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Overview of fluctuations at the QCD phase transition in heavy-ion collisions

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Abstract. I will summarize our current understanding of the QCD phase diagram with special emphasis on what we can expect to learn from heavy-ion collision experiments. With the upcoming Beam Energy Scan Phase II at RHIC and the experimental facilities FAIR, GSI, and NICA, Dubna, several challenges need to be met both experimentally and theoretically. I will discuss in which aspects theoretical advances are necessary in order to be able to interpret the experimental results.

1. Introduction

Heavy-ion collisions have proved to be an excellent experimental tool to study strongly interacting matter under extreme conditions such as high temperatures and high densities for a couple of decades. While the available beam energies increased steadily from one generation of accelators to the next with the LHC soon to operate at $\sqrt{s_{\rm NN}} = 5$ TeV, a revived interest in lower beam energies is seen. This is predominently due to the possibility of discovering a critical point in the phase diagram of QCD.

In the chiral limit, where the up and down quarks are massless, the phase transition between a quark-gluon plasma (QGP) and the hadronic phase at vanishing baryochemical potential $\mu_B = 0$ is of second order if the strange quark has mass $m_s = \infty$ and turns into first-order at some finite value of m_s [1, 2]. It has nowadays been established by lattice QCD calculations that the phase transition at $\mu_B = 0$ and at the physical quark masses is an analytic crossover [3, 4]. Being the only available exact method to solving QCD in the nonperturbative regime, current lattice QCD methods are only feasible at $\mu_B = 0$. Experimentally this corresponds to LHC and top-RHIC energies. Due to the stopping of the incoming baryon currents of the two colliding nuclei, one can study strongly-interacting matter at finite baryonic densities in the laboratory by decreasing the beam energy. This fascinating idea is the main motivation behind the beam energy scan (BES) at RHIC. Phase I has been completed by runs in 2010 and 2011 with data taken at at $\sqrt{s} = 7.7$, 11.5, 19.6, 27, 39, 62.4 and 200 GeV and in 2014 with an additional run at $\sqrt{s} = 14.5$ GeV. Phase II with larger statistics and additional energies is planned for 2018/19.

This brief overview is organised as follows: in the next section I will turn to theoretical studies that conjecture the existence of a critical point and an adjacent line of first-order phase transition at finite μ_B. Sections 3 and 4 discuss the potentials to discover the critical point and the first-order phase transition in heavy-ion collisions via possible signatures and point out the necessity of dynamical modeling of the phase transition. In section 5 some challenges for the upcoming BES phase II will be highlighted.

2. Indications for the existence of a critical point

As there is not yet any exact method to solve QCD thermodynamics at finite baryochemical potential, our current knowledge is limited to model studies or asymptotic behavior. While lattice QCD gives very reliable results at $\mu_B = 0$, a finite baryochemical potential makes it impossible to perform Monte-Carlo importance sampling due to the complex fermionic determinant, often called the "sign problem".

Functional methods, including Dyson-Schwinger equations (DSE) [5, 6] and functional renormalization group (FRG) [7, 8], do not suffer from the sign problem and can be applied in the entire phase diagram and in the nonperturbative regime. In the DSE approach the temperature-dependent quenched gluon propagator is usually taken as input from lattice QCD calculations and the back reaction of the quark loops is systematically included. For the quark-gluon vertex a general ansatz has to be made with parameters fixed to match the gluon input, the quark condensate and to obtain the correct asymptotic behaviors in the infrared and ultraviolet. Solving the gluon and quark DSE excellent agreement with lattice calculations for the order parameters (chiral condensate and Polyakov-loop) in the crossover transition at $\mu_B = 0$ is found. At finite baryochemical potential a critical point and a first-order phase transition are found [5, 6]. In the FRG the thermodynamic potential is solved momentum shell by momentum shell, thus obtaining a solution for the QCD flow equation. This method achieves excellent agreement with lattice calculations for vacuum QCD [9] and for pure glue at finite temperature [10] and has widely been applied to low-energy effective models, see below. Full QCD calculations in the (T, μ_B) plane are work in progress.

From both functional methods combined one can so far conclude that there is no critical point for $T/\mu_B \lesssim 2$. For larger values a more thorough treatment of baryonic degrees of freedom needs to be included.

