

On the renormalization of ultraviolet divergences in the inflationary angular power spectrum

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Abstract.

We revise the role of ultraviolet divergences of cosmological observables and the corresponding renormalization from a space-time perspective. We employ the two-point function of primordial perturbations generated during inflation to derive an analytic expression for the multipole coefficients C_ℓ in the Sachs-Wolfe regime. We analyze the ultraviolet behavior and stress the fact that the standard result in the literature is equivalent to a renormalization of the two-point function at zeroth adiabatic order. We also argue that renormalization at second adiabatic order seems to be more appropriate from a physical point of view. This may change significantly the predictions for C_ℓ , while maintaining scale invariance.

1. Introduction and motivation

The theory of quantum fields interacting with gravity [1, 2], applied to the very early and rapidly expanding universe, explains considerably well the pattern of temperature anisotropies of the cosmic microwave background (CMB) as well as the large scale structure (LSS) of the universe [3]. The assumption of a slow-roll inflationary universe [4, 5] is particularly efficient to account for the huge and detailed cosmological data observed today [6]. Particle creation [7, 8] is the fundamental mechanism driving primordial perturbations that eventually seeded the tiny fluctuations in the temperature of the CMB. It can also be regarded as the source for the gravitational clumping of matter that gave rise to galaxies and structure formation.

In a curved space-time, new ultraviolet (UV) divergences arise in the computation of vacuum expectation values, and these infinities can not be removed by standard methods in Minkowski space-time. Specific methods to define regularization and renormalization in expanding universes have been constructed to account for the new UV divergences sourced by curved backgrounds [1, 2, 9] (for more recent works, see [10]).

Let φ represent a generic free field living in an homogeneous and isotropic *Friedmann-Lemaître-Robertson-Walker* (FLRW) spacetime, with line element $ds^2 = dt^2 - a^2(t)d\vec{x}^2$. The field φ will later describe scalar (or tensor) perturbations during inflation. In the quantum theory the free field operator is most generally studied by its expansion in Fourier k -modes $\varphi_k(t)$,

$$\varphi(t, \vec{x}) = \int d^3k \left[A_{\vec{k}} \varphi_k(t) + A_{-\vec{k}}^\dagger \varphi_k^*(t) \right] e^{i\vec{k} \cdot \vec{x}}, \quad (1)$$

where $A_{\vec{k}}$ and $A_{\vec{k}}^\dagger$ are creation and annihilation operators, such that $A_{\vec{k}}|0\rangle = 0$, and satisfy



canonical commutation relations, $[A_{\vec{k}}, A_{\vec{k}'}] = 0$, $[A_{\vec{k}}^\dagger, A_{\vec{k}'}^\dagger] = 0$ and $[A_{\vec{k}}, A_{\vec{k}'}^\dagger] = \delta^3(\vec{k} - \vec{k}')$. A basic object in quantum field theory is the two-point function. For future purposes, we will consider it at equal times $t = t'$

$$\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle = \int d^3k |\varphi_k(t)|^2 e^{i\vec{k}(\vec{x} - \vec{x}')} . \quad (2)$$

We can formally construct local physical observables from the above two-point function. For instance, the local quantum fluctuations of φ can be quantified by the mean square fluctuation in the vacuum state

$$\langle \varphi^2(t, \vec{x}) \rangle = \int d^3k |\varphi_k(t)|^2 . \quad (3)$$

In either case, it is quite common in cosmology to refer the quantity $\Delta_\varphi^2(k, t) \equiv 4\pi k^3 |\varphi_k(t)|^2$ as the primordial power spectrum.

A proper definition of the physical power spectrum in inflationary cosmology is not free of subtleties, as first pointed out in [11], and subsequently studied in [12, 13, 14]. In momentum-space, and for a single mode k , the power spectrum $\Delta_\varphi^2(k, t)$ is well defined. However, the formal variance $\langle \varphi^2(\vec{x}, t) \rangle$, which is a sum in all modes, diverges in the ultraviolet. There is no doubt that the self-correlator needs renormalization when it is used to quantify the amplitude of quantum perturbations at a single space-time point, as in (3). However, the two-point function (2) may not need renormalization when used to quantify physical observables involving correlations. One could argue that the two-point function has a well-defined definition in the distributional sense and there is not mathematical need for any regularization [15]. However, as the spatial points get close together, the two-point function will grow without bound and produce divergences in physical observables [16, 1] (see expression (6) for this reasoning). In [11, 12, 13, 14, 16] it was argued that the physical power spectrum should be defined in terms of renormalized quantities. The challenging of this proposal for inflationary cosmology and quantum gravity has been recently stressed in [17]. The purpose of this work is to reanalyze these issues, specially from a spacetime viewpoint.

