

Soft Interactions at High Energies: Amplitudes and Cross-Sections

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Abstract. I briefly specify elements of the GLM model that successfully describes soft hadronic interactions at energies from ISR to LHC. The model is based on a single Pomeron with a large intercept $\Delta_P = 0.23$ and slope $\alpha'_P = 0.028$, and so provides a natural matching with perturbative QCD. I analyze the elastic, single diffractive and double diffractive amplitudes, and compare the behaviour of the GLM amplitudes to those of other parameterizations. I then summarize the main features and results of the GLM model and compare with those of the Kaidalov-Poghosyan and other models for soft interactions at high energies.

1. Introduction

The recent measurements of the proton-proton cross sections at the LHC at an energy of $W = 7$ TeV, allows one to appraise the numerous models that have been proposed to describe soft interactions. The classical Regge pole model *à la* Donnachie and Landshoff [1], which provided a reasonable description of soft hadron-hadron scattering upto the Tevatron energy, fails when extended to LHC energies [2]. In addition, it has the intrinsic problem of violating the Froissart-Martin bound [3].

At present there are a number of models based on Reggeon Field Theory that provide a reasonable description of proton-proton scattering data over the energy range from ISR to LHC. I will describe the essential features of the GLM model [4] as an example of a model of this type, before comparing its results with other competing models on the market.

1.1. Basic features of the GLM model

We utilize the simple two channel Good-Walker (GW) [5] model, to account for elastic scattering and for diffractive dissociation into states with masses that are much smaller than the initial energy, and impose the unitarity constraint by requiring that

$$2\text{Im } A_{i,k}(s, b) = |A_{i,k}(s, b)|^2 + G_{i,k}^{in}(s, b)$$

where, $A_{i,k}$ denotes the diagonalized interaction amplitude and $G_{i,k}^{in}$, the contribution of all non GW inelastic processes.

A general solution for the amplitude satisfying the above unitarity equation is:

$$A_{i,k}(s, b) = i \left(1 - \exp \left(-\frac{\Omega_{i,k}(s, b)}{2} \right) \right) \quad (1)$$



the opacities $\Omega_{i,k}$ are arbitrary. In the eikonal approximation $\Omega_{i,k}$ are assumed to be real, and taken to be the contribution of a single Pomeron exchange.

GLM parameterize the opacity :

$$\Omega_{i,k}(s, b) = g_i(b) g_k(b) P(s)$$

where $P(s) = s^\Delta$, and $g_i(b)$ and $g_k(b)$ are the Pomeron-hadron vertices given by:

$$g_l(b) = g_l S_l(b) = \frac{g_l}{4\pi} m_l^3 b K_1(m_l b).$$

$S_l(b)$ is the Fourier transform of $\frac{1}{(1+q^2/m_l^2)^2}$, where q is the transverse momentum carried by the Pomeron, $l = i, k$. The form of $P(s)$ used by GLM, corresponds to a Pomeron trajectory slope $\alpha'_{\mathbb{P}} = 0$. This is compatible with the exceedingly small fitted value of $\alpha'_{\mathbb{P}}$, (0.028 GeV⁻²) and in accord with $N=4$ SYM.

For the case of $\Delta_{\mathbb{P}} \rightarrow 0$, the Pomeron interaction leads to a new source of diffraction production with large mass ($M \propto s$), which cannot be described by the Good-Walker mechanism. Taking $\alpha'_{\mathbb{P}} = 0$, allows one to sum all diagrams having Pomeron interactions [6, 7]. This is the advantage of such an approach. The GLM model only takes into account triple Pomeron interaction vertices ($G_{3\mathbb{P}}$), this provides a natural matching to the hard Pomeron, since at short distances $G_{3\mathbb{P}} \propto \alpha_s^2$, while other vertices are much smaller. A full description of the procedure for summing all diagrams (enhanced + semi-enhanced) is contained in [6, 7, 8]. We would like to emphasize that in the GLM model, the GW sector contributes to both low and high diffracted mass, while the non-GW sector contributes only to high mass diffraction ($\log(M^2/s_0) \approx 1/\Delta_{\mathbb{P}}$).

