial conditions for the hydrodynamic phase. This feature is often missing in the present initial state models, such as (Monte Carlo) Glauber approach, CGC-based models, IP-Glasma. We use UrQMD cascade [2, 3] for the pre-thermal phase of the evolution, which includes and employs Pythia for the calculation of initial hard scatterings. The interactions during the pre-thermal phase are calculated until a hypersurface of a constant Bjorken proper time $\tau = \sqrt{t^2 - z^2} = \tau_0$, the latter being a parameter of the model.

On the $\tau = \tau_0$ hypersurface the system is fluidized in a following way. The energy, momenta, baryon and electric charges of hadrons crossing the hypersurface are distributed to the corresponding quantities of the (geometrically) neighbouring fluid cells using Gaussian profiles:

$$\left\{\Delta P^{\mu}_{ijk}, \Delta N^{0}_{ijk}\right\} = \left\{P^{\mu}, N^{0}\right\} \cdot C \cdot \exp\left(-\frac{\Delta x_i^2 + \Delta y_j^2}{R_\perp^2} - \frac{\Delta \eta_k^2}{R_\eta^2}\gamma_\eta^2 \tau_0^2\right) \tag{1}$$

where Δx_i, Δy_j, $\Delta \eta_k$ denote the particle's position with respect to the center of the hydrodynamic cell $\{i, j, k\}$, and $\gamma_\eta = \cosh(y_p - \eta)$ is the longitudinal Lorentz factor of a particle moving with respect to the reference frame with rapidity η. The Gaussians are normalized so that the total energy, momentum and charge deposition to the fulid from each particle equals to its energy, momentum and charge.

The second key ingredient of the model is finite baryon density, which is consistently taken into account. It is naturally present in the pre-thermal phase via UrQMD dynamics, and initial baryon and electric charge densities for the hydrodynamic expansion are calculated as mentioned above, whereas initial strangeness density is taken to be zero for all hydrodynamic system. Next, in hydrodynamic phase the additional charge conservation equations are solved, which provides densities distributions at the end of the hydro phase. The equation of state for hydrodynamics depends on the baryon density as well, provided by Chiral model [5].

The Israel-Stewart framework [4] is employed for the causal relativistic viscous hydrodynamic equations. In particular, the energy-momentum conservation equations and additionally the evolutionary equations for the shear stress tensor:

$$\langle u^{\gamma} \partial_{;\gamma} \pi^{\mu\nu} \rangle = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{\text{NS}}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_{;\gamma}u^{\gamma} \tag{2}$$

are solved, see [11] for the details. For the purpose of the current study neither bulk viscosity nor baryon diffusion are taken into account. The numerical 3+1D viscous hydrodynamic solution is performed with `vHLLE` code, which is extensively described in [6] and is tested through.

The particlization, i.e. transition from hydrodynamic medium to a hadron gas, is fixed to happen at the hypersurface of constant energy density in the rest frame of the fluid, $\epsilon = \epsilon_{\text{crit}} = 0.5$ GeV/fm^3. A Cooper-Frye prescription [7] is used to produce hadron distribution at the hypersurface. Numerically, the particlization hypersurface is assembled from many finite flat elements $\Delta\sigma_\mu$, which are found via Cornelius subroutine [8]. This results in a following distribution for the hadron species i:

$$p^0 \frac{d^3 N_i}{d^3 p} = \sum_k \Delta\sigma_{k,\mu} p^{\mu} f(p \cdot u_k, T_k, \mu_{k,i}), \tag{3}$$

where the distribution function f is taken as the one for local equilibrium with the non-equilibrium corrections according to shear the shear stress tensor $\pi^{\mu\nu}$ [11].

The produced particles are inserted as an input to the UrQMD cascade, which calculates its interactions and decays short-lived and long-lived resonances, serving as an afterburner component of the model.

