SPMCS2014 IOP Publishing
Journal of Physics: Conference Series **604** (2015) 012007 doi:10.1088/1742-6596/604/1/012007

Recent Progress in Some Active Topics on Complex Networks

J Gu[1,2], Y Zhu[3,2], L Guo[4], J Jiang[5], L Chi[2], W Li[2,*], Q A Wang[3], and X Cai[2]

[1] School of Science, Jiangnan University, Wuxi 214122, China
[2] College of Physical Science and Technology, Huazhong Normal University, Wuhan 430079, China
[3] Laboratoire de Physique Statistique et Systemes Complexes, ISMANS, 44 ave. Bartholdi, Le Mans 72000, France
[4] School of Mathematics and Physics, China University of Geosciences (Wuhan), Wuhan 430074, China
[5] Research Center of Nonlinear Science and College of Mathematics and Computer Science of Wuhan Textile University, Wuhan 430200, China
[*] Corresponding author

E-mail: `liw@mail.ccnu.edu.cn`

Abstract. Complex networks have been extensively studied across many fields, especially in interdisciplinary areas. It has since long been recognized that topological structures and dynamics are important aspects for capturing the essence of complex networks. The recent years have also witnessed the emergence of several new elements which play important roles in network study. By combining the results of different research orientations in our group, we provide here a review of the recent advances in regards to spectral graph theory, opinion dynamics, interdependent networks, graph energy theory and temporal networks. We hope this will be helpful for the newcomers of those fields to discover new intriguing topics.

1. Introduction

Complex network describes the interaction among the elements of complex systems, which consider particular nature of beings as vertices and perform specified function through interactions among vertices, such as scientific collaboration network[1, 2], language network [3], spreading network [4, 5], earthquake network [6, 7], stock network [8, 9], evolutionary network [10, 11] and so on. The study of networks, network theory, is an useful tool for analyzing complex system, which is an emerging area of science [12], and has many applications, especially in the interdisciplinary areas [13, 14, 15]. Even though there exist myriads of real networks in the world, scientists are more willing to believe most of networks are marked by several features and governed by countable laws. In recent years, many progresses have drawn continuously increasing attentions and brought us different understandings and new insights [16, 17, 18, 19, 20, 21, 22, 23].

One of the amazing points in the research of complex networks is that the objects in networks are not separated, but interact with each other through links. This interaction can be investigated from the viewpoint of the basic structural feature of complex networks. For example, spectral graph theory studies the properties of graphs through the spectra of their representing matrices [24, 25]. Graph energy is associated with the graph structure in the sense of building cost [26, 27]. Entropy is a general measure of probabilistic uncertainty and proves to be related to the mathematical structure of a complex network [28, 29, 30, 31, 32].

Besides the exploration on the structure of complex networks, the study of dynamics behaviors focuses on the interaction among individuals directly. Individual changes its state through the influence of its neighbors, and also exerts its influence to its neighbours to cause the possible change of their states. For example, opinion dynamics is one of the collective dynamical phenomena in the society [33]. The convergence (or divergence) of opinions among participants of a system is realized through the interactions among participants. Moreover, the interaction phenomenon sometimes does not only happen among individuals, but among independent networks. For example, the failure of nodes in interdependent networks generally not only leads to the failure of their neighbors, but leads to the failure of their dependent nodes in other networks, which may in turn cause further damages to the first network, resulting in cascading failures and catastrophic consequences in the entire system [34].

Additionally, interaction itself is also an interesting object. Take a simple example, in real-world complex systems, many relationships are (relatively) permanent, such as the existing collaboration relationship between two authors in scientific collaboration networks. However, there also exist some relationships which do not persist over long time, but only exist during short periods of time. This temporal effects of complex systems can be taken into account by the temporal networks [35] (also called time-varying graphs [36], or evolving graphs [37]).

2. Spectral graph theory

For a long time, graph theory has played a vital role in analyzing and understanding networks structures. All the topological information of one network can be found in its connectivity matrices. As a related object to matrices, eigenvalue is naturally used to explore the structural properties of a graph. The initial question is that how much information of a graph is contained in its eigenvalues sequence. This problem is described by spectral graph theory, which is the study of properties of a graph in relationship to eigenvalues of its associated matrices.