As we have said, (3) is UV divergent and needs to be renormalized according to standard rules

$$\langle \varphi^2(t, \vec{x}) \rangle = \lim_{\vec{x} \rightarrow \vec{x}'} [\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle - {}^{(N)}G_{Ad}((t, \vec{x}), (t, \vec{x}'))] , \quad (4)$$

where ${}^{(N)}G_{Ad}((t, \vec{x}), (t, \vec{x}'))$ is the N th order adiabatic subtraction term. It is defined via the adiabatic regularization method, or, equivalently, using the DeWitt-Schwinger scheme (for more details see [18].) To properly cancel the UV divergences in our 4-dimensional spacetime the second adiabatic order $N = 2$ is the right choice for the mean square fluctuation $\langle \varphi^2(t, \vec{x}) \rangle$. It must be noted that the proper adiabatic order of the subtraction term depends on the particular physical quantity to evaluate. For instance, the computation of the renormalized expectation value of the stress-energy tensor $\langle T_{\mu\nu}(t, \vec{x}) \rangle$ needs subtraction up to the fourth adiabatic order, using $[\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle - {}^{(4)}G_{Ad}((t, \vec{x}), (t, \vec{x}'))]$ instead of $\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle$, and taking the coincident limit.

Apart from these fundamental objects, we can also be interested in integrated quantities from the two-point function, like

$$\langle \varphi_{\vec{p}} \varphi_{\vec{p}'} \rangle = \int d^3\vec{x} d^3\vec{x}' e^{i(\vec{p} \cdot \vec{x} + \vec{p}' \cdot \vec{x}')} \langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle = |\varphi_k(t)|^2 \delta^3(\vec{p} + \vec{p}') , \quad (5)$$

or, assuming rotational invariance, we can also construct

$$\begin{aligned} C_\ell^{\varphi\varphi} &= \frac{1}{4\pi} \int d^2\hat{n} d^2\hat{n}' P_\ell(\hat{n} \cdot \hat{n}') \langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle \\ &= 2\pi \int_{-1}^1 d\cos\theta P_\ell(\cos\theta) \langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle = 16\pi^2 \int_0^\infty \frac{dk}{k} |\varphi_k(t)|^2 j_\ell^2(k|\vec{x}|) , \end{aligned} \quad (6)$$

where P_ℓ are the Legendre Polynomials, and $\cos\theta = \vec{n} \cdot \vec{n}'$ is the angle formed by the two directions $\vec{n} = \vec{x}/|\vec{x}|$ and $\vec{n}' = \vec{x}'/|\vec{x}'|$. For completeness, we have also added the equivalent and familiar expression in momentum space. It involves the spherical Bessel functions j_ℓ . Although the former integral (5) is UV finite and no renormalization is needed, the latter (6) has an UV divergence. A detailed inspection reveals that the divergence is of $N = 0$ adiabatic order. This conclusion can be deduced from the short-distance (adiabatic) expansion ($\theta \rightarrow 0$)

$$\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle \sim \frac{1}{1 - \cos\theta} - \frac{(\frac{1}{6} - \xi)R}{2} \log(1 - \cos\theta) + \dots \quad (7)$$

and $P_\ell(\cos\theta) \sim 1$. Therefore, while (5) should be kept unaltered, expression (6) must be modified according to the renormalization prescription. One should then replace $\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle$ by $[\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle - {}^{(0)}G_{Ad}((t, \vec{x}), (t, \vec{x}'))]$ in expression (6) to guarantee the UV finiteness of the integral.

