

# Investigation of toroidal flow effects on $L$ - $H$ transition in tokamak plasma based on bifurcation model

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**Abstract.** The impacts of toroidal flow on the  $L$ - $H$  transition phenomenon in tokamak plasmas based on bifurcation concept are investigated. A set of pressure and density transport equations with both neoclassical and anomalous effects included is considered. The anomalous transport suppression mechanism considered is only from a flow shear, which is calculated based on a force balance equation with both pressure gradient and toroidal velocity components included. The toroidal velocity can be calculated using four different models. The first model is an empirical model in which the velocity is dependent on a local ion temperature. The second model is based on neoclassical toroidal viscosity theory in which the velocity is driven by ion temperature gradient. In the third model, the velocity is dependent on current density flow in plasma. The fourth model is essentially the third model including effect of an intrinsically generated bootstrap current. It is found that inclusion of toroidal velocity can substantially increase the plasma pressure and density, mainly due to an increase of the pedestal width. It is also found that the pedestal for pressure tends to form first. After the pedestal forms, it expands inwards with the characteristic of super-diffusive nature in initial state and become sub-diffusive nature in final state before reaching steady state. The expansion speed depends sensitively on the strength of flow shear effect. It is also found that the time required plasma to reach steady state after the  $L$ - $H$  transition is much longer than that for  $L$ - $H$  transition.

## 1. Introduction

Discovery of high confinement mode ( $H$ -mode) in tokamak plasma is one of the greatest achievement in fusion research [1]. Unlike low confinement mode ( $L$ -mode), the  $H$ -mode can provide significant improvement in plasma performance i.e. high central temperature, high density and pressure, and sufficient long enough confinement time. Consequently, a fusion output power is largely increased. For that reason, a modern fusion project, such as International Thermonuclear Experimental Reactor (ITER), is designed based on  $H$ -mode operation [2]. Evidently, the  $H$ -mode plasma occurs as a result of formation of an edge transport barrier (ETB), which is defined as a local narrow region near the edge of plasma, called a “pedestal” [3]. The pedestal has several key characteristics, such as a transport reduction and relatively strong gradients. The sharp gradient at plasma edge causes the plasma profiles at its core to rise up because of its stiffness property. The success of ITER depends largely on the achievement of  $H$ -mode plasma and its performance level relies on both the pedestal values and pedestal width. Historically, the  $H$ -mode plasma was first discovered over 30 years ago in

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ASDEX, Germany [1]. Since then, several tokamaks around the world have also been able to achieve the mode. Experimentally, it is known that the plasma can reach  $H$ -mode if two criteria are satisfied; firstly, the operation is equipped with either a divertor or a limiter, and secondly, plasma heat injection, regardless of heating method, surpasses a threshold value. When these two criteria are met, the plasma makes a sudden transition from  $L$ -mode to  $H$ -mode, called  $L$ - $H$  transition. Recently, an intermediate confinement mode ( $I$ -mode) occurring during the transition was discovered and it has been extensively investigated [4]. The mechanism of this mode is still not clear. Therefore, it will not be considered in this work.

Even though the  $H$ -mode was found several decades ago and many experiments have successfully operated in this mode on a daily basis, the comprehensive understanding of physics behind  $L$ - $H$  transition is still not completely developed. Many theoretical and empirical concepts revolve around the idea that the transition is caused by radial electric field in the plasma. Generally, it is known that the plasma transport is composed of neoclassical and anomalous contributions; the latter being much more dominant except possibly near plasma center. It was found from experiments that the total plasma transport is reduced to the predictable neoclassical value inside the transport barrier [5], which could imply that the anomalous transport is quenched. Theoretically, it is explained that the anomalous transport can be stabilized by flow velocity shear because of a convection cell breaking up [6]. Moreover, there are some experimental evidence which show that the anomalous transport fluxes can be suppressed by a sheared flow within the local region of transport barrier [7-8]. Therefore, many ideas propose that the transport barrier forms as a result of anomalous transport suppression by magnetic shear or/and flow shear [7], which is related to the radial electric field. In general, the flow effect can have different impacts on an unstable mode, such as strong effect on ITG, but no effect on ETG.