In the early days, matrix theory were used to analyze the adjacency matrix of graph. There is a large amount of literatures on algebraic aspects of spectral graph theory, such as *Biggs* [38], *Cvetković* [39] and *Godsil et al.* [40]. In a way, spectral graph theory can be considered as the well-developed theory of matrices, of which the purpose is to be related to graph theory and applications with its own characteristics.

In the recent years, many developments in spectral graph theory have the geometric favors on graph, as shown by *Chung* [24] and *Jost* [25]. An important development of spectral graph theory is the interaction with Riemannian geometry. For example, the Cheeger constant from Riemannian geometry has a discrete analogue, which can provide estimation for the first non-zero eigenvalues of the Laplacian matrix [41].

In network theory, besides modeling of complex networks, considerable attention has been given to the problems of capturing topological properties. In particular, it was proved that the important information on the topological properties of a network can be extracted from its spectra [24, 39]. For example, in 1955, Wigner introduced Wigner semicircle law, for certain special classes of random matrices [42, 43]. According to this, the distribution of eigenvalues of a large real symmetric random matrix follows a semicircle distribution. Specifically, the eigenvalues of adjacent matrix and Laplacian matrix of Erdős-Rényi random networks follow the semicircle distributions [44]. The eigenvalues distributions of Watts-Strogatz small-world networks and some of power-law networks are quite far form semicircle [44, 45]. Besides, Banerjee summed up Laplacian spectra for different types of networks and introduced a tentative classification scheme for empirical networks based on their Laplacian spectra [46]. He also revealed more understanding about Laplacian spectra and some particular dynamical processes on networks.

The search on distinguishing the unique characteristics for a given system and uncovering the universal features on large set of systems attracts great interests of scientists [47, 48, 49]. Three generic models support rough classification among networks, but can not provide further classification, especially for networks within the same class. Motif supplies a deep insight into the networks functional abilities associated with evolutionary process, of which analysis implies a similarity measurement based on a comparison of subgraphs frequencies on networks [50]. One problem with this method returns back to the same one shown in graph case, if motifs are too large, then the isomorphism problem appears again. Furthermore, for one important class of networks in our world, biological systems, there are lots of evidences showing the evolutionary process on them can result in their structure changes [51, 52]. The comparison of structures of biological networks may bring us new insights about evolutionary mechanism in biological systems. Without analyzing sequence data of gene or genome, this comparison, focusing on the whole network structure, may support us one way in understanding the functions and evolutions of biological system more conjointly and more intuitively. There is no doubt that graph spectra theory becomes one important and useful tool for analyze networks and brings crucial insights on hiding geometry information of networks.

3. Graph energy

The energy of a graph is an important parameter related to the building cost of the graph. Generally speaking, low energy means low cost, and then, which can be connected to the graph with better structure. Briefly, the energy of a graph is defined as the sum of the absolute values of the eigenvalues of its adjacency matrix. This definition, introduced by *Gutman* in the 1970s [26], is coming from chemistry where it was used to approximate the total π-electron energy of molecules [53]. Initially, graph energy did not draw many attentions of either mathematicians or physicists. Until the coming of the 21st century, extensive research about the graph energy started.

Researches related to the energy of graph can be traced to the 1940s or even 1930s [54, 55, 56, 57]. A finite and undirected graph G with N nodes is denoted to be "hyper-energetic" if its energy is larger than $2(N-1)$ [58], which corresponds to the energy of a complete graph with N vertices. However, if the energy of a connected graph G is less than N, then the graph will be called "hypo-energetic" graph [59]. And another one that corresponds to the "hypo-energetic" graph is the strong graph with energy being smaller than $(N-1)$ [60]. The structure of graph G on N vertices with energy N can be constructed by $N/2$ isolated edges, or $N/4$ isolated quadrangles or something else. In all graphs with N vertices, star graph owns the lowest energy $2\sqrt{N-1}$, except the trivial empty graph whose energy is zero. Actually, the "hypo-energetic" graphs are not so common as *Li* illustrated all this class of graphs with the maximum degree at most 3 [61]. But the "hyper-energetic" graphs are very probable. *Gutman* has found that "hyper-energetic" graphs of order N exist for all $N \geq 8$ [58], which means that there are no "hyper-energetic" graphs with less than 8 vertices. And all graphs with more than $(2N-1)$ edges are "hyper-energetic" [62].

