

# Shear viscosity in $SU(2)$ lattice gluodynamics

V. V. Braguta<sup>1,2,3</sup>, A. Yu. Kotov<sup>1</sup>

<sup>1</sup> Alikhanov Institute for Theoretical and Experimental Physics, ul. Bolshaya Cheremushkinskaya 25, Moscow, 117218 Russia

<sup>2</sup> Institute for High Energy Physics, Protvino, Moscow region, 142281 Russia

<sup>3</sup> Moscow Institute of Physics and Technology (State University), Institutskii per. 9, Dolgoprudnyi, Moscow region, 141700 Russia

E-mail: [braguta@mail.ru](mailto:braguta@mail.ru), [kotov@itep.ru](mailto:kotov@itep.ru)

**Abstract.** The calculation of the gluon plasma viscosity has been performed by lattice simulations in the  $SU(2)$  gluodynamics at  $T/T_c = 1.2$  on the supercomputers. The evaluation is based on the Kubo formula that relates viscosity to the spectral function of the correlation functions of the energy-momentum tensor. For the extraction of the spectral function from the Euclidean correlator the linear method is applied.

## 1. Introduction

One of the most important results obtained at RHIC experiment on heavy ion collisions is the measurement of the elliptic flow of final particles[1, 2]. Obtained value can be explained within the hydrodynamical approximation if one assumes that quark-gluon plasma formed during heavy ion collisions is almost a superfluid liquid[3, 4, 5]. Numerical simulations of a relativistic liquid lead to the following bound on the ratio of the viscosity to the entropy density of the system[6]:  $\eta/s \leq 0.4$ , which is the smallest ratio for all systems known to date. Thereby one of the most important problems in the modern theoretical physics is a first-principle calculation of the viscosity of quark-gluon plasma. One of the most powerful approaches to this problem is the numerical simulation of quark-gluon plasma. It is worth mentioning, that there were the works, in which the viscosity of quark-gluon plasma was studied on the lattice[7, 8, 9], but due to the complicated calculations, the results obtained are not exhaustive.

In the present paper the calculation of the viscosity of the  $SU(2)$  gluodynamics is performed by means of lattice modeling. Instead of the  $SU(3)$  QCD we consider the  $SU(2)$  model as both theories are rather similar physically and at the same time numerical calculations in the  $SU(2)$  gluodynamics are much simpler than in the  $SU(3)$  theory. This leads to the significant advances in the study of transport coefficients of quark-gluon plasma.

## 2. A brief description of the method

Kubo formulas [10] relating transport coefficients to the correlators of energy-momentum tensor are used in this calculation. In particular, for the viscosity the Kubo formula reads:

$$\eta = \pi \lim_{\omega \rightarrow 0} \frac{\rho_{12,12}(\omega, \mathbf{q} = 0)}{\omega}, \quad (1)$$



where  $\rho_{12,12}$  is a spectral function, which is determined as an imaginary part of the two-point retarded Green function:  $\rho_{12,12}(\omega, \mathbf{q}) = \frac{1}{\pi} \text{Im} \langle T_{12} T_{12} \rangle_{ret}(\omega, \mathbf{q})$ . The Green function of energy-momentum tensor measured in lattice calculations is defined as follows:

$$C_{12,12}(x_0, \mathbf{p}) = \beta^5 \int d^3 \mathbf{x} e^{i\mathbf{p}\mathbf{x}} \langle T_{12}(0) T_{12}(x_0, \mathbf{x}) \rangle, \quad (2)$$

where  $\beta = 1/T$  is an inverse temperature of the system. Function (2) is an analytical continuation of the retarded Green function:  $p_0 \rightarrow i\omega$  and is related to the spectral function in the following way:

$$C_{12,12}(x_0, \mathbf{p}) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, \mathbf{p}) \frac{\cosh \omega(\frac{1}{2}\beta - x_0)}{\sinh \frac{\omega\beta}{2}} d\omega \quad (3)$$

In order to determine the value of viscosity one needs to measure on the lattice the correlator  $C_{12,12}$  (1) as a function of  $x_0$  for  $\mathbf{p} = 0$ , and then invert (3) to find the spectral function  $\rho_{12,12}(\omega, \mathbf{q} = 0)$ . Low-frequency behaviour of spectral function  $\rho_{12,12}$  determines the value of the viscosity.

For the inversion of (3) we use the linear method proposed in [9], which is based on the assumption that the spectral function is rather smooth and can be decomposed into certain basis functions.

