Influence of the functional form of nonlinearity in the Modulational Instability spectra of relaxing saturable nonlinear system

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Abstract. We investigate the modulational instability (MI) of the optical beam propagating in the relaxing saturable nonlinear system. We identify and discuss the salient features of various functional forms of saturable nonlinear responses such as exponential, conventional and coupled type on the MI spectrum. Using Debye relaxation model, the relaxation of nonlinear response is effectively included along with the saturable nonlinear response (SNL). Using linear stability analysis, an explicit dispersion relation is determined for considering different functional forms of SNL. Firstly, we analyze the impact of SNL on the MI spectrum and found that the MI gain and bandwidth is maximum for exponential nonlinearity in comparison to other types of SNL's. Latter the relaxation of the nonlinearity is included, the inclusion of the finite value of the response time extends the range of the unstable frequencies literally down to infinite frequencies. In the regime of slow response, the MI inevitably suppressed regardless of the sign of the dispersion coefficient. To give insight into the MI phenomena, the maximum MI gain and the optimum modulation frequency is drawn as a function of the delay. Thus the MI dynamics in the system of relaxing saturable nonlinear media is emphasized and the significance of various functional forms of SNL are highlighted.

1. Introduction

In the classical optics, the intensity dependent process in the optical system seeds the emergence to the field of nonlinear optics [1], which deserves a considerable attention for more than five decades in terms of both fundamental and application perspectives. One such exciting prospect that earns serious scientific interest nearly for three decades is the so-called modulational instability (MI) [1]. MI is an ubiquitous good old phenomena studied in diverse fields such as fluid mechanics[2], plasma [3], nonlinear optics [1, 4–6] etc., The dynamics of MI is governed by the nonlinear Schrödinger equation (NLSE), which admits soliton or solitary wave through a conservative interaction between anomalous group velocity dispersion (GVD) and self focusing Kerr nonlinearity [4–6]. The potential applications of MI in various disciplines such as optical communication and signal processing systems, ultrafast pulse generation, supercontinuum generation, new laser sources, optical amplification of weak signal, material absorption and loss compensation, all-optical switching etc., are some of the examples of MI, which provides a continuing interest. [7–12].

In the context of optical fiber, MI admits natural means of generating ultrashort pulses was theoretically proposed by Hasegawa in 1984 [4]. Latter, the experimental demonstration was given by Tai *et al* [5] in 1986. MI and four wave mixing are recognized as the time/frequency analog of the same physics, such that the two degenerate pump beams under nonlinear interaction evolves into a pair of parametric sidebands that are frequency downshifted (Stokes) and upshifted (anti-Stokes) relative to the pump [1, 7, 13]. The parametric sideband grow exponential with a gain given by G $= [(\gamma P_0)^2 - (\rho/2)^2]^{1/2}$, where $\rho = 2\gamma P_0 + 2\sum_{n=1}^{\infty}[\beta_{2n}/(2n)!]\Omega^{2n}$ is the phase mismatching condition, and Ω is the angular frequency shift from the pump and β_{2n} are the even order dispersion coefficients of the propagation mode. The maximum gain $G_{max} = \gamma P_0$ corresponds to $\rho = 0$.

The perception of analyzing MI takes different dimension at different stages depending upon the nonlinear contribution of the refractive index. As a consequence, system with higher order nonlinearity like quintic and non-Kerr nonlinearity, saturable nonlinearity [14–18], relaxing nonlinearity [19–21] and relaxing saturable nonlinearity [22] are subject of high interest due to its potential applications.The resonant materials such as semiconductor-doped glass fibers (SDF) are the modern examples of scientific curiosity. In SDF, in addition to the fast response due to electronic contribution, thermal mechanisms may lead to additional contribution to the nonlinear response whose time scale ranges from tens of picoseconds to hundreds of nanoseconds [12]. Such system are identified as the relaxing nonlinear system due to the delayed nature of its nonlinear response. In the ultrashort pulse regime, besides the slow response, due to the high nonlinearity of SDF, at high intensities there is an upper limit for the optically induced refractive index variation, beyond which the nonlinear response of the medium saturates and such type of material is known to be saturable nonlinear media [14–18, 23]. Thus index relaxation and saturation are integral part of the high index material in the ultrashort regime, and hence both need to be considered together for any dynamical study [22].