The GLM model has 14 parameters describing the Pomeron and Reggeon sectors. The values of these parameters are determined by fitting to data for σ_{tot} , σ_{el} , σ_{sd} , σ_{dd} and B_{el} in the ISR-LHC range [8]. We find the best fit value for $\alpha_{\mathbb{P}} = 0.21$, however, to be in accord with the LHC data we have tuned $\alpha_{\mathbb{P}}$ to 0.23. The fitted values for $\alpha'_{\mathbb{P}}$ is 0.028 GeV⁻², while the triple Pomeron vertex $G_{3\mathbb{P}} = 0.03$ GeV⁻¹.

2. Amplitudes

The Good-Walker formalism [5] provides an explicit form for the various elastic and diffractive amplitudes. Until recently most of the comparison of models has been made on the level of cross-sections (which are areas), and only reveal the energy dependence, and do not display other features. Having the behaviour of the various amplitudes as functions of impact parameter (momentum transfer) would be more revealing. Unfortunately, there is a paucity of material available on amplitudes, and most refer only to the elastic amplitude.

In Fig.1 (left panel) we show elastic amplitudes emanating from the GLM model for various energies. We note the overall gaussian shape of the elastic amplitudes for all energies $0.545 \leq W \leq 57$ TeV, with the width and height of the gaussian growing with increasing energy. For small values of b the slope of the amplitudes decreases with increasing energy. The elastic amplitude (as $b \rightarrow 0$) becomes almost flat for $W = 57$ TeV, where it is still below the Unitarity limit $A_{el} = 1$.

In Fig. 1 (right panel) we show the energy behaviour of the GLM (G-W contribution) of the single diffractive amplitude as a function of impact parameter for different energies. In Fig. 2 (left panel) we display the behaviour of the double diffractive amplitude. A common feature of both diffractive amplitudes is that with increasing energy the peaks broaden and become more peripheral. In Fig. 2 (right panel) we show the elastic, single diffraction and double diffraction amplitudes as functions of b for $W = 7$ TeV. Note the completely different shapes of the three amplitudes, the elastic amplitude $A_{el}(b)$ is gaussian in shape, while the single

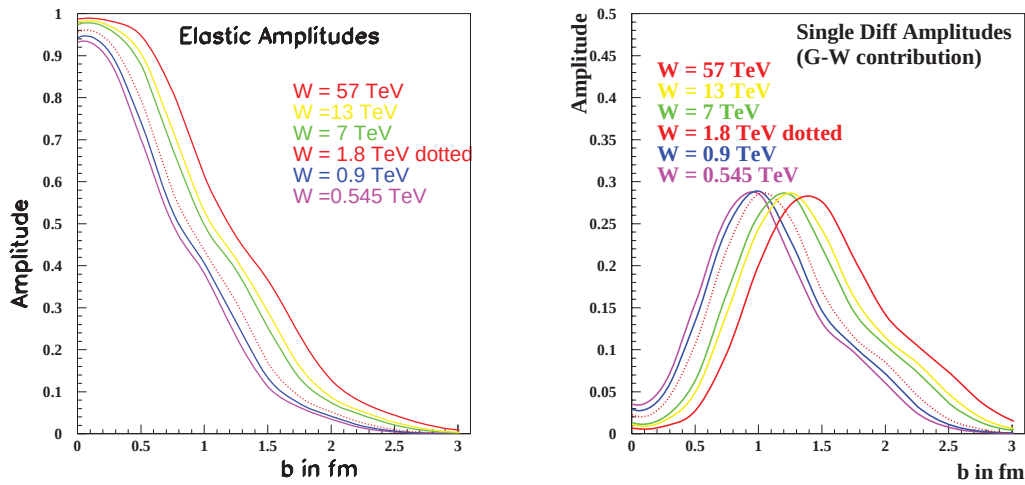


Figure 1. Left panel: GLM elastic amplitudes and Right panel: GLM-GW single diffraction amplitudes.