### 3. Measurement of the viscosity

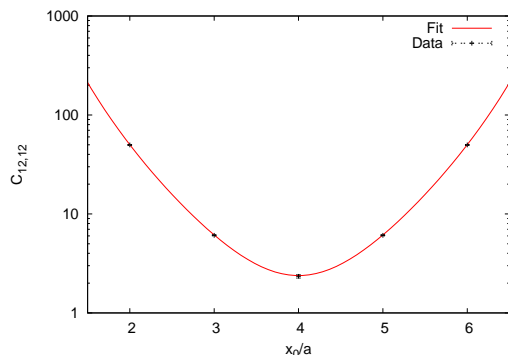


Figure 1: Correlation function  $C_{12,12}(x_0)$  of the energy-momentum tensor. The points show the data measured on the lattice. The red line corresponds to the spectral function extracted from this data.

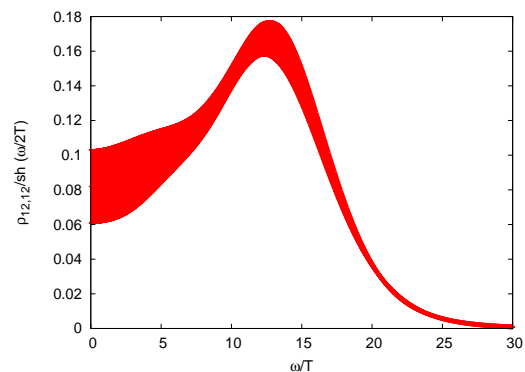


Figure 2: Spectral density  $\rho_{12,12}(\omega)$  divided by  $\sinh(\frac{\omega}{2T})$ . The filled region corresponds to statistical uncertainties of our data.

The measurement of energy-momentum tensor was performed in lattice gluodynamics with the  $SU(2)$  gauge group and the Wilson action:

$$S_g = \beta \sum_{x, \mu < \nu} \left( 1 - \frac{1}{N_c} \text{tr} U_{\mu, \nu}(x) \right). \quad (4)$$

Energy-momentum tensor in gluodynamics has the form:

$$T_{\mu\nu} = F_{\mu\alpha}^a F_{\nu\alpha}^a - \frac{1}{4} \delta_{\mu\nu} F_{\rho\sigma}^a F_{\rho\sigma}^a \quad (5)$$

For the chromo-electromagnetic tensor the clover-shaped discretization is used:

$$\begin{aligned} F_{\mu\nu}(x) &= \frac{1}{4}(V_{\mu,\nu}(x) + V_{\nu,-\mu}(x) \\ &\quad + V_{-\mu,-\nu}(x) + V_{-\nu,\mu}(x)) \\ V_{\mu,\nu}(x) &= \frac{1}{2}(U_{\mu,\nu}(x) - U_{\nu,\mu}(x)). \end{aligned} \quad (6)$$

The two-level algorithm is used for measuring correlator (1). It significantly improves the accuracy of the calculations [12]. The simulations are performed on the lattice  $8 \times 32^3$ ,  $\beta = 2.643$ , which corresponds to the temperature  $T/T_c \approx 1.2$ . The results of the evaluation of correlator (2) are shown in Fig. 1.

For the extraction of spectral function the linear method described in [9] is used. The following function that describes the behaviour of the spectral function at large frequencies (which is determined by the asymptotical freedom) is used as an initial approximation:

$$m(\omega) = \frac{A\omega^4}{\tanh^2 \frac{\omega}{4T} \tanh^2 \frac{\omega}{2T}} \quad (7)$$

In Fig. 2 the obtained data for the spectral function is shown. Statistical errors are determined in accordance with the method described in [9].

The results for the measuring of the viscosity are presented as the ratio  $\eta/s$ , where  $s$  is an entropy density. The latter is determined using the relation:  $s = \frac{\epsilon+p}{T}$  and the method for the evaluation of  $\epsilon + p$ , described in [13]. Applying this method we obtain the following value for this ratio:

$$\frac{\eta}{s} = 0.111 \pm 0.032 \quad (8)$$

An error in this result corresponds to the statistical uncertainty of our data. Value (8) is close to the so-called *KSS* bound  $\eta/s \geq 1/4\pi \approx 0.08$ , obtained by means of *AdS/CFT*-correspondence [14]. Moreover, result (8) satisfies the bound obtained from the description of RHIC experiments  $\eta/s < 0.4$  [6].

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