The role of delayed Raman response on the MI spectrum was illustrated in [24]. Recently, the impact of finite response time in the MI spectrum was explained by Liu *et al.* using a simple model called as Debye relaxational model [19]. The authors identified that for any finite relaxation time the range of unstable frequencies extends literally upto infinite frequencies [19]. Similar analysis can be seen in [22], where the authors reported the role of relaxation in the saturable nonlinear media. The existence of two unstable bands in the anomalous GVD regime, namely, instantaneous and Raman band was reported in ref. [22]. In [22], what the authors have considered as saturable response is known to be the conventional type (CSN). But the earlier report of Lyra *et al.* reveals the existence of two other types of functional forms of SNL, which we named as exponential (ESN) and third one known as coupled type saturable nonlinearity (TSN). Thus, in this article we intent to investigate the influence of different functional form of SNL on the MI.

The organization of the paper is as follows: Following the detailed introduction in the Section I. Section II features the theoretical model, followed by the linear stability analysis in Section III. Section IV is meant for the detailed MI analysis under different sings of the dispersion coefficients. Section V concludes the paper with a detailed summary of the results.

2. Theoretical model

The optical beam propagation in SNL is governed by modified nonlinear Schrödinger equation (MNLSE). The MNLSE for the slowly varying envelope $E(z, t)$ along the z-axis in a retarded reference time is given by [19, 22, 25]

$$i\frac{\partial E}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 E}{\partial t^2} - \gamma\frac{f(\Gamma|E|^2)}{\Gamma}E, \tag{1}$$

where, t and z are the time and longitudinal coordinate in the moving reference frame, respectively. β_2 is the group velocity dispersion coefficient and $\gamma = \frac{n_2\omega_0}{cA_{eff}}$ is the Kerr parameter, A_{eff} is the effective core area, n_2 is the nonlinear index coefficient.

The term on the extreme right accounts for the saturation of the nonlinear response. The commonly studied saturable nonlinear response in the literature are as follows
Type -I is the CSN of the form [26, 27]

$$f_1(\Gamma|E|^2) = \frac{\Gamma|E|^2}{1+\Gamma|E|^2}, \tag{2a}$$

Type -II is the ESN takes the from [28, 29]

$$f_2(\Gamma|E|^2) = 1 - \exp(-\Gamma|E|^2) \tag{2b}$$

Type - III is the TSN of the form [30]

$$f_3(\Gamma|E|^2) = \frac{\Gamma|E|^2(2+\Gamma|E|^2/2)}{2(1+\Gamma|E|^2/2)^2} \tag{2c}$$

where, $\Gamma = 1/P_s$ is the saturation parameter and P_s is the saturation power. It should be noted that for $\Gamma \to 0$, the nonlinear response of the medium switch back to the conventional Kerr type response *(i.e.)* $f(\Gamma|E|^2) \to \Gamma|E|^2$. The Eq. (1) corresponds to the instantaneous response of the nonlinearity, but for the case of ultrashort pulses the instantaneous nonlinear response is considered as a limiting case, but in strict sense the response of the medium is indeed delayed in nature. Therefore, to account for the finite response time of the nonlinearity, the above equations have to be suitably modeled. One such model is the so-called Debye relaxational Model [19, 22]. Therefore, the equation corresponding to the propagation of the optical beam in the relaxing saturable nonlinear media is given by

$$i\frac{\partial E}{\partial z} = \pm\frac{\beta_2}{2}\frac{\partial^2 E}{\partial t^2} + \gamma\, N\, E, \tag{3a}$$

$$\frac{\partial N}{\partial t} = \frac{1}{\tau}[-N + \tilde{f}_j(\Gamma, |E|^2)]. \tag{3b}$$

where, $\tilde{f}(\Gamma, |E|^2) = f(\Gamma|E|^2)/\Gamma$, and N represents the nonlinear index of the medium [22].

