

Teaching the gravitational redshift: lessons from the history and philosophy of physics

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Abstract. The equivalence principle and the notion of an ideal clock running independently of acceleration suggest that clocks are unaffected by gravity. The apparent contradiction with the gravitational redshift points to a subtlety in general relativity theory. Indeed, early attempts for a clear derivation of the gravitational redshift were fraught with errors and ambiguities, and much confusion endured for the next two decades. This suggests that the subject should be treated carefully in introductory textbooks on relativity theory. I analyze the weaknesses of the presentation in five otherwise excellent modern introductory general relativity books (by Rindler, Schutz, Hobson et al., Weinberg, and Carroll). I also present some analysis from an history and philosophy of physics article, which proves to be a great resource to learn about, anticipate, and clarify problems in teaching the redshift.

1. Introduction

Several authors have made the case that general relativity GR can and should be taught to undergraduate physics students [1–4] and many other articles can be found offering advice in this endeavor [5–10]. Recurring themes are to motivate the mathematics by first introducing the physics [2] and use an interactive approach [10]. Surely another important two components of good teaching are to identify the subtle areas and to present them clearly using careful language. I found that help can be found for both these good-teaching components from an unexpected source, namely the history and philosophy of physics. In this short article I focus on the gravitational redshift as an important example.

The gravitational redshift provides an example of an especially subtle topic. Okun *et al.* [11] note that in the literature there are two interpretations of the gravitational redshift in a static gravitational field: either the photon frequency is modified en route between emitter and receiver (and the clock rate unaffected by gravitational potential) or the clocks at lower potential are slowed down (and the photon unaffected en route). They advocate strongly the clocks-slow-down view, stating that the gravitational redshift should be taught in a way that “centers on the universal modification of the rate of a clock exposed to a gravitational potential” [11][p. 119]. But the situation is subtle and confusing because an important heuristic principle in GR is that the local effects of gravity can always be eliminated with a coordinate transformation. This follows from a version of the principle of equivalence of gravity and inertia that asserts that inertial forces and gravitational forces are one and the same physical effect Møller(1952, §83)[12]. So do clocks slow down? According to Weinberg

Consider a [ideal] clock in an arbitrary gravitational field . . . The equivalence principle



tells us that its rate is unaffected by the gravitational field ..., see Weinberg(1972, §3.5)[13].

The history of physics also alerts us to the subtlety surrounding the gravitational redshift. Indeed, when GR was introduced by Einstein in the first two decades of the XXth century, there was great confusion among the leading physicists of the day especially around the derivation of the gravitational redshift. (If leading physicists could be confused when first confronted with relativity, we should not be surprised when a fresh crop of students are confused too.) This history was reviewed by two philosophers, John Earman and Clark Glymour [14] and makes for both informative and entertaining reading. Furthermore their derivation is admirably clear; we give the full derivation in §3.2.

The gravitational redshift and the equivalence principle are basic concepts and standard material for an introductory GR theory course. Given the subtlety and historical confusion surrounding these issues one would expect surely to find careful discussion of these points in the textbooks and pedagogical literature. Surprisingly I found this not to be the case. I provide a brief survey of the interpretation of the gravitational redshift in my five favourite introductory GR textbooks in §2. In none of them is the tension between ideal clocks/equivalence principle and the gravitational redshift explicitly presented and is at best implicit in two of them [13, 15]. I then turn to the history and philosophy of physics literature and review the lessons from the analysis of the gravitational redshift [14]. First I present the erroneous derivation due to Eddington [16] and show how this was repeated in Weinberg's book [13]. Then I present the admirably clear Earman-Glymour derivation. This reveals in which sense clocks do *not* slow down in a gravitational field. I conclude with a summary in §4.

2. Interpretation of the redshift in introductory textbooks

In §5.1 of his well-known introductory GR textbook [17], Bernard Schutz derives the gravitational redshift first using a conservation of energy argument applied to the setting of the Pound-Rebka-Snider experiment. A very similar argument can be found in advanced textbooks, e.g. §7.2 of ref. [18]. He offers the interpretation:

...the gravitational redshift implies that time itself runs slightly faster at the higher altitude than it does on the [surface of] Earth. (Schutz, 2009, § 5.1, p. 113)[17]

Naively one might infer that gravity somehow compresses spacetime. No where is it mentioned that this appears to contradict the idea that the equivalence principle tells us that clock rate is unaffected by the gravitational field.