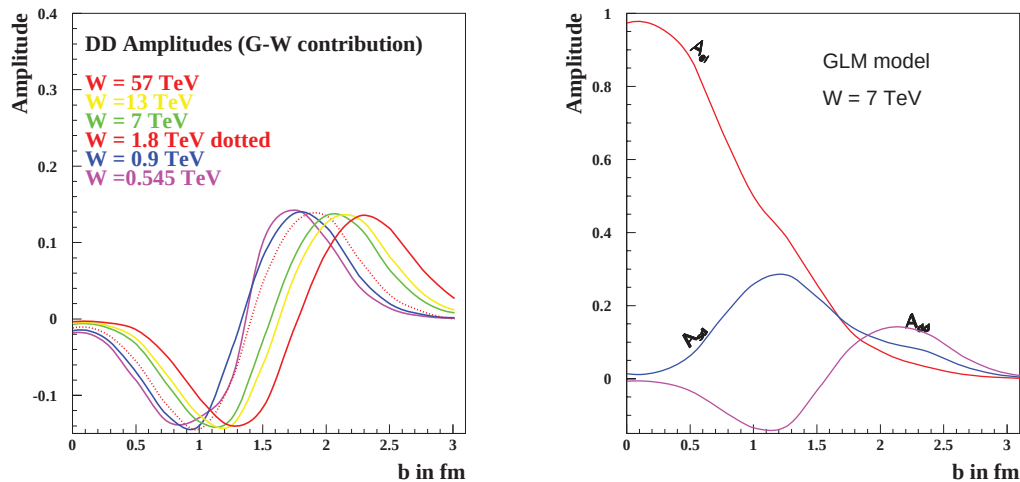


Figure 2. Left panel: GLM double diffractive amplitudes and Right panel: GLM elastic, single diffractive and double diffractive amplitudes for $W = 7$ TeV.

diffractive amplitude $A_{sd}(b)$ and the double diffractive amplitude $A_{dd}(b)$ are very small at small b . A_{sd} has a peak at 1.25 fm, while A_{dd} 's maximum is at $b = 2.15$ fm.

The Durham group [9] have attempted to extract the form of the Elastic Opacity directly from the data. They assume that at high energies the real part of the scattering amplitude is very much smaller than the imaginary part, then to a good approximation

$$A(b) = i[1 - \exp(-\Omega(b)/2)]$$

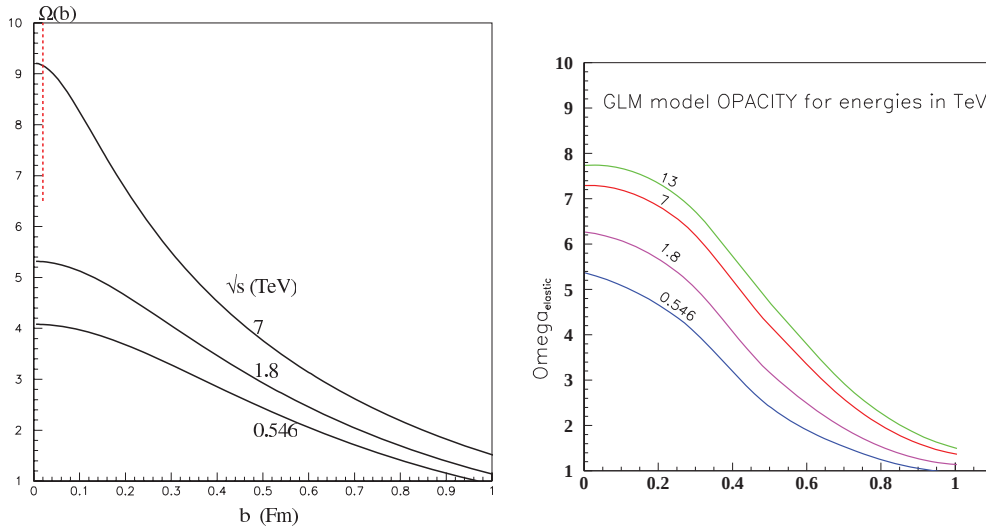


Figure 3. Left panel: The proton opacity $\Omega(b)$ determined directly from the pp $d\sigma_{el}/dt$ data at 546 GeV , 1.8 TeV and 7 TeV data. The uncertainty on the LHC value at $b = 0$ is indicated by a dashed red line. This figure is taken from [9] which should be consulted for details. Right panel: Opacities calculated using the GLM model.