3. Linear Stability Analysis

The stability of the steady-state solution in the presence of small perturbations is studied using linear stability analysis [1]. The steady-state solution, (i.e.) the CW solution is given by

$$E_s = E_0 \exp[-i\tilde{f}_j(\Gamma|E_0|^2)z], \tag{4}$$

$$N_s = \tilde{f}_j(\Gamma, |E_0|^2). \tag{5}$$

Here, $a(z,t)$ and $n(z,t)$ are the perturbation corresponding to the stationary solutions of the electric field and nonlinear index, respectively. The resulting perturbed field is of the form

$$E_p = (E_0 + a(z,t)) \exp[-i\tilde{f}_j(\Gamma|E_0|^2)z], \tag{6}$$

$$N_p = \tilde{f}_j(\Gamma, |E_0|^2) + n(z,t). \tag{7}$$

After some mathematical manipulation, the linearized equation for the perturbation can be written as

$$i\frac{\partial a}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 a}{\partial t^2} + \gamma\, n E_0, \tag{8}$$

$$i\frac{\partial n}{\partial t} = \frac{1}{\tau}[-n + \Gamma\tilde{f}'(\Gamma, |E_0|^2)\, E_0(a + a^*)]. \tag{9}$$

We assume the following ansatz for the perturbation, where Ω is the frequency detuning, and K will be the wavenumber of the perturbation.

$$a(z,t) = U\exp[-i(Kz - \Omega t)] + V\exp[i(Kz - \Omega t)], \tag{10}$$

where, U and V are the amplitude of the perturbation corresponding to the anti-Stokes and Stokes sidebands, respectively. Solving the above equation leads to two homogenous equation for U and V. Collecting the linear terms in U and V, results in a 2 x 2 stability matrix as follows.

$$\begin{bmatrix} K + \beta_2\frac{\Omega^2}{2} + \tilde{\gamma}_j E_0^2 & \tilde{\gamma}_j E_0^2 \\ \tilde{\gamma}_j E_0^2 & -K + \beta_2\frac{\Omega^2}{2} + \tilde{\gamma}_j E_0^2 \end{bmatrix}, \tag{11}$$

where,

$$\gamma_1 \equiv \frac{\gamma}{(1+\Gamma E_0^2)^2}, \quad \tilde{\gamma_1} \equiv \frac{\gamma_1}{(1+i\Omega\,\tau)} \tag{12a}$$

$$\gamma_2 \equiv \gamma\exp[-\Gamma E_0^2], \quad \tilde{\gamma_2} \equiv \frac{\gamma_2}{(1+i\Omega\,\tau)} \tag{12b}$$

$$\gamma_3 \equiv \frac{\gamma}{(1+\Gamma E_0^2/2)^3}, \quad \tilde{\gamma_3} \equiv \frac{\gamma_3}{(1+i\Omega\,\tau)} \tag{12c}$$

The above matrix leads to non-trivial solutions only when the determinant of the matrix vanishes. The stability matrix determine the wavenumber K of the perturbation given by the dispersion relation

$$K = \pm\frac{1}{2}|\beta_2\Omega|[\Omega^2 + sgn(\beta_2)\Omega_c^2]^{1/2} \tag{13}$$

where, $\beta_2 = \pm 1$ depending upon the sign of β_2, the lower sign (-) corresponds to the anomalous dispersion regime and the upper sign (+) corresponds to the normal dispersion regime. Ω_c is the critical modulation frequency (CMF) given by

$$\Omega_c = \left[\frac{4\tilde{\gamma}_j E_0^2}{|\beta_2|}\right]^{1/2} \tag{14}$$