The phase diagram of low-energy effective models has extensively been studied in chiral mesonic models with constituent quarks like the Quark-Meson (QM) or the Nambu-Jona-Lasino (NJL) model, and show the expected structures of crossover, critical point and first-order phase transition with increasing baryonic density. These models can further be extended to include the temporal gauge field A_0 in terms of the Polyakov-loop

$$\ell = \frac{1}{N_c}\langle tr_c \mathcal{P}\rangle_\beta \ , \quad \bar{\ell} = \frac{1}{N_c}\langle tr_c \mathcal{P}^\dagger\rangle_\beta \ , \tag{1}$$

for QCD with N_c number of colors and with the operator $\mathcal{P}$

$$\mathcal{P} = P \exp\left(ig_{\mathrm{s}} \int_0^\beta \mathrm{d}\tau A_0\right) \ , \tag{2}$$

where g_{s} is the strong coupling and $\beta = 1/T$ the inverse temperature. The Lagrangian of the Polyakov-Quark-Meson (PQM) model [11] reads

$$\mathcal{L} = \overline{q}\left[i\left(\gamma^\mu\partial_\mu - ig_{\mathrm{s}}\gamma^0 A_0\right) - g\left(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}\right)\right]q + \frac{1}{2}\left(\partial_\mu\sigma\right)^2 + \frac{1}{2}\left(\partial_\mu\vec{\pi}\right)^2 - U\left(\sigma,\vec{\pi}\right) - \mathcal{U}_T(\ell,\bar{\ell}) \ . \tag{3}$$

Here, the potential of the mesonic fields $U\left(\sigma,\vec{\pi}\right)$ is the classical Mexican hat potential and the temperature-dependent effective potential for the Polyakov loop is a parametrization of quenched QCD calculations from the lattice

$$\frac{\mathcal{U}_T}{T^4}\left(\ell,\bar{\ell}\right) = -\frac{b_2(T)}{4}\left(|\ell|^2 + |\bar{\ell}|^2\right) - \frac{b_3}{6}\left(\ell^3 + \bar{\ell}^3\right) + \frac{b_4}{16}\left(|\ell|^2 + |\bar{\ell}|^2\right)^2 \ , \tag{4}$$

for which the parameters can for example be found in [11].

The inclusion of the Polyakov-loop leads to the suppression of one- and two-quark states in the thermodynamic potential of the PQM model, the so-called statistical confinement. This is due to the change of the numerical value of the Polyakov-loop. It vanishes below the transition temperature and has a finite value at larger temperatures. It is thus an order parameter of the confinement/deconfinement transition. The exact position of the critical point in these models depends among other aspects on the way fluctuations are treated, on the mean-field level or beyond [11, 12].

The existence of a first-order phase transition is further backed by work on color superconductivity at low T and very large μ_B, where a rich phase structure of color-flavor locking and 2SC phases is expected [13, 14, 15], and on neutron stars [16, 17].

3. Potential to discover the critical point in heavy-ion collisions

A critical point is a very special point in the phase diagram of any system. Here, fluctuations grow very large and the thermodynamic characteristics of the systems are governed by only some macroscopic parameters, like e.g. the dimension. Microscopically, very different systems show a common, universal, behavior at their respective critical point. One of the particular features is the divergence of the correlation length of the order parameter, $\xi \to \infty$, which gives rise to the divergence of the ensemble fluctuations themselves.

By coupling the order parameter, e.g. the sigma field σ for chiral symmetry, to measurable particles like pions, ie. $g_\pi \sigma\pi\pi$, or protons, ie. $g_p \bar{p}\sigma p$, the diverging fluctuations of the sigma field should translate into event-by-event fluctuations of pion or proton multiplicities in heavy-ion collisions [18, 19]. This was realized not without noting that in small systems the correlation length can obviously not become infinitely large beyond the size of the system. In a heavy-ion collision one can assume that due to spatial limits the correlation length can maximally grow up to $\xi = 6 - 10$ fm. First fluctuations were measured at the NA49 experiment at the SPS. It was specifically looked for nonmonotonic behavior of the fluctuation measure as a function of the beam energy. Results based on second-order moments did, however, not show the expected nonmonotonic behavior [20].