(see Eqn(1)). As $\Omega_{el} = -2\ln(1 - A_{el})$, they determine the Opacity directly from the data since

$$\text{Im}A(b) = \int \sqrt{\frac{d\sigma_{el}}{dt} \frac{16\pi}{1 + \rho^2}} J_0(q_t b) \frac{q_t dq_t}{4\pi},$$

where $q_t = \sqrt{|t|}$ and $\rho \equiv \text{Re}A/\text{Im}A$. Their results are shown in Fig. 3 (left panel) .

The Durham group [9] find that at $SppS$ and Tevatron energies the Opacity distributions have approximately a Gaussian form. The analogous GLM model results are shown in Fig. 3 (Right panel), are in agreement with [9] regarding the shape of $\Omega_{el}(b)$, and in addition suggest that this is also true for the LHC energies. GLM find that with increasing energy, the intercept of the Opacity at $b=0$ increases, while the slope at small b decreases.

Kopeliovich, et al [10] have calculated the proton-proton elastic amplitude within the framework of a two scale dipole model. We show their result in Fig. 4 (Left panel). As well as that of Ferreira, Kodama and Kohara [11] who have recently made a detailed study of the proton-proton elastic amplitude for center of mass energy $W = 7$ TeV, based on the Stochastic Vacuum Model.

In Fig. 4 (Left panel) we compare the GLM, KPPS and FKK elastic amplitudes at $W = 7$ TeV as a function of the impact parameter. Although the shapes are similar, the KPPS and FKK amplitudes have lower intercepts at $b = 0$. If we normalize the FKK amplitude to the GLM value at $b = 0$, we note that the amplitudes which are gaussian in shape, have very similar behaviour as a function of the impact parameter. In Fig.4 (Right panel) we show the single diffraction amplitude as given by the DIPSY Monte Carlo [12] (dashed line) at $W = 1.8$ and 14 TeV. This includes contributions both from the Good-Walker sector and enhanced and semi-enhanced sector. The full line is the GLM amplitude which only contains the Good-Walker contribution. Note, although the amplitudes for the same value of W , peak at the same value of b , the DIPSY amplitudes are broader and higher, due to the additional enhanced contributions. For historical purposes we mention that the impact parameter behaviour of our

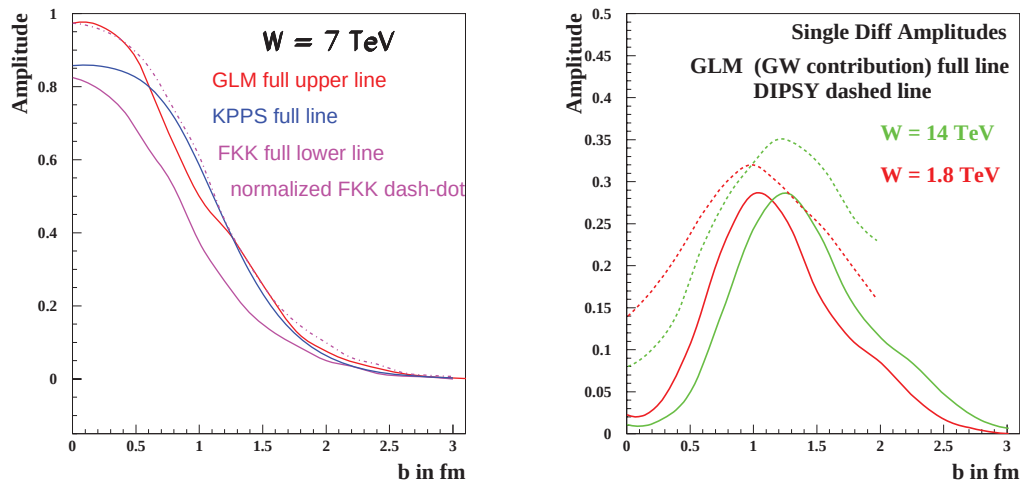


Figure 4. Left panel: Comparison of the elastic amplitude at 7 TeV determined by FKK [11], KPPS [10] and GLM. Right panel: Comparison of single diffraction amplitude, dashed line DIPSY [12] (which includes enhanced and semi-enhanced contributions) and full line GLM (only GW contribution)

diffractive amplitudes are in accord with the estimates of Miettinen and Pumplin [13] made over 35 years ago.