The MI gain is given by $2Im(K)$ and the corresponding expression for the versatile class of SNL can be written in a general form as

$$G(\Omega) \;=\; |\beta_2\Omega|[\Omega_c^2 \pm \Omega^2]^{1/2} \tag{15}$$

The dispersion relation K corresponds the stability of the steady state against the harmonic perturbation. It is a well established fact that in the instantaneous limit ($\tau = 0$) of nonlinear response, the MI occurs only if K takes non-zero imaginary part and therefore the steady state becomes unstable which leads to the exponential growth of the weak perturbation along the length of the fiber. However, in the case of relaxing nonlinear system, the response is not instantaneous and therefore there is a finite delay in the nonlinear response, which is found to be a crucial factor in the MI dynamics.

This finite delay in nonlinear response is equivalent to assuming complex nonlinearity, which consists both real part and imaginary parts. The real part of the nonlinearity accounts for the parametric MI process through the phase matching condition, on the other hand the imaginary part of the nonlinearity models the Raman like process and leads to instability band due to self phase matching. Thus the instability band in the relaxing nonlinear system is of two kinds, (i) Parametric MI bands and (ii) Raman band, respectively. As a consequence, unlike the instantaneous case, where the unstable frequency is finite ($\Omega < \Omega_c$), here any finite value of nonlinear response time leads to the non-zero imaginary part for all frequencies. The saturation on the other hand suppress the MI due to the depletion in the effective nonlinear index of the medium, which is identified to be dependent on the functional form of the type of SNL system.

4. MI Analysis

4.1. Anomalous Dispersion regime ($\beta_2 < 0$)

Here in this case, the system is in the anomalous dispersion ($\beta_2 < 0$) and the phase matching for the parametric MI band can be achieved through a balance between the nonlinearity and negative dispersion coefficient. For the case of saturable system at the instantaneous limit $\tau \to 0$, the nonlinearity coefficient is $\gamma \to \gamma_j$. For simplicity, we change the notation for the pump power as P instead of E_0^2. From the dispersion relation (13), the MI is realized in the frequency region given by $\Omega^2 < \Omega_c^2$. The critical modulational frequency (CMF) corresponding to the MI is given by