Bifurcation concept can be used to explain formation of ETB and physics of  $L$ - $H$  transition. Several previous works used this approach to either qualitatively or quantitatively model the transition and study barrier mechanics. Bifurcation concept is based on the idea that the  $L$ - $H$  transition phenomenon is an intrinsic property of the plasma. Namely, there exists a regime in the plasma where two different plasma states can co-exist, i.e. low and high confinement states. This allows the plasma to abruptly bifurcate from one to another. The model is frequently illustrated as an  $s$ -curve, non-monotonic, in fluxes (heat/particle) versus gradients (pressure/density) space composing of one unstable and two stable, low and high, branches, as can be seen in reference. [9]. When the heat or particle flux reaches a certain threshold the plasma gradient jumps from low to high branch, implying  $L$ - $H$  transition. The radial location of an ETB is represented where the plasma has high value of gradient. The work in this paper is based on the two-field bifurcation model analyzed in reference. [9] where heat and particle transport equations are studied simultaneously. The setback of the original model was the assumption that both neoclassical and anomalous diffusivities to be constant. Also, in calculation of radial electric field the toroidal and poloidal rotation terms are neglected. In this work, the anomalous diffusivities are calculated based on profiles stiffness model, which means that in the low gradients regimes, the plasma transport is governed solely by neoclassical effect, while in the high gradients regimes, the dominating anomalous transport is driven by the gradients [10]. Moreover, it was found that in some regimes of the plasma, toroidal rotation can significantly dominates the calculation of radial electric field as can be seen in reference. [11]. In addition, it is known that the toroidal flow can play significant role in plasma performance improvement [12]. So in this paper, effects of toroidal rotation are included for investigation.

In this work, the pressure and density transport equations are numerically solved simultaneously for spatiotemporal profiles of plasma pressure and density,  $L$ - $H$  transition, pedestal width and its dynamics. Each equation is composed of neoclassical and anomalous transports, source term and the flow shear as suppression mechanism. The suppression only affects on the anomalous channel for the reason mentioned previously. Three calculations of toroidal velocity are used; the first one is based on empirical observation in which toroidal velocity is proportional to plasma temperature [12], the second one is based on neoclassical toroidal viscosity (NTV) physics [11], and the last one is based on empirical approximation [13]. The three models are based on different physical regimes in the plasma

which are not necessary mutually exclusive so they can partially coexist. The effects of each model toward this bifurcation picture of  $L$ - $H$  transition are studied and results are compared.

This paper is organized as follows: brief descriptions of bifurcation model as well as toroidal velocity models are presented in section 2; numerical results and analysis are discussed in section 3; and the conclusion is given in section 4.

## 2. Bifurcation concept and toroidal velocity models

This section introduces the models used in the work which include bifurcation and toroidal velocity models. The main transports equations are solved numerically using discretization method. The code is developed in MATLAB environment based on the built-in “*pdepe*” function which is appropriate for solving initial-boundary problems for parabolic-elliptic partial differential equations in one dimension.

### 2.1. Two-field bifurcation model

The model used in this work is based on the framework of bifurcation concept introduced in reference [9]. It conceptually explains that the  $L$ - $H$  transition is an intrinsic property of the plasma where its confinement mode can abruptly change providing that some criteria are satisfied, i.e. ratio of anomalous over neoclassical diffusivities and heat or particle fluxes must exceed threshold values [14]. Graphically, the model is depicted as bifurcation diagram illustrating an  $s$ -curve shape graph of pressure/density gradients against heat/particle fluxes, respectively. A jump of pressure and density gradients in the graph indicates that the plasma has entered  $H$ -mode. The model consists of coupled pressure and density transport equations which are expressed in slab geometry as shown:

$$\frac{3}{2} \frac{\partial p}{\partial t} - \frac{\partial}{\partial x} \left[ \chi_{neo} + \frac{\chi_{ano}}{1 + \alpha v_E'^2} \right] \frac{\partial p}{\partial x} = H(x), \quad (1)$$