More important than finiteness in space is actually the finiteness in time. It is not only the correlation length, which diverges at a critical point, but also the relaxation time. Even if a system is in equilibrium above the critical point it is necessarily driven out of equilibrium as it evolves through the critical point at any finite time. It was shown in [21] by using a phenomenological evolution equation for the mass of the sigma field $m_\sigma = 1/\xi$ that the correlation length only grows up to $\xi = 1.5-2.5$ fm. Experimentally, it thus turns out to be more favorable to look at higher-order moments of the fluctuations since they diverge proportionally to larger exponents of the correlation length [22].

At finite baryochemical potential the net-baryon density mixes with the sigma field. Due to its diffusive dynamics it becomes the true critical mode in terms of long-time dynamics [23, 24]. Therefore one expects more generally that at the critical point a non-monotonic behavior in the fluctuations of conserved charges such as net-baryon number and net-electric charge can be seen [13, 2, 18, 19, 25]. The fluctuations can be quantified by the susceptibilities, which are given by

$$\chi_l = \left. \frac{\partial^l (P/T^4)}{\partial (\mu/T)^l} \right|_T \tag{5}$$

where the derivative of the pressure P is taken with respect to the chemical potential corresponding to the conserved charge of interest. These susceptibilities can naturally be related

to the cumulants of measured multiplicity distributions of the net quantities via

$$\chi_1 = \frac{1}{VT^3}\langle N\rangle_c = \frac{1}{VT^3}\langle N\rangle\,, \tag{6}$$

$$\chi_2 = \frac{1}{VT^3}\langle(\Delta N)^2\rangle_c = \frac{1}{VT^3}\langle(\Delta N)^2\rangle\,, \tag{7}$$

$$\chi_3 = \frac{1}{VT^3}\langle(\Delta N)^3\rangle_c = \frac{1}{VT^3}\langle(\Delta N)^3\rangle\,, \tag{8}$$

$$\chi_4 = \frac{1}{VT^3}\langle(\Delta N)^4\rangle_c \equiv \frac{1}{VT^3}\left(\langle(\Delta N)^4\rangle - 3\langle(\Delta N)^2\rangle^2\right)\,, \tag{9}$$

where the first three cumulants are simply given by the corresponding central moments. The fourth cumulant is a combination of fourth and second central moments. The fluctuations are the event-wise deviation from the event-by-event mean: $\Delta N = N - \langle N\rangle$.

In the above equations for χ_l experimentally unknown quantities like the temperature and the volume of the system appear. It turns out that suitable ratios of the susceptibilities can be expressed in terms of the mean $M = \langle N\rangle$, the variance $\sigma^2 = \langle(\Delta N)^2\rangle$, the skewness $S = \langle(\Delta N)^3\rangle/\langle(\Delta N)^2\rangle^{3/2}$ and the kurtosis $\kappa = \langle(\Delta N)^4\rangle/\langle(\Delta N)^2\rangle^2 - 3$, by

$$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M}, \quad \frac{\chi_3}{\chi_2} = S\sigma, \quad \frac{\chi_4}{\chi_2} = \kappa\sigma^2\,. \tag{10}$$

The unknown volume and temperature and the simple dependence on the particle multiplicity according to the central limit theorm of statistics cancel in these ratios.

In the years 2010 and 2011, the beam energy scan phase I was run successfully at the RHIC facility. Data taken at $\sqrt{s}$ = 7.7, 11.5, 19.6, 27, 39, 62.4 and 200 GeV has been analysed and the first results of higher-order cumulants of the net-proton distribution [26] and net-charge [27] were reported.

In order to make a better connection between experimental data and the theoretical expectation for critical phenomena it will become increasingly important to develop dynamical models of the phase transition. One approach that can effectly treat fluctuations at the critical point dynamically is nonequilibrium chiral fluid dynamics (NχFD) [28, 29, 30, 31, 32, 33] where fluctuations of the sigma field and the Polyakov-loop are propagated explicitly via a coupling to a fluid dynamical expansion of heavy-ion collisions. After the discussion of some aspects of the first order phase transition in the next section I will highlight some challenges for dynamical models in section 5.