3. Experimental Data and GLM results

Our results for σ_{inel} , σ_{sd} and σ_{dd} , are contained in Fig. 5 which is taken from the talk given by Orlando Villalobos Baillie (for the ALICE collaboration) (see reference [14]), where the experimental data, our results and the results of other models are displayed.

The comparison of our results with experimental data for σ_{tot} , σ_{el} and for B_{el} is shown in Fig. 6.

To summarize our results at high energy, we obtain a very good reproduction of TOTEM's values for σ_{tot} and σ_{el} . The quality of our good fit to B_{el} is maintained. As regards σ_{inel} , our results are in accord with the higher values obtained by ALICE [15] and TOTEM [16]; ATLAS [17] and CMS [18] quote lower values with large extrapolation errors, see [19].

There are also recent results at $W = 57$ TeV by the Auger Collaboration [20] for σ_{tot} and σ_{inel} . In Table 1 we compare the experimental results at $W = 7$ and 57 TeV with the predictions the GLM model.

4. Alternative Models

4.1. Kaidalov-Poghosyan Model

Kaidalov and Poghosyan [21] in 2009 suggested a model based on Reggeon calculus, which predicted the soft diffractive results expected at the LHC. Their model took into account all possible non-enhanced absorptive corrections to three Reggeon vertices and loop diagrams. They apply AGK rules for calculating the discontinuity of the matrix element, and a generation of the optical theorem for the case of multi-Pomeron exchange. They utilize a single Pomeron with intercept $\Delta_P = 0.12$ and slope $\alpha'_P = 0.22 \text{ GeV}^2$, as well as secondary Regge poles. The KP model forms the basis of the Monte Carlo program used by the ALICE collaboration to analyze their soft scattering data [19].