$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial x} \left[ D_{neo} + \frac{D_{ano}}{1 + \alpha v_E'^2} \right] \frac{\partial n}{\partial x} = S(x), \quad (2)$$

where  $p$  is the plasma pressure,  $n$  is the plasma density,  $\chi_{neo}$  and  $D_{neo}$  represent thermal and particle neoclassical transport coefficients, respectively,  $\chi_{ano}$  and  $D_{ano}$  represent thermal and particle anomalous transport coefficients, respectively,  $v_E'$  is the flow shear suppression,  $\alpha$  is a positive constant representing strength of the suppression, and  $H(x)$  and  $S(x)$  are the thermal and particle sources. Essentially, these two equations represent conservation of energy and mass, respectively. Note that throughout this work, the thermal and particle sources are estimated to be localized at plasma center and edge, respectively, using Gaussian distribution. The slab geometry is justified because tokamak plasma is assumed to have toroidal and poloidal symmetries in circular shape limit. The stabilizing mechanic for anomalous transport is the flow shear  $v_E'$ , which accounts for the known reduction of turbulent transport by sheared radial electric field [6]. It couples the two transport equations based on force balance calculation:

$$v_E' \propto E_r' = \left( \frac{p'}{n} - v_\theta B_\phi + v_\phi B_\theta \right)', \quad (3)$$

where  $v_\theta$  and  $v_\phi$  are the poloidal and toroidal velocities, respectively, and  $B_\theta$  and  $B_\phi$  are poloidal and toroidal magnetic fields, respectively. The poloidal rotation term, which is a result from neoclassical calculation, is neglected in this work. Whereas, the toroidal rotation term was found to be dominating in some plasma regime as illustrated in reference [11] so its effect will be studied in this work because it was ignored in previous bifurcation related works. Models for toroidal velocity prediction are discussed in the next section. The neoclassical transport coefficients are simply set to be constant while the anomalous transport coefficients follows critical gradient transport model similar to that described in reference [10], with the functional forms:

$$\chi_{ano} = c_x (p' - p'_c) H(p' - p'_c), \quad (4)$$

$$D_{ano} = c_D (n' - n'_c) H(n' - n'_c), \quad (5)$$

where  $c_x$  and  $c_D$  are proportional constants,  $p'_c$  and  $n'_c$  are the critical gradients for pressure and density fields, respectively, and  $H$  represents a Heaviside step function. This local anomalous transport model realizes the physical plasma transport in which at low gradient, near plasma center, the neoclassical effect dominates the transport, while at high gradient, near plasma edge or transport barrier location, the anomalous transport is driven by the gradient itself.

### 2.2. Toroidal velocity models

This section discusses the toroidal velocity models used in this paper. Four different toroidal velocity models are employed, each governs different physics in tokamak plasma. These models provide explicit calculations of toroidal velocity which means the toroidal momentum equation is not included. This requires extensive modelling and analysis, and is left for future work. Model a) is based on an empirical observation which assumes that the toroidal flow is proportional to local plasma temperature  $T$  [12]:

$$v_{\phi,a} \propto T. \quad (6)$$

This model is applicable in tokamak which is directly driven by on-axis neutral beam injection (NBI) heating, for example JET. Model b) is based on neoclassical toroidal viscosity (NTV) physics which applies to plasma with symmetry breaking via the application of a non-axisymmetric field [11]. It has the form:

$$v_{\phi,b} \propto \frac{\partial T}{\partial r}. \quad (7)$$

So, the toroidal velocity in this model is proportional to the gradient of local plasma temperature gradient. Model c) is an empirical estimation discussed in Ref. [13], where the toroidal velocity is assumed to be proportional to plasma toroidal current density:

$$v_{\phi,c} \propto j_\phi = j_0 \left( 1 - \frac{r^2}{a^2} \right)^v, \quad (8)$$

where  $j_\phi$  and  $j_0$  are toroidal current density and its value at plasma center,  $r/a$  is normalized minor radius and  $v$  is a constant to control current shaping. This form of plasma current is a typical estimation for on-axis current drive system [15]. Model d) is similar to model c) except the extra bootstrap current  $j_b$  term. This bootstrap current is an intrinsic current generated in the plasma it has the form [15]:

$$j_b = -T \frac{\partial n}{\partial r}. \quad (9)$$

So, the last model has the form:

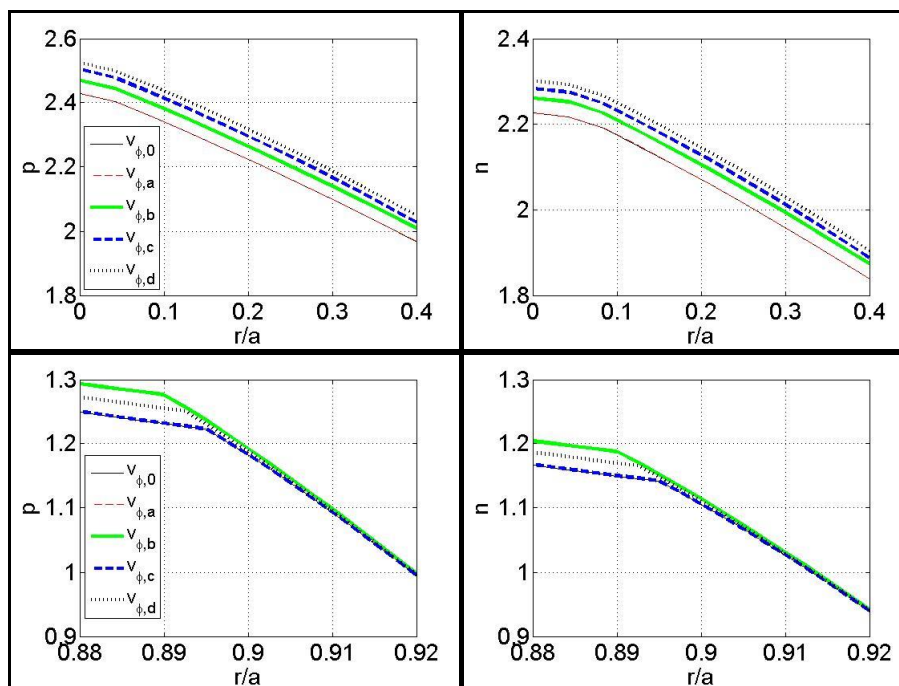
$$v_{\phi,d} \propto j_\phi + j_b, \quad (10)$$

## 3. Numerical results and discussions

The numerical simulations in this work are results of solving transport equations (1) and (2) simultaneously and self-consistently. In accordance with physical observations, the anomalous transports are assumed to be one order of magnitude over the neoclassical transports, while particle diffusivities are assumed to be one third of the thermal diffusivities. The heat and particle sources are assured to be large enough so the plasmas can access the  $H$ -mode.