4. ... and the first order phase transition

An indirect discovery of the critical point could go via a disappearance of signals of a first-order phase transition toward higher $\sqrt{s}$. The first-order phase transition is characterized by a coexistence region where the two phases have the same pressure. Above and below the transition temperature, meta-stable states exist, in which the system can be trapped when nonequilibrium effects become important. For example the fast collective expansion of matter produced in a heavy-ion collision can lead to supercooling [34, 35, 36, 30]. If the nucleation rates are low enough, this phase will decay via spinodal instabilities [37, 38, 39]. It was shown in recent works that this effect can lead to domain formation in the net-baryon density [40, 41, 32]. In [40, 41] a nonequilibium equation of state constructed by joining a QGP equation of state via a spline with a hadron gas equation of state was applied in a fluid dynamical simulation of heavy-ion collisions. This is different from the usual equations of state with a first-order phase transition, as they are taken in the equilibrium mode, which corresponds to a Maxwell construction over the coexistence and spinodal region. It was seen that initial inhomogeneities, present in the

fluctuating initial conditions from UrQMD, get amplified in the spinodal region. In NχFD fluctuations are generated dynamically which means that even for smooth initial conditions spinodal instabilities may arise during the evolution through the phase transition, as was shown in [32].

If fluctuations are to be seen from a critical point scenario or a first-order phase transition is a question if the system is sufficiently in or out of equilibrium. One can perform a similar analysis of susceptibilities in Eq. (5) in the spinodal region instead of restricting oneselves to the thermodynamically stable states. In [42] it was found that the quark number susceptibility diverges along the isothermal spinodal lines due to the convex structure of the pressure in the presence of mechanically instable regions.

5. Challenges for the Beam Energy Scan Phase II

The following points are some important challenges for experiment and theory for current and future investigation of the QCD phase diagram.

- *Net-baryon versus net-proton fluctuations*: If net-baryon number is the driving force behind the critical fluctuations then the critical signals in the experimentally measured fluctuations in net-proton number are significantly obscured due to isospin randomization. Assuming that the number of (anti-)protons and neutrons among a certain number of (anti)baryons are distributed according to a binomial distribution, the net-baryon number fluctuations can be reconstructed from the measured (anti-)proton distributions via formulas in [43, 44]. The isospin randomization can also be included in benchmark calculations starting from a hadron resonance gas model [45].
- *Efficiency corrections*: Additional fluctuations can come from the experimental setup itself. Limited reconstruction efficiency will affect all higher-order moments of measured distributions [46]. It turns out that if the momentum coverage is extended the efficiency corrections need to be treated momentum dependent [47] and they will also affect the experimental errors [48].
- *Global charge conservation*: In heavy-ion collisions the net-baryon number, net-electric charge and net-strangeness are conserved not on average but exactly in each collision. Due to the limited experimental acceptance fluctuations are observed. It remains a challenging question if the system can at all be described in a grandcanonical ensemble or if a canoncial ensemble is more suitable. Experimental cuts can strongly affect the fluctuations [49, 50]. In transport model studies, that account for the microcanonical character of the individual scatterings, it was shown that net-proton fluctuations (in contrast to net-baryon number) are affected only at lower $\sqrt{s}$ [49]. Observing only a small fraction of all net-baryon in each event leads also to a strong bias toward observing a Poisson distribution although the underlying distribution might be very different from Poisson [50].
- *Initial state and final state*: In theoretical modeling of the fluctuations at the phase transition it cannot be forgotten that there are other sources of fluctuations coming from the initial and the final state. Especially the initial state at high baryonic densities is only very poorly understood so far. Since we are interested in fluctuations in net-baryon number, the initial fluctuations due to baryon stopping at lower beam energies need to be treated more carefully.
- *Equation of state and transport coefficients*: It is an open question if the QCD phase transition at high baryonic densities is predominantly a liquid-gas or a hadron-quark phase transition, for which the (pseudo-)critical pressure either increases or decreases as a function of the temperature [51, 52]. This will have significant phenomenological consequences. In addition to the equation of state, any realistic simulation at finite baryochemical potential requires transport coefficients as an input.

- *Phase transition and freeze-out conditions*: It needs to be emphasized that an interplay of the location of freeze-out conditions [53] and variations in $\sqrt{s}$ can generate various nonmonotonic fluctuation patterns, which need to be investigated thoroughly. In order to distinguish between fluctuations at the crossover [54], critical point and first-order phase transition a combined analysis of all higher-order cumulants needs to be performed.

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