$$\Omega_c = \left[\frac{4\gamma_j P}{|\beta_2|}\right]^{1/2} \tag{16}$$

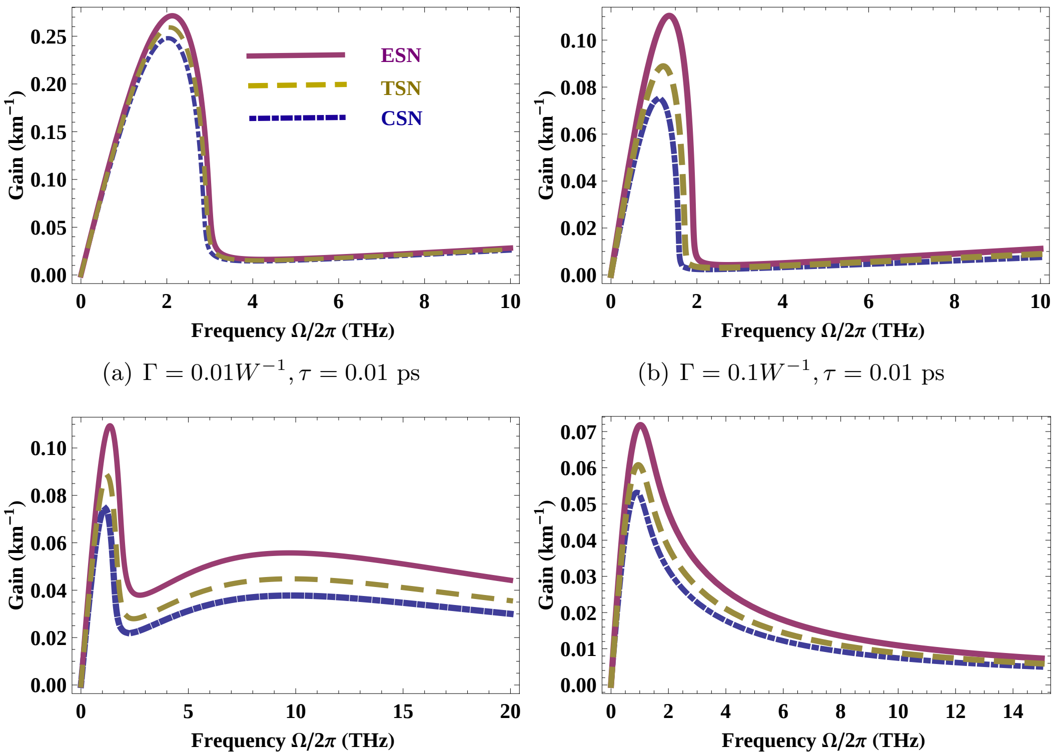
$$\Omega^2 < \Omega_c^2 \tag{17}$$

The frequency at which the MI gain reaches maximum is known to be the optimum modulation frequency (OMF). The OMF corresponding to the spectral band can be written as

$$\Omega_{OMF} \approx \sqrt{\frac{2\gamma_j P}{|\beta_2|}} \tag{18a}$$

where $j = 1, 2, 3$, takes into account ESN, TSN, CSN respectively. The above two equations find huge value in the technical point of view and provides rich variety of information about the various features of SNL in the MI spectrum. For typical system parameters, the maximum MI gain using the expression Eq. (15) for different SNL can be sorted out as $Gmax_{ESN} > Gmax_{TSN} > Gmax_{CSN}$. This difference in the G_{max} is a consequence of the depletion in the nonlinear index depending upon the functional form of the SNL. This change in the nonlinear index leads to the subsequent change in the nonlinear length ($L_{nl} = \frac{1}{\gamma P_0}$), which is identified to be the reason for the difference in CMF and G_{max} among the individual cases of SNL. Thus among the cases of SNL, ESN possess the maximum value of CMF and G_{max} than the rest.

Now, we consider the case of relaxing nonlinear system, due to the finite value of nonlinear response (τ) the effective nonlinear coefficient becomes complex ($\gamma \to \tilde{\gamma}$), which leads to the non-zero imaginary part for all frequencies. Moreover, unlike the case of instantaneous system,

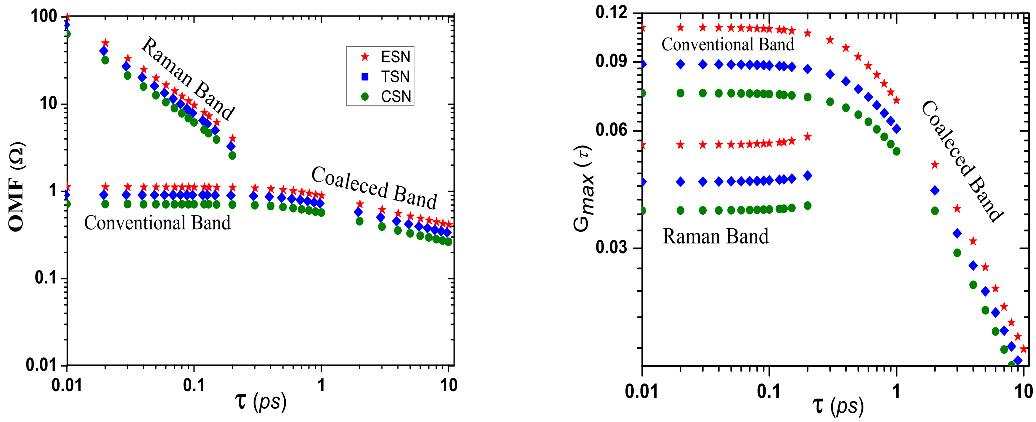


(a) $\Gamma = 0.01W^{-1}, \tau = 0.01$ ps
(b) $\Gamma = 0.1W^{-1}, \tau = 0.01$ ps
(c) $\Gamma = 0.1W^{-1}, \tau = 0.1$ ps
(d) $\Gamma = 0.1W^{-1}, \tau = 1$ ps