Similarly, Wolfgang Rindler in his §1.16 of textbook [15] also derives the gravitational redshift first using a conservation of energy argument. He also replaces the gravitational field with a uniform acceleration and finds a Doppler shift for the receiver because of his velocity relative to an inertial frame initially at rest. A similar argument can be found in §7.4 of ref. [18]. Price considers the same calculation but without approximation [19]. For the interpretation Rindler states

That result (known as the *gravitational frequency shift*) has the important consequence that standard clocks fixed in a *stationary* gravitational field at low potential go slower than clocks fixed at higher potential. (Rindler, 2001, § 1.16, p. 25)[15]

But Rindler continues:

... So let a standard clock at some point *A* of low potential (for example, on the surface of a dense planet) be seen from some point *B* of higher potential, and let ν_A be **the universal rate at which all standard clocks tick**. Let the rate at which the standard A-clock *is seen* to tick at *B* be ν_B . This is, by [his] Eq. (1.11), less than the

rate ν_A at which the standard clock at B ticks, by the factor on the RHS. But if the clock at A is *seen* to go slow, then it really *does* go slow.

(Rindler, 2001, § 1.16, pp. 25 ... 26, italics in the original, bold added by author)[15]

So Rindler appears to contradict himself by referring to the universal rate at which *all* standard clocks tick, and yet emphasizing that these clocks really do slow down. For static and stationary spacetimes he explains that a ‘world-movie’ running at constant rate in global coordinate time would display the events slowed down near heavy masses and indeed frozen at the horizon, §9.4 of [15]. My two other favourite introductory GR textbooks [20, 21] avoid an interpretation, as does the advanced book [18]. As we saw from the quote in §1, Weinberg [13] clearly states the clock-rate-gravity-independent view. However, he offers the reader no help in reconciling this with the interpretation of the gravitational redshift nor does he even hint that there is something subtle to be helped with. Surprisingly, he also repeats a fundamental error in deriving the gravitational redshift that we discuss in detail in §3.1.

3. Lessons from the history of physics

The confusion started with an ambiguous derivation of the gravitational redshift by Einstein [22–24] and the confusion was compounded with an erroneous derivation by Eddington [16, 25, 26].

3.1. Erroneous Eddington gravitational redshift derivation

Eddington was concerned with the gravitational redshift of the Sun’s spectra received on Earth. He starts his derivation with what Earman and Glymour call the Eddington premise that the proper period $d\tau$ of vibration of a given type of atom (acting as an ideal clock) is independent of its environment. Let A be a point on the surface of the Sun, and B a point on the surface of the Earth. For simplicity we can ignore the relative motions of the source and receiver and focus on the effect of the gravitational potential.

With $(d\tau)_A$ the proper period of a vibrating atom at location A and $(d\tau)_B$ the proper period of the same type of atom at location B , the Eddington premise becomes

$$(d\tau)_A = (d\tau)_B. \quad (1)$$

The proper time period $(d\tau)_A$ is related to the coordinate time interval $(dt)_A$ at some fixed location A (so $dx^i = 0$) via

$$\sqrt{g_{00}(r_A)}(dt)_A = (d\tau)_A, \quad (2)$$

where of course $g_{00}(r_A)$ is the lapse squared or temporal component of the metric tensor at location r_A in the approximately static spacetime. Unfortunately Eddington used his premise Eq. (1) as follows,

$$\sqrt{g_{00}(r_A)}(dt)_A = (d\tau)_A = (d\tau)_B = \sqrt{g_{00}(r_B)}(dt)_B. \quad (3)$$

And for $g_{00}(r_A) < g_{00}(r_B) \Rightarrow (dt)_A > (dt)_B$. The frequency of vibration $\nu = 1/dt$, and, Eddington infers

$$\frac{1}{(dt)_A} = \nu_A < \nu_B = \frac{1}{(dt)_B}. \quad (4)$$

Eddington concludes from Eq. (4) that “the solar atom thus vibrates more slowly, and thus its spectral lines will be displaced towards the red” [16, 25].