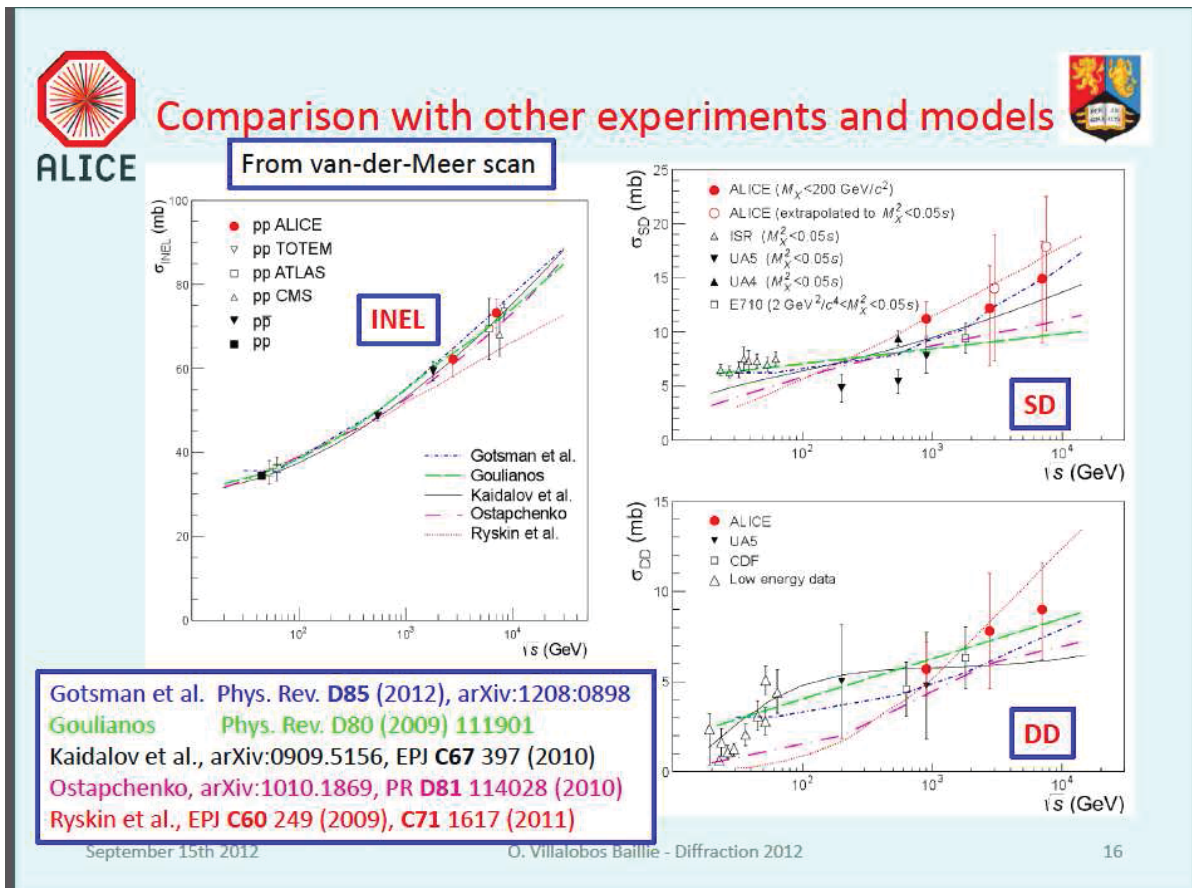


Figure 5. Comparison of Models with LHC data from Villalobos Ballie's talk at Diffraction 2012. [14]

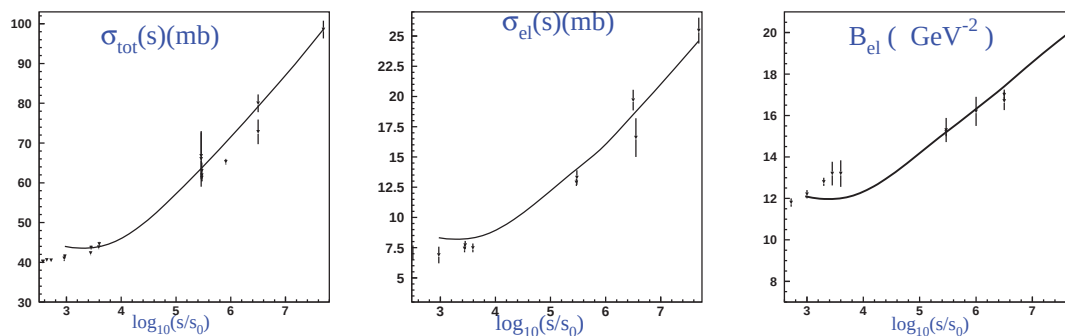


Figure 6. The GLM results compared to data for σ_{tot} , σ_{elas} and B_{elas}

The predictions of Kaidalov-Poghsyan [21] appear in Table 2, in the column KP.

4.2. Other Models on the Market

There are several models on the market today that reproduce the LHC experimental results. The most promising of these are summarized here, and their results are compared with those of GLM [4] in Table 1.

Table 1. Comparison of the values obtained from the GLM model with experimental results at $W = 7$ and 57 TeV.

W	$\sigma_{tot}^{model}(\text{mb})$	$\sigma_{tot}^{exp}(\text{mb})$	$\sigma_{el}^{model}(\text{mb})$	$\sigma_{el}^{exp}(\text{mb})$
7 TeV	98.6	TOTEM: 98.6 ± 2.2	24.6	TOTEM: 25.4 ± 1.1

W	$\sigma_{in}^{model}(\text{mb})$	$\sigma_{in}^{exp}(\text{mb})$	$B_{el}^{model}(\text{GeV}^{-2})$	$B_{el}^{exp}(\text{GeV}^{-2})$
7 TeV	74.0	CMS: $68.0 \pm 2^{syst} \pm 2.4^{lumi} \pm 4^{extrap}$ ATLAS: $69.4 \pm 2.4^{exp} \pm 6.9^{extrap}$ ALICE: $73.2 (+2./ - 4.6)^{model} \pm 2.6^{lumi}$ TOTEM: $73.5 \pm 0.6^{stat} \pm 1.8^{syst}$	20.2	TOTEM: 19.9 ± 0.3