Researchers are striving to find the upper and lower bounds of energy for some special graphs. Firstly, for a graph on N vertices, the energy is less than or equal to $\frac{N}{2}(1+\sqrt{N})$, where the upper bound is related to a strongly regular graph with some special parameters [63], which are equivalent to a certain type of Hadamard matrices. *Haemers* surveyed constructions of the corresponding Hadamard matrices and the related strongly regular graphs [64]. While the energy of a graph with given vertices N and edges m will be less than or equal to $\sqrt{2mN}$ according to the Cauchy-Schwarz inequality with equality holding if and only if the graph is either empty or 1-regular [65]. And for a regular graph with given vertices and average degree, the energy will also have a largest value [66], which is deduced from the energy of a very general graph on N vertices with $m(\geq N/2)$ edges, whose upper bound for the energy is $\frac{2m}{N}+\sqrt{(N-1)[2m-(\frac{2m}{N})^2]}$, where equality holds for some special networks, see Ref. [63]. The results we have mentioned are limited to a general graph. However, most lower or upper bounds are just corresponding to some special graphs, such as the bipartite graph, whose eigenvalues are symmetrical about zero.

The upper bounds for graph energy we have introduced are for the graph with given vertices. However, if a graph is just given the number of edges m but with unknown vertices, the graph energy will also be restricted to a region $[2\sqrt{m}, 2m]$ [67]. The lowest value will be taken if a graph consists of a complete bipartite graph and some arbitrarily isolated vertices. The energy will reach the upper bound while the graph is consisting of m copies of P_2 (a path of two connected vertices) and some isolated vertices. Except for the energy, the largest eigenvalue of adjacency matrix for a general graph will be larger than or equal to the second moment of degree sequence [68]. And this equality will be held only by regular or semiregular bipartite graph. In 2003, *Koolen* and *Moulton* strictly proved that there is a upper bound for the energy of a general bipartite graph according to the number of vertices [69]. So, for all bipartite graphs with the same vertices, there will be a graph with special topological structure corresponding to the maximal energy.

We know that the lower or upper bounds for the graph energy are just connected to some special graphs. That means that the minimal or maximal energy graphs are very limited. However, this limitation is inapplicable for those graphs without the minimal or maximal energy. In 2004, *Balakrishnan* established the existence of equienergetic non-cospectral graphs [66]. He defined that two graphs on the same vertices are called equienergetic if they have the same energy. When two graphs own the same spectra, then they will be called cospectral. Take as a simple example, two simple graphs with two isolated edges and with a quadrangle that are equienergetic non-cospectral with the same energy 4. While the spectra of them are $\{-1, -1, +1, +1\}$, and $\{-2, 0, 0, 2\}$respectively. There is no doubt that two cospectral graphs are also equienergetic.

In recent years, many researches have been studying the graph energy related to the regular graphs [70, 71]. This may be driven from the isotropic of regular graphs that can be easily analyzed from the theoretical view. *Gutman* firstly pointed out that the energy of k-regular graph G on any N-vertices is greater than or equal to N [72]. Therefore, a regular graph with degree $k > 0$ will be never hypo-energetic. If G is also triangle-and quadrangle-free, the minimal value for the energy will depends on k. In Ref. [72], Gutman also showed a remarkable narrow interval for the energy of fullerene or nanotube which can be represented as a 3-regular graph without triangles and quadrangles [73].

4. Opinion dynamics with social diversity

Opinion dynamics is one of the collective dynamical phenomena in society. Convergence (or divergence) of opinions among participants of a debate is a very important social process [74], which is similar to the phase transition from disorder to order of Ising model in statistical physics. Here, we review some works about the effect of social diversity of agents in the opinion dynamics briefly.

In real society, the diversity of agents can be described from different aspects, including the social status, the psychological attitude and mental path. The agents are diverse in their wealth and social status and have diverse influence on others. For simplicity, *Guan et al.* considered the two types of agents A and B with different influence activity in the majority rule model and find that the role of the heterogeneous influence in the order-disorder transition [75]. For example, social leaders have stronger influence compared to normal populations and have a better chance to be followed [76]. And in social networks, the social power of agent may also be quantified as a proportion of its connection degree [76, 77, 78, 79, 80], since the social leader has many followers, namely vertices to which he/she is linked generally. *Kandiah* and *Shepelyansky* introduced the PageRank method to weight the nodes' social power and proposed the PageRank opinion formation (PROF) model [81].