Figure 1: The MI spectrum for different combination of Γ and τ in the anomalous dispersion regime. The system parameters are $\beta_2 = -60\,ps^2km^{-1}$, $\gamma = 15W^{-1}km^{-1}$, $P = 10W$.

where there exist unique conventional parametric band, here in the case of relaxing nonlinear system, the unstable frequencies are continuous and run from the center frequency literally down to the infinite frequencies. Interestingly, there exist another band at the higher detuning frequency, which has been recognized as the Raman band. The behavior of SNL is very much in a perceptible manner, where the G_{max} and CMF follows similar order as in the case of instantaneous case. The Raman band due to the delayed nonlinear contribution behaves similar to the conventional band, such that the G_{max} and CMF is maximum for ESN than the rest. To illustrate the influence of relaxation and saturation, the MI spectrum for some representative values of delay (τ) and saturation parameter (Γ) are considered. One can straightforwardly observe from Figs. (1(a)) and (1(b)) that increase in Γ certainly suppresses the MI and also reduces the width of the MI band. This is attributed to the fact that saturation parameter depletes the nonlinear contribution ($\gamma_j < \gamma$), thereby suppress the MI gain and the CMF.

In the limit of slow response time (Fig. 1(d)), the dynamics evolve dramatically in different manner, where there is no signs of distinct instability bands and the MI spectrum consists of only a single coalesced band. The two instability bands (one parametric and one Raman band) that appears for fast response (Figs. 1(a),1(b),1(c)) is no longer distinguishable. The absence of characteristics MI band for higher value of τ is a clear manifestation of the relative dominance of Raman process over the parametric process in the regime of slow response. From Fig. (1(d)), one can also observe that for slow response time, the MI gain strongly decreases and the wide instability band that appears for fast response also decrease in width.



(a) The OMF as a function of delay (b) The variation in G_{max} as a function of delay

Figure 2: The variation of OMF and delay for different types of SNL as a function of delay in anomalous dispersion regime.