The problem with this derivation is that t is coordinate time and hence ν is the coordinate frequency, but observed frequencies are related to $1/d\tau$, the proper time intervals. The result is of course correct in that the observational frequency is redshifted. Two mistakes have cancelled.

Only James Rice queried Eddington on why we observe a redshift if $(d\tau)_A = (d\tau)_B$. Eddington's reply in the *Proceeds of the Royal Society* (1920), and especially in *Observatory* (1920), while confusing, indicates that he has all the elements of the corrected derivation, according to Earman and Glymour[14]. And yet strangely Eddington kept the erroneous redshift derivation in his influential textbook [26]! Sadly Rice repeated the erroneous redshift derivation in *his* textbook [27][p. 287–289]. Importantly the erroneous derivation was repeated consistently in the most influential presentations of GR before 1935, including, Earman and Glymour claim, in all five editions of Weyl's monograph *Raum-Zeit-Materie*, [28–32]. Recently Harvey and colleagues [33, 34] advocate the use of the definition of the gravitational redshift as presented in the fifth edition of *Raum-Zeit-Materie* [32]. Harvey [34] suggests that this definition has been overlooked because it is only available in German.

It is strange that Earman and Glymour [14] fail to mention that Weinberg repeated the erroneous Eddington derivation in his very authoritative and influential textbook [13][§3.5, pp. 79–80]. Earman and Glymour almost certainly noticed, since they cite [13][§3.5, pp. 84–85] in a different context. But there is no ambiguity that Weinberg repeated the erroneous Eddington derivation above; his notation is clear from the equations. His Eq. (3.5.1) is

$$\frac{dt}{\Delta t} = \left(-g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \right)^{-1/2}, \quad (5)$$

which clearly reveals that Δt is the proper time interval, while dt is the coordinate time interval. He considers the case of observing light at point 1 (like our point B above) that came from an atomic transition at point 2 (like our point A above), both points being at rest in a stationary gravitational field. In this case his Eq. (3.5.1) reduces to his Eq. (3.5.2)

$$\frac{dt}{\Delta t} = (-g_{00})^{-1/2}, \quad (6)$$

from which he derives his next two unnumbered equations

$$dt_2 = \Delta t (-g_{00}(x_2))^{-1/2} \quad \text{and} \quad dt_1 = \Delta t (-g_{00}(x_1))^{-1/2}. \quad (7)$$

Taking their ratio dt_1/dt_2 he finds his Eq. (3.5.3)

$$\frac{\nu_2}{\nu_1} = \left(\frac{g_{00}(x_2)}{g_{00}(x_1)} \right)^{1/2}. \quad (8)$$

Just as Eddington did, Weinberg mistakes the coordinate time interval dt with the observable time period. Yet he gets the correct answer by setting the proper time interval Δt to a constant. The text of this section of Weinberg's book is more correct but there is a disconnect with the equations. Weinberg's bibliography for this chapter includes Eddington's and Weyl's books [26, 31], suggesting his presentation was misled by them.

3.2. Earman-Glymour gravitational redshift derivation

The Earman-Glymour [14] derivation starts with the Eddington premise Eq. (1). A seemingly small but actually very helpful point is their notation; they introduce the notation $(d\tau)_{A,B}$ for the proper time interval between vibrations of atom at A , as measured at B . So then $(d\tau)_{A,A}$ denotes the proper time period of vibrations of the atom on the Sun, as measured on the Sun, while $(d\tau)_{A,B}$ is the proper time interval between makers of the vibration of the atom on the Sun, as measured on Earth. And the Eddington premise in Earman-Glymour notation becomes

$$(d\tau)_{B,B} = (d\tau)_{A,A}. \quad (9)$$

Earman & Glymour are concerned that we'll find this notation "fussy and pedantic"; I find it clarifies a confusing situation. Then, with constant coordinate time interval dt :

$$\frac{(d\tau)_{A,A}}{(d\tau)_{A,B}} = \frac{dt\sqrt{g_{00}(r_A)}}{dt\sqrt{g_{00}(r_B)}} = \frac{\sqrt{g_{00}(r_A)}}{\sqrt{g_{00}(r_B)}}. \quad (10)$$