W	$\sigma_{sd}^{model}(\text{mb})$	$\sigma_{sd}^{exp}(\text{mb})$	$\sigma_{dd}^{model}(\text{mb})$	$\sigma_{dd}^{exp}(\text{mb})$
7 TeV	$10.7^{GW} + 4.18^{nGW}$	ALICE : $14.9(+3.4/-5.9)$	$6.21^{GW} + 1.24^{nGW}$	ALICE: 9.0 ± 2.6

W	$\sigma_{tot}^{model}(\text{mb})$	$\sigma_{tot}^{exp}(\text{mb})$
57 TeV	130	AUGER: $133 \pm 13^{stat} \pm 17^{sys} \pm 16^{Glauber}$
	$\sigma_{inel}^{model}(\text{mb})$	$\sigma_{inel}^{exp}(\text{mb})$
	95.2	AUGER: $92 \pm 7^{stat} \pm 11^{syst} \pm 7^{Glauber}$

The Durham group's approach for describing soft hadron-hadron scattering [22] is similar to the GLM [4] approach, they include both enhanced and semi-enhanced diagrams. The two groups utilize different techniques for summing the multi-Pomeron diagrams. The Durham Group [23] to be consistent with the TOTEM result [16], have a model, based on a three channel eikonal description, with three diffractive eigenstates of different sizes, but with only one Pomeron. $\Delta_P = 0.14$; and $\alpha'_P = 0.1 \text{ GeV}^{-2}$. Which we will refer to as KMR3C.

Recently KMR [24] suggested a two channel eikonal model where the Pomeron couplings to the diffractive eigenstates depend on the collider energy. They have four versions of the model. The parameters of the Pomeron of their "favoured version" Model 4 are: $\Delta_P = 0.11$; and $\alpha'_P = 0.06 \text{ GeV}^{-2}$. We refer to this as KMR2C.

Ostapchenko [25] [pre LHC] has made a comprehensive calculation in the framework of Reggeon Field Theory, based on the resummation of both enhanced and semi-enhanced Pomeron diagrams. To fit the total and diffractive cross sections he assumes two Pomerons: (for his solution set C) "Soft Pomeron" $\alpha^{Soft} = 1.14 + 0.14t$ and a "Hard Pomeron" $\alpha^{Hard} = 1.31 + 0.085t$. His results are quoted in Table 2, in the column Ostap(C).

Ciesielski and Goulianos have proposed an "event generator" [26] which is based on the MBR-enhanced PYTHIA8 simulation. In Table 2 their results are denoted by MBR.

5. Conclusions

We [4] have succeeded in building a model for soft interactions, which provides a very good description of all high energy data, including the LHC measurements. The model is based on a Pomeron with a large intercept ($\Delta_P = 0.23$) and very small slope ($\alpha'_P = 0.028$). We find no need to introduce two Pomerons: i.e. a soft and a hard one. The Pomeron in our model provides a natural matching with the hard Pomeron in processes that occur at short distances. The qualitative features of our model are close to what one expects from $N=4$ SYM [6, 7], which

Table 2. Comparison of results of the different models for $W = 1.8, 7$ and 14 TeV.

W = 1.8 TeV	GLM	KMR3C	KMR2C	Ostap(C)	BMR*	KP
$\sigma_{\text{tot}}(mb)$	79.2	79.3	77.2	73.0	81.03	75.0
$\sigma_{\text{el}}(mb)$	18.5	17.9	17.4	16.8	19.97	16.5
$\sigma_{SD}(mb)$	11.27	5.9(LM)	2.82(LM)	9.2	10.22	10.1
$\sigma_{DD}(mb)$	5.51	0.7(LM)	0.14(LM)	5.2	7.67	5.8
$B_{\text{el}}(GeV^{-2})$	17.4	18.0	17.5	17.8		
W = 7 TeV	GLM	KMR3C	KMR2C	Ostap(C)	BMR	KP
$\sigma_{\text{tot}}(mb)$	98.6	97.4	96.4	93.3	98.3	96.4
$\sigma_{\text{el}}(mb)$	24.6	23.8	24.0	23.6	27.2	24.8
$\sigma_{SD}(mb)$	14.88	7.3(LM)	3.05(LM)	10.3	10.91	12.9
$\sigma_{DD}(mb)$	7.45	0.9(LM)	0.14(LM)	6.5	8.82	6.1
$B_{\text{el}}(GeV^{-2})$	20.2	20.3	19.8	19.0		19.0
W = 14 TeV	GLM	KMR3C	KMR2C	Ostap(C)	BMR	KP
$\sigma_{\text{tot}}(mb)$	109.0	107.5	108.	105.	109.5	108.
$\sigma_{\text{el}}(mb)$	27.9	27.2	27.9	28.2	32.1	29.5
$\sigma_{SD}(mb)$	17.41	8.1(LM)	3.15(LM)	11.0	11.26	14.3
$\sigma_{DD}(mb)$	8.38	1.1(LM)	0.14(LM)	7.1	9.47	6.4
$B_{\text{el}}(GeV^{-2})$	21.6	21.6	21.1	21.4		20.5