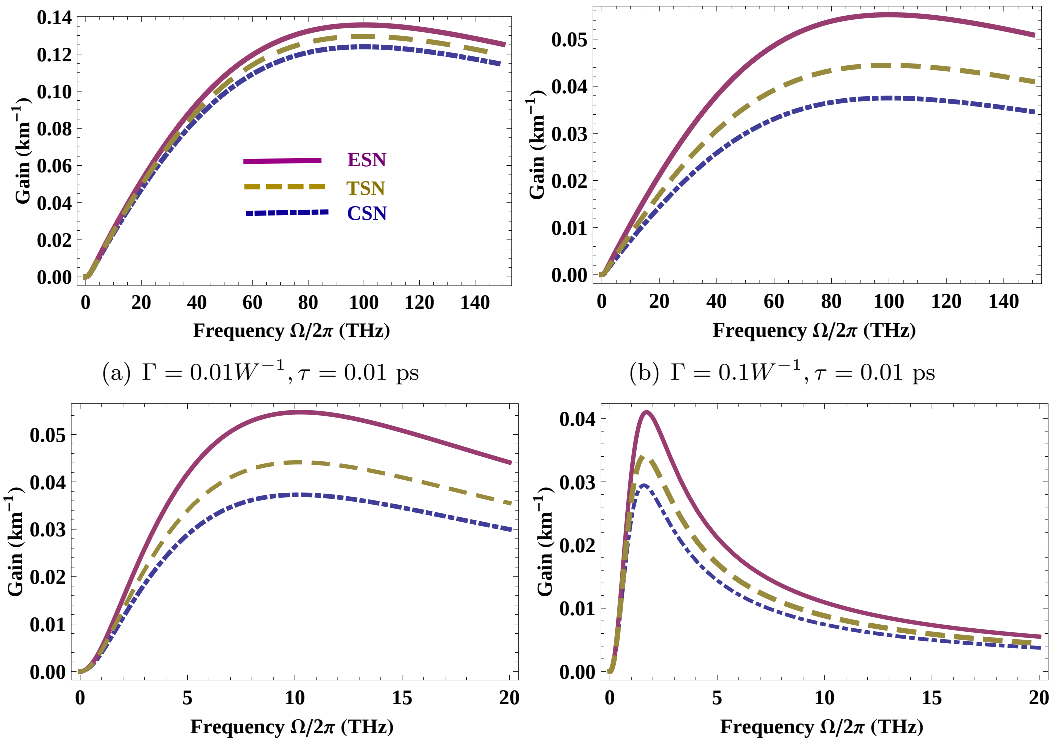
To put things in perspective, we plot the frequency corresponding to the maximum gain known as optimum modulation frequency (OMF) and the maximum gain (G_{max}) as a function of delay. Fig. (2(a)) shows the OMF as a function of delay time, where one can infer that the nature of the variation of OMF with respected to delay is similar for all class of SNL's. As dealt earlier the OMF of SNL follows the order $OMF_{ESN} > OMF_{TSN} > OMF_{CSN}$. In the regime of fast response (low values of τ) the conventional band remain unaffected and separated from the Raman band. Although the parametric instability bands are found to be independent of τ, the Raman band is extremely sensitive to the delay and decreases by a factor $1/\tau$, as shown in the Fig. (2(a)).

The behavior of MI in the regime of slow response is interesting since the parametric band corresponding to the individual type of SNL vanishes and a single coalesced band can only be observed. From Fig. (2(a)), we infer that with increase in the delay time, the Raman band downshift towards the center frequency and gets coalesced with the instantaneous band at the delay time corresponding to the order of the inverse of the CMF of SNL at the instantaneous limit [22]. Further increase in the delay time leads to the continuous downshift of coalesced band towards the center frequency at a rate of $1/\tau^{1/3}$.

The maximum MI gain associated with the instability band and the Raman band is analyzed in the Fig. (2(b)). As discussed earlier, among the types of SNL, ESN register higher value of G_{max} than the rest. The gain of the Raman band is found to be nearly half of the gain of the conventional band and it is found to be insensitive to the response time. In the regime of the slow response, for modest value of the delay (typically $\tau = 0.1 - 1$ ps), the G_{max} of the conventional band falls rapidly and coalesced with the Raman band as evident from the Fig. (2(b)). Further increase in delay time, G_{max} of the resulting coalesced band decreases by a factor $1/\tau^{2/3}$.

4.2. Normal Dispersion regime ($\beta_2 > 0$)

In the case of normal dispersion regime, the dispersion coefficient β_2 is positive, therefore there is no possibility of phase matching between the nonlinear and dispersion effects and hence, thus ruled out the parametric MI process. However, Saleh *et al.*, demonstrated that the formation