They note that Max von Laue [35] showed that in a static spacetime Maxwell's equations have solutions proportional to $\exp(i\nu t)$, with constant coordinate frequency ν . This justifies cancelling the coordinate time interval dt in the numerator and denominator in Eq. (10).¹ Here, according to Earman & Glymour, is the appropriate place to apply Eddington's premise Eq. (9). We want to compare the interval $(d\tau)_{A,B}$ with a similar atom vibrating on Earth for which $(d\tau)_{B,B} = (d\tau)_{A,A}$. Substituting the Eddington premise into Eq. (10) allows us to relate local observables on Earth:

$$\frac{(d\tau)_{A,A}}{(d\tau)_{A,B}} = \frac{(d\tau)_{B,B}}{(d\tau)_{A,B}} = \frac{\nu_{A,B}}{\nu_{B,B}} = \frac{\sqrt{g_{00}(r_A)}}{\sqrt{g_{00}(r_B)}} < 1, \quad (11)$$

where $\nu \equiv 1/d\tau$ is the proper frequency.

The Eddington premise can be seen as a succinct statement of the clock-rate-gravity-independent view point of §1. And yet, with its careful application, we arrive at the gravitational redshift. This supports the compatibility of the two apparently contradictor two statements of Rindler in §2, and suggests that the redshift is a phenomenon related to message-passing.

4. Conclusion

History repeats itself every year in the classroom; when new students are introduced to subtle concepts they are confused exactly on those points that the leading scientists were confused about when the subject was newly discovered. Indeed the historical analysis of Earman and Glymour [14] warns us that the gravitational redshift is notoriously subtle and confusing.

The gravitational redshift derivation of Earman and Glymour [14] is admirably clear. A key point was their notation; even if a bit cumbersome it pays for itself in clarity. In particular their notation $(d\tau)_{A,B}$ allows one to distinguish between the markers of the beginning and end points of a proper time interval, say a tick of a given atomic clock at event A , as observed at different events B in spacetime.

The statement that "clocks slow down at lower gravitational potential" must be used with caution. In some sense it is true and in another sense it is not. Clarifying exactly in what sense we mean clocks slow down is one of the challenges of teaching introductory GR. It is tempting to play it safe, as do many textbooks [18, 20, 21]. But I think this is a disservice to the most inquisitive students who may become discouraged by the confusion surrounding an interpretation of the gravitational redshift. Recall the wisdom of John A. Wheeler when he said "Sad the week without a paradox, a difficulty, an apparent contradiction! For how can one then make progress?" [37]. We should lead our students to the paradox and then help resolve it.

An important limitation of the Earman-Glymour analysis [14] is that they forgo interpretation of the gravitational redshift, wishing to avoid a "windy warfare" over the Doppler vs. non-Doppler interpretations. But one can and should offer a more detailed interpretation in terms of the structure of spacetime. Furthermore the gravitational time dilation phenomenon has been observed with two clocks at different altitude [38, 39], wherein there is no message-passing. How can this be understood? This will be taken up in our next publication.

¹ Of course one could also appeal at this point to the time independence of the static gravitational field; see [36] and [17][Fig. 5.2].