### 3.1. Plasma response on profile and pedestal width

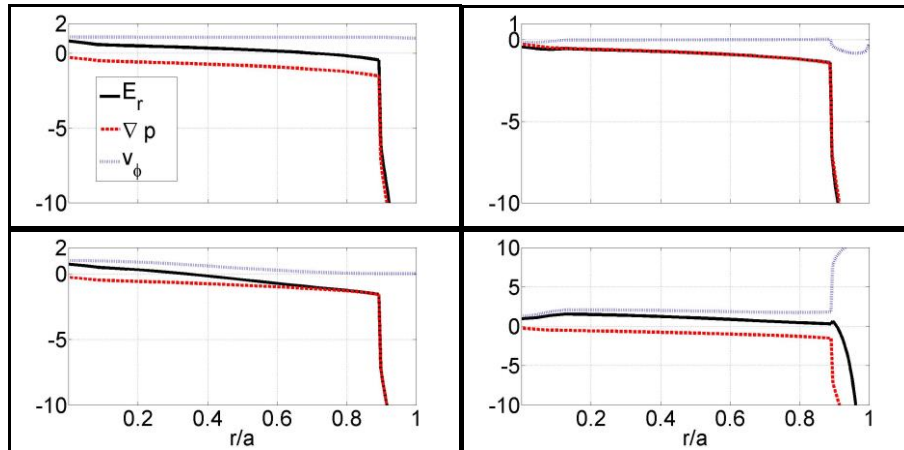
Effects of toroidal rotation on plasma profiles are investigated in this section. This can be seen in figure 1, which shows the simulation results for plasma pressure and density at plasma core and edge as a function of normalized minor radius ( $r/a$ ) at steady state. The four toroidal velocity models are used for simulations in order to compare with results without toroidal flow included ( $v_{\phi,0}$ ). In general, inclusion of toroidal flow tends to increase plasma pressure and density except at its edge where transport barrier is formed. This is not surprising because the plasma in the edge region is governed solely by neoclassical effect, which remains unaffected by the toroidal term. Moreover, the toroidal term facilitates the enhancement of transport reduction, thus higher plasma profiles. The top panels show plasma profiles near its center, it can be estimated that the increases are approximately 0.04%, 1.72%, 3.09% and 3.96% of center pressure, and 0.03%, 1.58%, 2.60% and 3.39% of center density for simulations using model a), b), c) and d), respectively.



**Figure 1.** Plasma core profiles for pressure (top left) and density (top right), and plasma edge profiles for pressure (bottom left) and density (bottom right) at steady state. Each line represents simulation results without toroidal flow (thin solid), using model a) (thin dashed), model b) (thick solid), model c) (thick dashed) and model d) (dotted).

The toroidal effects yield somewhat different results at pedestal area (area with high gradients) as seen in the bottom panels of figure 1. First of all, the results show that the pedestal width remains unchanged from simulation without toroidal effect when using model a) and c) with pedestal width of 0.105. On the other hand, the simulations using model b) and d) yield pedestal widths of 0.110 and 0.108, respectively. Note that the pedestal width appears to be the same for pressure and density channels regardless of the model used. The values at the top of pedestal ( $p_{ped}$ ,  $n_{ped}$ ) are also increased by inclusion of toroidal flow. The increases are approximately 0.09%, 4.40%, 0.04% and 2.29% of  $p_{ped}$ , and 0.08%, 4.07%, 0.04% and 2.12% of  $n_{ped}$  for simulations using model a), b), c) and d), respectively. The reasons for the increase being difference from model to model at plasma center and edge can be seen in figure 2 which shows radial electric field, pressure gradient and toroidal flow contributions as a function of normalized minor radius. First of all, equation (3) implies that the shear or gradient of toroidal velocity profile is an essential ingredient for transport reduction calculation. It can be seen that in model a), the profile is relatively flat throughout the whole plasma so it does not