The diversity of agents can also be described according to the psychological attitude and mental path. In real life, people are always rational and make decision through team collaboration or group debate. Then they update their opinion following the rule of peer pressure in majority-rule model. However, some people are inflexible and contrarians, which play an important role in the opinion formation [82]. The inflexible reflects the inertia effect of human during making decision. In contrast to the floater agent who updates its opinion according to the rule of opinion model, inflexible agents keep their opinion always unchanged. *Galam* and *Jacobs* [82] studied the role of inflexible minorities in the democratic opinion model following the local majority firstly. *Biswas* and *Sen* introduced the inflexible in a model of binary opinions in which the updating of agent's opinion according to the state of their neighboring domains with the probability ρ [83]. *Masuda et al.* introduce the parameter ϵ quantifying the strength of the intrinsic preference or partisanship in the voter model [84]. When $\epsilon = 1$, each voter becomes a zealot that never changes opinion (i.e., inflexible) after aligning with its innate preference, when $\epsilon = 0$ reduces to the classic voter model. Here, the zealot effect of voters that never change opinion (i.e., the inflexible effect) has been studied in Refs. [85, 86]. Moreover, the opinion leaders also are considered as the special inflexible [87].

The other psychological attitude is the contrarians, who are the agents that deliberately decides to oppose the prevailing choice of others. *Mobilia* and *Redner* introduced a model of opinion formation according to the majority versus minority with the probability p [88]. *Borghesi* and *Galam* introduced the contrarian effect in the Galam model with a constant density of contrarian a for both opinions to study the chaotic, staggered and polarized dynamics [89]. *Ding et al.* introduced the application of game theory to model the opinion dynamics [90]. Furthermore, the contrarian effect has been introduced in the Sznajd model [91, 92] and q-voter model [93] through a stochastic parameter p. However, those previous works describe the contrarian effect as a constant stochastic parameter, which is too simple to describe the heterogeneous property of agents in social networks. Probably, the contrarian effect can also be determined by the present status of agents, such as the change of its local environment. *Grauwin* and *Jensen* propose a natural, thermal noise which allows for a small probability of interaction between agents when the opinion difference $\Delta > \epsilon$ in Deffuant model with the form of $p_{conv} = [1 + exp((\Delta/\epsilon - 1)/T)]^{-1}$, where T resembles a temperature and characterize the steepness of the convergence and called interaction noise that shows the contrarian effect indirectly [94].

5. The robustness study of interdependent networks

The robustness of interdependent networks has attracted a great deal of attention and understanding how robustness is affected by the interdependence is one of the main challenges faced when designing resilient infrastructures. For example, the robustness of critical infrastructure is one of the most important topics all over the world: specially, different kinds of infrastructure have become more and more interactive under modern technology, like communication and power grid systems, water and food supply systems [95].

In 2010, the seminal model of interdependent networks has defined a one-to-one correspondence between nodes of network A and nodes of networks B [34]. Suppose that two networks have the same number of nodes N. Each node $A_i (i = 1, 2, \ldots, N)$ in network A depends on a functioning node B_i in network B, and if node A_i stops functioning owing to attack or failure, node B_i stops functioning, and vice versa. Based on the generating function formalism and percolation theory, a first-order discontinuous phase transition was found in this model, which is totally different from the second -order continuous phase transition found in isolated networks. In Ref. [96], it was shown that, when the strength of coupling between networks is reduced, the percolation transition becomes second-order transition at a critical coupling strength, which enhanced the robustness of the system. In addition, the vulnerability of the system could be increased by the clustering and assortativity within the network components [97, 98]. And a more realistic case with both strength of coupling and connectivity links between the coupled networks was studied in Ref. [99].

However, the assumption that one node in network A depends on only one node in network B is not valid sometimes. In 2011, Ref. [100] investigated a theoretical framework to study the robustness of two interdependent networks with multiple support dependent relations.

Real interdependent networks are usually not randomly coupled: for example, well-connected ports tend to couple to well-connected airports. So Ref. [101] proposed two inter-similarity measures between the interdependent networks and found that the more inter-similar the entire network is, the more robust the system is [102]. The case in which all pairs of interdependent nodes in both networks have the same degree was studied in Ref. [103]. In the real world, a network is not always attacked randomly. Ref. [104] investigated the robustness of fully and partially interdependent networks under targeted attack, respectively.