(a) $\Gamma = 0.01W^{-1}, \tau = 0.01$ ps

(b) $\Gamma = 0.1W^{-1}, \tau = 0.01$ ps

(c) $\Gamma = 0.1W^{-1}, \tau = 0.1$ ps

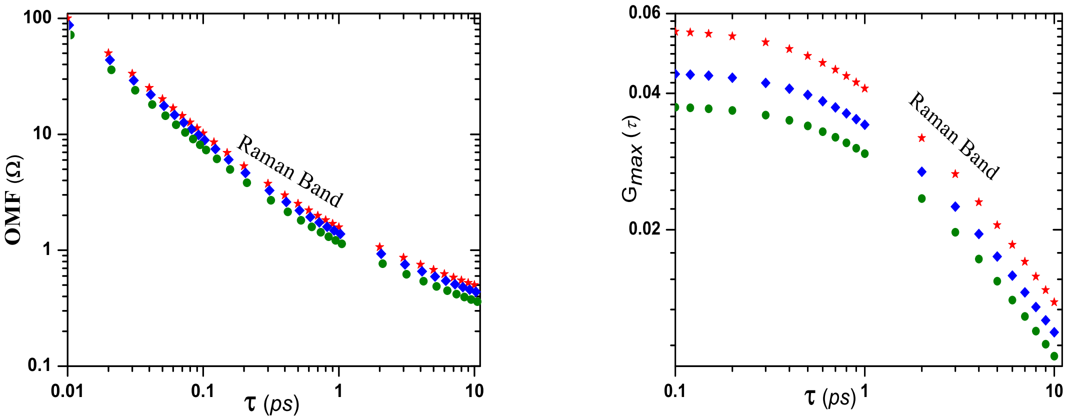
(d) $\Gamma = 0.1W^{-1}, \tau = 1$ ps

Figure 3: The MI spectrum for different combination of Γ and τ in the normal dispersion regime. The system parameters are $\beta_2 = 60\, ps^2 km^{-1}$, $\gamma = 15W^{-1}km^{-1}$, $P = 10W$.

of plasma inside gas-filed photonic crystal fibers might yield to the onset of MI in the normal dispersion regime [3]. Here in the case of relaxing system, the incorporation of finite response time leads to instability band (Raman band) even all the possibilities for parametric process is cut out. This is due to the imaginary part of the nonlinearity, which accounts for the MI gain.

Fig. (3) shows the instability band for different combination of delay and saturation parameter. The Fig. (3) illustrates that the G_{max} and the OMF of the Raman band is maximum for ESN than the rest. One can also observe that the G_{max} and the OMF is high for fast response than the slow response. Also the the increase in saturation suppress the instability gain of the Raman band.

For the further insight into the dynamics of MI in the normal dispersion regime of the relaxing nonlinear system we drawn in Fig. (4) the OMF and the G_{max} as a function of delay time. Fig. (4(a)) shows that at fast response the OMF of the Raman band decreases by a factor $1/\tau$ and further increase in the delay takes to the slow response regime. The behavior of G_{max} is quite in a perceptible manner where in fast response case the gain of the Raman band is insensitive to delay, followed by a continuous decrease in the gain at the slow response regime by a factor $1/\tau^{2/3}$ as shown in the Fig. (4(b)).



(a) The OMF as a function of delay

(b) The variation in G_{max} as a function of delay

Figure 4: The variation of OMF and delay for different types of SNL as a function of delay in normal dispersion regime.

5. Summary and Conclusion

In summary, we have investigated the MI in the relaxing nonlinear system under the combined action of nonlinear saturation and relaxation. We suitably modeled the standard NLSE, to incorporate the relaxation and saturation of nonlinear response using modified Debye relaxational model. Using the linear stability analysis, the exact dispersion relation corresponding to the stability of the steady solution against harmonic perturbation is found and the MI analysis is performed for different signs of dispersion coefficient. The saturation in general suppresses the MI by depleting the nonlinear contribution and it is found to be dependent on the functional form of the saturable media. In anomalous dispersion regime, there exist two unstable bands (*i.e.*) the conventional MI band due to parametric process and the Raman band due to the imaginary contribution of the nonlinear response. Interestingly, unstable band is still observed even in normal dispersion regime, which is purely due to the delayed nature of the nonlinear response. Among the different functional form of SNL, ESN is found to be dominant and register maximum value of OMF and G_{max}. The general order of MI identities like CMF, OMF, G_{max} can be sorted as $ESN > TSN > CSN$. To conclude, through our theoretical analysis we brings to the light that the characteristic functional form of SNL is a crucial factor in the MI phenomena. We believe that our theoretical result will be useful and can stimulate new experiments in the context of MI in relaxing nonlinear systems.

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