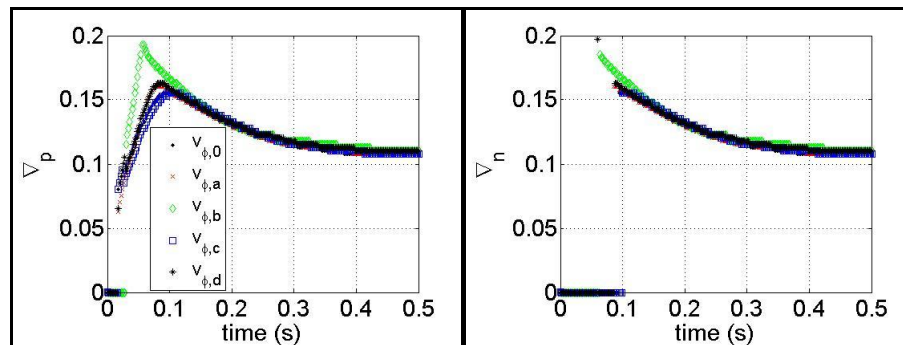
affect the flow shear much. In model b), the profile appears to have slight gradient at plasma center and strong gradient at plasma edge. Model c) has somewhat high gradient near plasma center and becomes flat near plasma edge. And model d) has both high gradients at plasma center and plasma edge. These results agree with the increase of plasma values at its center and pedestal.



**Figure 2.** Profiles of radial electric field (solid), pressure gradient term (dashed) and toroidal rotation term (dotted) at steady state for simulations using model a) (top left), b) (top right), c) (bottom left) and d) (bottom right).

### 3.2. Pedestal dynamics

This section illustrates the pedestal dynamics once  $L-H$  transition occurs. Figure 3 demonstrates the change of pedestal width after it is formed. Initially, the transport barrier only forms at pressure channel. After forming, it starts to expand. It appears that the pedestal growth is categorized as superdiffusive behaviour ( $\Delta_{ped} \propto t^b, b > 0.5$ ), agreeing with the turbulent nature of the plasma because in this phase the suppression effect is still not too high, especially at the particle channel suppression, thus turbulent transport still plays a considerable role. The speed of pedestal width expansion appears to be related to the gradient of toroidal velocity profile near pedestal area. Left panel of figure 3 shows that the barrier expands fastest for simulations using model b) with expansion rate of  $2.9 \text{ s}^{-1}$ , then model d) with rate of  $1.4 \text{ s}^{-1}$ , their toroidal velocity gradients near the pedestal are 3.2 and 2.4, respectively. The correlation is not clear in model a) and c) which yield very small negative value of toroidal velocity gradients while the expansion rate is slightly less than that of model d). At some time later, the transport reduction effect becomes large enough to suppress the density channel so the transport barrier in density channel starts to form. Note that the pedestal width of both pressure and density channels are about the same regardless of the model used. The onset of transport barrier formation in density channel also improves density profiles significantly. This actually reduces the first contribution in the radial electric field calculation of equation (3) resulting in the flow shear being less effective. As a result, both barrier widths slowly decrease or collapse until the plasma reaches steady state. The decrease is slower than the width expansion because now the plasma is governed solely by the neoclassical transport which is a slower process than turbulent transport. In this process the width dynamics becomes subdiffusive or even slower. There is an interesting point worth mentioning here, which is the time it takes the plasma to evolve during H-mode is around one order of magnitude slower than the time it takes for the plasma to evolve from L to H mode.



**Figure 3.** Time evolution of pedestal width of pressure (left) and density (right) channels.

#### 4. Conclusion

Numerical method is used to solve the coupled pressure and density transport equations based on bifurcation concept. The transport effects included are neoclassical transport which is assumed to be constant and anomalous transport which is inspired by the critical gradient transport model. The suppression mechanism is the flow shear calculating from the force balance equation which includes toroidal flow effect. Four simple toroidal velocity models are considered in the calculation of radial electric field based on the force balance equation. It is found that inclusion of toroidal velocity can substantially increase the plasma pressure and density, mainly due to an increase of the pedestal width. It is also found that the pedestal for pressure tends to form first. After the pedestal forms, it expands inwards with the characteristic of super-diffusive nature in initial state and become sub-diffusive nature in final state before reaching steady state. The expansion speed depends sensitively on the strength of flow shear effect. It is also found that the time required plasma to reach steady state after the  $L$ - $H$  transition is much longer than that for  $L$ - $H$  transition.

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