3. Results

The model parameters: Gaussian radii $R_\perp$, R_η, shear viscosity over entropy rati η/s and starting time for the hydrodynamic evolution τ_0 have to be fixed from the fit to the experimental data.

$\sqrt{s}$ [GeV]	τ_0 [fm/c]	$R_\perp$ [fm]	R_η [fm]	η/s
7.7/8.8(SPS)	3.2/2.83	1.4	0.5	0.2
11.5	2.1	1.4	0.5	0.2
19.6/17.3(SPS)	1.22/1.42	1.4	0.5	0.15
27	1.0	1.2	0.5	0.12
39	0.9	1.0	0.7	0.08
62.4	0.7	1.0	0.7	0.08
200	0.4	1.0	1.0	0.08

Table 1. Collision energy dependence of the model parameters, which is used to adjust the model to the experimental data in BES region and higher RHIC energies.

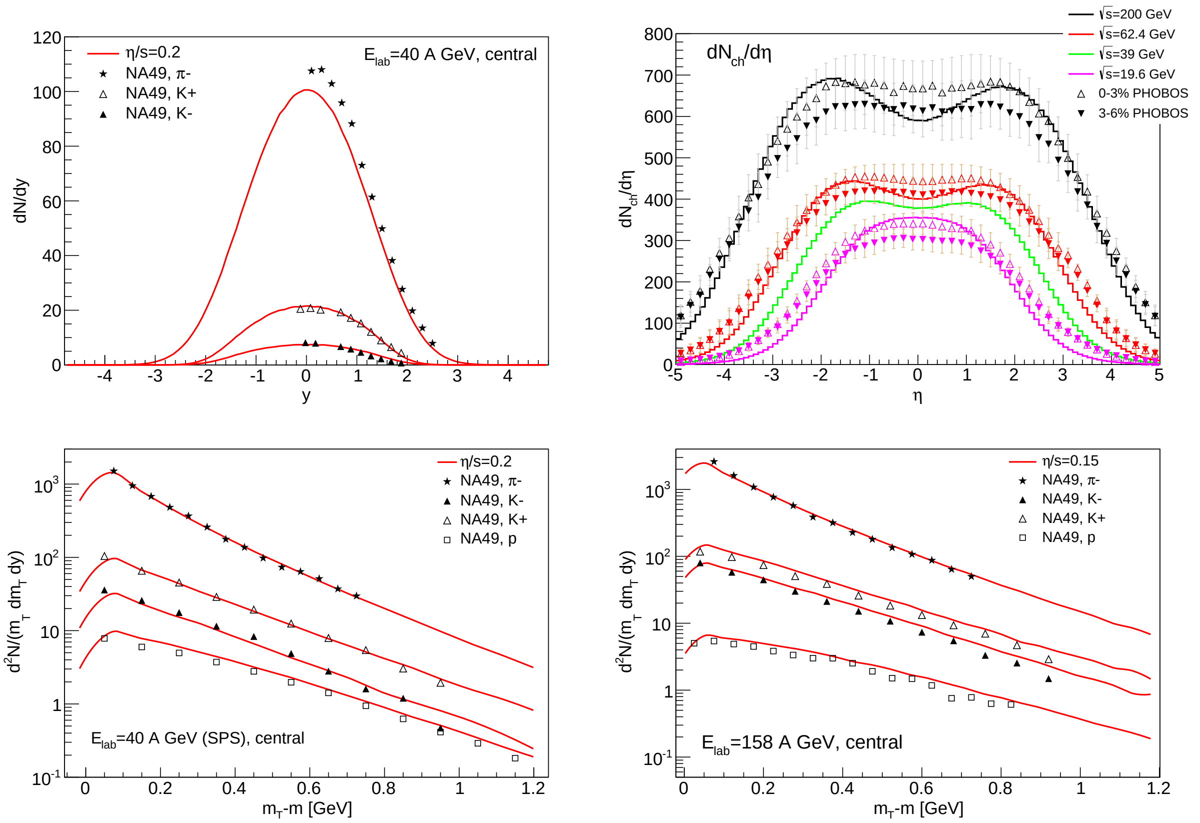

Figure 1. Rapidity distributions of pions and kaons at $E_{\mathrm{lab}} = 40$ A GeV Pb-Pb collisions (top left), pseudorapidity distributions of all charged hadrons at $\sqrt{s} = 19.6, 62.4$ and 200 GeV Au-Au collisions (top right), calculated in the model and compared to the experimental data from NA49 [9] and PHOBOS [10]