As most of real systems are not randomly but spatially embedded, it is reasonable to consider the factor of space limitation. In interdependent lattice networks, Ref. [105] found that there is a change from first to second order phase transition at the critical length of dependency links and Ref. [106] concluded that there is no critical dependency and any small fraction of interdependent nodes leads to an abrupt collapse. Moreover, transport process has been explored in coupled spatial networks in Ref. [107].

There are also some other considerations beyond above ones. A network of networks (NON) is taken into account in Refs. [108, 109] with more realistic consideration that there are more than two interdependent networks in many real systems. Antagonistic interaction [110, 111], autonomous nodes [112] and node-weighted [113] are considered in study of interdependent networks, respectively, which are leading to a better understanding of the effect of dependence between networks on the dynamics of interdependent networks.

6. Temporal networks

Temporal networks, compared to static networks, emphasize on the times *when* and *how long* contact events (edges) are present. The addition of time dimension provides a new sight into the framework of complex network theory. In temporal networks, structural properties and spreading dynamics are constrained by the time ordering of edges. Consequently, the concepts and methods for temporal networks need to be extended or redefined based on the top of static graphs.

In the research of temporal networks, aggregated static networks play a crucial role in understanding the temporal effect of network structure due to the lack of methods to uncover full contact patterns. We can aggregate temporal networks to a list of snapshots of static graphs if the topological characteristics are more relevant than the temporal properties, since it is usually easier to analyze static networks. A temporal network is described as $G(V, E, t, \delta t)$, in which the contact event happens at time t and δt is its duration. An edge is formed in the aggregated network if there is at least one contact happening in time window $[t, t + \Delta t]$. It is noticed that the aggregation time interval Δt has critical consequences on the structural properties emerging from temporal networks. Many existing tools of static graphs have been adopted to analyze temporal networks. For instance, the error and attack vulnerability of temporal networks [114], optimal way for constructing static snapshots in temporal networks [115], and so on.

The adjacency matrix in a temporal network is defined as $a(i, j, t) = 1$ if there exists an edge between node i and node j at time t, and $a(i, j, t) = 0$ otherwise. Given the adjacency matrix as a function of time, the path of temporal networks has two distinct definitions [36]. One corresponds to topological distance, which is analogous to shortest path length in static networks. The other is referred to temporal distance, which means the path of minimum duration to reach each other. Keep these time-respecting paths in mind, we can redefine temporal degree, betweenness, centrality, closeness, component, motif, to name a few. However, not all the structural concepts have their counterparts in static graphs. *Holme* introduced the concept of reachability as a time-ordered chain of contact leading from one node to the other [116]. *Lentz et al.* proposed accessibility to measure temporal networks [117]. See Ref [35] for a review of more details.

In order to find the fundamental role of time ordering and duration, the other direction is to gain insights into the effects of different time correlations. Null temporal models are served as a reference, in which the original time sequences are randomized. It allows us to distinguish between different contributions to the time correlations coming from randomizing contact times, nodes, edges, or combinations of the three.

Besides of this, scientists have also interest in the following question: how will the temporal structures affect dynamical processes on temporal networks, and vice versa? For example, concerning contact events exhibit heterogeneous inter-event time distributions, bursty characters [118] have a strong influence on dynamical processes on temporal networks.

The other effort along this line is to control or to avoid the spread. *Lee et al.* have introduced the concepts of Recent and Weight to investigate the immune strategy [119]. Recent and Weight are specified as the most recent contact and the most often contact, respectively. The research towards spreading dynamics of temporal networks has grown in various aspects ranging from cascades [120] and random walks [121] to synchronization [122], and so on.

Despite the promoting results in temporal networks, this field is still in its infancy and there is not yet a general framework for describing and analyzing it. For example, what is the proper (or characteristic) aggregated time window to reflect network structure over time? Dynamical approaches remain rare in describing spreading processes. Standard models are still lacking for the study of temporal networks. By extending theory and analyzing data to account for temporal networks, we can approach a better understanding of time-stamped complex system.

Acknowledgments
This work was partially supported by the National Natural Science Foundation of China under grant No. 10975057, the Programme of Introducing Talents of Discipline to Universities under Grant No. B08033. J. Jiang wants to thank the support of the National Natural Science Foundation of China under grant No. 11405118. L. Chi wants to thank the support of the Fundamental Research Funds for the Central Universities.

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