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References

- [1] Wald R M 2006 *Am. J. Phys.* **74** 471–477
- [2] Hartle J B 2006 *Am. J. Phys.* **74** 14–21
- [3] Bertschinger E and Taylor E F 2008 *Am. J. Phys.* **76** 103–105
- [4] Hobson A 2008 *Am. J. Phys.* **76** 607–607
- [5] Silvert W 1973 *Am. J. Phys.* **41** 926–927
- [6] Lambourne R 2005 *Eur. J. Phys.* **26** S135
- [7] Baez J C and Bunn E F 2005 *Am. J. Phys.* **73** 644–652
- [8] Drake S P 2006 *Am. J. Phys.* **74** 22–25
- [9] Zimmerman S 2010 *Eur. J. Phys.* **31** 1377
- [10] Christensen N and Moore T 2012 *Phys. Today* **65** 41–47
- [11] Okun L B, Selivanov K G and Telegdi V L 2000 *Am. J. Phys.* **68** 115–119
- [12] Møller C 1952 *The Theory of Relativity* (Oxford Clarendon Press) ISBN 978-1124104133
- [13] Weinberg S 1972 *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (New York, NY: John Wiley & Sons)
- [14] Earman J and Glymour C 1980 *Studies in History and Philosophy of Science* **11** 175–214
- [15] Rindler W 2006 *Relativity: Special, General and Cosmological, 2nd Ed.* (Oxford, UK: Oxford University Press)
- [16] Eddington A S 1918 *Report on the Relativity Theory of Gravitation* (Fleetwood Press, London UK)
- [17] Schutz B 2009 *A first course in General Relativity* (Cambridge UK: Cambridge University Press)
- [18] Misner C, Thorne K and Wheeler J 1973 *Gravitation* (San Francisco: W.H. Freeman)
- [19] Price H E 1974 *Am. J. Phys.* **42** 336–337
- [20] Hobson M, Efstathiou G and Lasenby A 2006 *General Relativity: An introduction for physicists* (Cambridge University Press, UK: Cambridge)
- [21] Carroll S 2004 *Spacetime and Geometry: An introduction to General Relativity* (San Francisco: Addison Wesley) 513 + xiv pp
- [22] Einstein A 1907 *Annalen der Physik* **23** 371–384 ISSN 0003-3804
- [23] Einstein A 1911 *Annalen der Physik* **35** 898 translation by W. Perrett and G. B. Jeffery, *The Principle of Relativity* (New York: Dover, 1952)
- [24] Einstein A 1916 *Annalen der Physik* **49** 720–822 translation by W. Perrett and G. B. Jeffery, *The Principle of Relativity* (New York: Dover, 1952)
- [25] Eddington A S 1920 *Report on the Relativity Theory of Gravitation, Second Edition* (Fleetwood Press, London UK)
- [26] Eddington A S 1923 *The Mathematical Theory of Relativity* (Chelsa, New York, NY 1975)
- [27] Rice J 1923 *Relativity: A systematic treatment of Einstein's Theory* (London, UK: Longmans, Green) 397 + xv pp
- [28] Weyl H 1918 *Raum, Zeit, Materie, 1st edition* (Verlag von Julius Springer, Berlin)
- [29] Weyl H 1918 *Raum, Zeit, Materie, 2nd edition* (Verlag von Julius Springer, Berlin)
- [30] Weyl H 1919 *Raum, Zeit, Materie, 3rd edition* (Verlag von Julius Springer, Berlin)
- [31] Weyl H 1921 *Space-Time-Matter, 4th ed.* (Dover, New York) ISBN 978-0486602677
- [32] Weyl H 1923 *Raum, Zeit, Materie, 5th edition* (Verlag von Julius Springer, Berlin)
- [33] Harvey A, Schucking E L and Surowitz E J 2006 *Am. J. Phys.* **74** 1017–1024
- [34] Harvey A 2012 *Eur. J. Phys.* **33** 823
- [35] von Laue M 1920 *Phys. Z.* **21** 659–662
- [36] Schild A 1967 Lectures on general relativity theory *Relativity Theory And Astrophysics* ed Ehlers J (American Mathematical Society) pp 1–105
- [37] Wheeler J A 1996 *At Home in the Universe* (American Institute of Physics, New York) ISBN 978-0883188620
- [38] Briatore L and Leschiutta S 1977 *Il Nuovo Cimento B Series 11* **37** 219–231 ISSN 0369-3554
- [39] Alley C O 1983 Proper time experiments in gravitational fields with atomic clocks, aircraft, and laser light pulses *Quantum Optics, Experimental Gravity, and Measurement Theory (NATO Advanced Science Institutes Series vol 94)* ed Meystre P and Scully M O (Springer US) pp 363–427 ISBN 978-1-4613-3714-0