is the only theory that is able to treat long distance physics on a solid theoretical basis.

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References

- [1] Donnachie A and Landshoff P V 1992 *Phys. Lett.* **296** 222
- [2] Donnachie A and Landshoff P V 2011 "*Elastic Scattering at the LHC*", arXiv:hep-ph/1112.2485
- [3] Froissart M 1961 *Phys. Rev.* **123** 1053
Martin A 1966 *Nuovo Cimento* **42A** 930
Martin A 2008 "*Can one improve the Froissart bound?*", arXiv:hep-ph/0812.0680v3
- [4] Gotsman E, Levin E and Maor U 2012 *Phys. Lett. B* **716** 425 and references quoted there
- [5] Good M L and Walker W D 1960 *Phys. Rev.* **120** 1857
- [6] Gotsman E, Levin E and Maor U 2011 *Eur. Phys. J. C* **71**, 1553
- [7] Gotsman E, Levin E, Maor U and Miller J S 2008 *Eur. Phys. J. C* **57**, 689
- [8] Gotsman E, Levin E and Maor U 2012 *Phys. Rev. D* **85**, 094007
- [9] Ryskin M G, Martin A D and Khoze V A 2012 *Eur. Phys. J. C* **72** 1937
- [10] Kopeliovich B Z, Potoshnikova I K, Povh B and Schmidt I 2007 *Phys.Rev. D* **76** 044020
- [11] Ferreira E, Kodama T and Kohara A K 2013 *Eur. Phys. J. C* **73** 2326
- [12] Flensburg C and Gustafson G 2010 *JHEP* **1010** 014
- [13] Miettinen H I and Pumplin J 1978 *Phys. Rev. D* **18** 1696
- [14] Villalobos Baillie O, talk at Diffraction 2012, "*Measurements of Diffractive and Inelastic Cross Section with ALICE at the LHC*" Lanzarote September 2012
- [15] Poghosyan M G 2011 *et al* [ALICE Collaboration] *J. Phys. G* **38** 124044
- [16] Ferro F [TOTEM Collaboration] 2011 *Europhys. Lett.* **96** 21002
- [17] Aad G *et al.* 2011 [ATLAS Collaboration] *Nature Commun.* **2** 463
- [18] Chatrchyan S *et al.* [CMS Collaboration] 2013 *Phys. Lett. B* **722** 5

- [19] Revol J P 2012 " *Proton-proton cross sections with multiplicities with ALICE at the LHC*" talk at the Low-x Meeting Phafos Cyprus, June 27- July 1
- [20] Abreu P *et al.* [Auger Collaboration] 2012 *Phys. Rev. Lett.* **109** 062002
- [21] Kaidalov A B and Poghosyan M G 2009 " *Description of soft diffraction in the framework of reggeon calculus: Predictions for LHC*" arXiv:[hep-ph] 0909.5156
- [22] Ryskin M G, Martin A D and Khoze V A 2011 *Eur. Phys. J. C* **71** 1617
- [23] Ryskin M G, Martin A D and Khoze V A 2012 *Eur. Phys. J. C* **72** 1937
- [24] Khoze V, Martin A D and Ryskin M G 2013 *Eur. Phys. J. C* **73** 2503
- [25] Ostapchenko S 2010 *Phys. Rev. D* **81** 114028
- [26] Ciesielski R and Goulianos K 2012 " *MBR Monte Carlo Simulation in PYTHIA8*" arXiv:[hep-ph] 1205.1446