However, doing a χ^2 fit is hardly feasible since evaluating every point in the parameter space, i.e. running an event-by-event hydrodynamic+cascade simulation typically requires $\propto 10^3$ CPU hours. Thus, the parameters are manually adjusted instead, basing on a visual reproduction of the experimental observables: rapidity spectra, transverse momentum spectra and p_T integrated elliptic flow, at every particular collision energy. The resulting values of the parameters are listed in Table 1. Note that at $\sqrt{s} < 20$ GeV collision energy the value of the starting time τ_0 is taken to be equal to the time for two nuclei to completely pass through each other on average, $\tau_0 = 2R/(\gamma v_z)$, where R is a radius of the nucleus and v_z is its velocity in the laboratory frame. However, at $\sqrt{s} > 40$ GeV the latter formula leads to the values of the starting time $\tau_0 < 0.5$ fm/c, which is too early time for the hydrodynamic picture to be applicable. Thus for higher collision energies τ_0 is adjusted as well as another free parameter of the model.

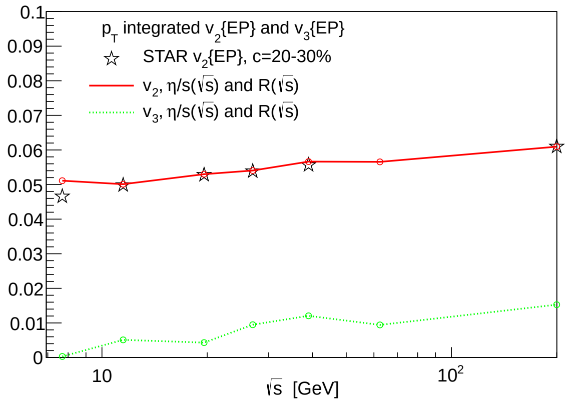

Figure 2. p_T integrated elliptic flow of all charged hadrons for 20-30% central Au-Au collisions at $\sqrt{s_{NN}} = 7.7-200$ GeV. The data points are from STAR collaboration [12].

The resulting (pseudo)rapidity and transverse momentum distributions are shown in Fig. 1. Note that, whereas the parameters are chosen so that only the overall height of the $dN/d\eta$ approaches the experiment, the trends in the shape change as a function of collision energy (going from baryon compression to baryon transparency) are generally reproduced as well. However, for higher collision energies there is a shallow dip in the pseudorapidity distribution (see Fig. 1, top right), which is not much visible in the data. The shapes of p_T spectra for pions, kaons and protons are reproduced as well, which indicates a correct interplay between the radial flow and thermal motion of the produced hadrons in the model.

Finally we turn to the results for the flow coefficients in Fig. 2. Whereas the elliptic flow v_2 is used to adjust the parameters, and is therefore following the experimental data points, the triangular flow v_3 can be interpreted as a model prediction.

In summary, event-by-event analysis in the 3+1D viscous hydrodynamic+cascade model is performed for heavy ion collisions at $\sqrt{s} = 7.7 - 200$ GeV. As a result of the model adjustment to the experimental data the effective values of η/s for the hydrodynamic phase are found for the set of BES collision energy points. A decrease of the effective values of η/s from 0.2 to 0.08 is observed as collision energy increases from $\sqrt{s} \approx 7$ to 39 GeV. This hints that in a more realistic hydrodynamic setup, where η/s would be taken temperature- and chemical potential dependent, it should increase with increasing baryon chemical potential.

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