However, we should remark that the appropriate choice of the adiabatic subtraction order N depend on the physically relevant object. In CMB cosmology the direct physical observables are temperature correlations, which are linked to the space-time two-point function $\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle$. For instance, in the Sachs-Wolfe regime we have $\langle \Delta T(\vec{n}) \Delta T(\vec{n}') \rangle_{SW} = \frac{T_0^2}{25} \langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle$, where $\varphi(t, \vec{x}) \equiv \mathcal{R}(t, \vec{x})$ is the comoving curvature perturbation. These temperature correlations are the ones upon which other observables like C_ℓ^{TT} are constructed

$$C_\ell^{TT} = \int_{-1}^1 d\cos\theta P_\ell(\cos\theta) \langle \Delta T(\vec{n}) \Delta T(\vec{n}') \rangle . \quad (8)$$

Since physical correlations are direct observables that can be measured in an experiment, it seems natural to demand that they must always be finite, even at coincidence $\vec{x} = \vec{x}'$. To achieve that, one should then relate $\langle \Delta T(\vec{n}) \Delta T(\vec{n}') \rangle$ with the quantity $[\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle - {}^{(2)}G_{Ad}((t, \vec{x}), (t, \vec{x}'))]$ to ensure UV finiteness at coincidence. Therefore, the actual related multipole coefficients should be constructed with second order adiabatic subtractions

$$C_{\ell, N=2}^{\varphi\varphi} = 2\pi \int_{-1}^1 d\cos\theta P_\ell(\cos\theta) [\langle \varphi(t, \vec{x}) \varphi(t, \vec{x}') \rangle - {}^{(2)}G_{Ad}((t, \vec{x}), (t, \vec{x}'))] . \quad (9)$$

2. Spacetime correlators in slow-roll inflation

Let us focus now on scalar perturbations during slow-roll inflation. For simplicity we will also consider the Sachs-Wolfe regime, for which the transfer functions are trivial. Scalar fluctuations are described through the comoving curvature perturbation field $\mathcal{R}(t, \vec{x})$. For single-field inflation, the modes $\mathcal{R}_k(t)$ defining the Bunch-Davies type vacuum, are [5]

$$\mathcal{R}_k(t) = \sqrt{\frac{-\pi\eta}{4(2\pi)^3 z^2}} H_\nu^{(1)}(-\eta k) , \quad (10)$$

where $H \equiv \dot{a}/a$ is the Hubble rate, $H_\nu^{(1)}$ is the Hankel function with $\nu = \frac{3}{2} + \frac{2\epsilon + \delta}{1 - \epsilon}$, $\epsilon \equiv -\dot{H}/H^2 \ll 1$, and $\delta \equiv \ddot{H}/2H\dot{H}$ is a second slow-roll parameter. In this expression we

introduced the so-called proper time η , defined by $d\eta = \frac{dt}{a(t)}$. Moreover, $z \equiv a\dot{\phi}_0/H$, where $\phi_0(t)$ is the homogeneous part of the inflaton field, responsible for the period of inflation itself.

Using the modes (10) one can work out analytically the corresponding two-point function $\langle \mathcal{R}(t, \vec{x}), \mathcal{R}(t, \vec{x}') \rangle$. It is given by

$$\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle = \frac{1}{16\pi^2 z^2 \eta^2} \Gamma\left(\frac{3}{2} + \nu\right) \Gamma\left(\frac{3}{2} - \nu\right) {}_2F_1\left(\frac{3}{2} + \nu, \frac{3}{2} - \nu; 2; 1 - \frac{(\Delta x)^2}{4\eta^2}\right), \quad (11)$$

where ${}_2F_1$ is the hypergeometric function, and Γ is the gamma function. The two-point separation is described by $\Delta x \equiv |\Delta \vec{x}| = 2^{\frac{1}{2}} |\vec{x}| (1 - \cos \theta)^{1/2}$. For very short separations we get

$$\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle = \frac{GH^2(1 - \epsilon)^2}{4\pi\epsilon} \left\{ \frac{4}{\Delta \bar{x}^2} + \left[\frac{1}{4} - \nu^2 \right] \log \frac{\Delta \bar{x}^2}{4} + O(\Delta \bar{x}^0) \right\} \quad (12)$$

where we defined $\Delta \bar{x} = \Delta x a H(1 - \epsilon)$, and G is the Newton's gravitational constant. It is clear then that, as expected, the scalar two-point function diverges when $\theta \rightarrow 0$. This produces an ultraviolet divergence in the (unrenormalized) expression for multipole coefficients in the Sachs-Wolfe regime, for which one makes the identification $\langle \Delta T(\vec{n}) \Delta T(\vec{n}') \rangle_{SW} = \frac{T_0^2}{25} \langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle$,

$$C_\ell^{SW} = \frac{2\pi T_0^2}{25} \int_{-1}^1 d \cos \theta P_\ell(\cos \theta) \langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle = \infty, \quad (13)$$

One could cure this divergence with the replacement $\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle \rightarrow [\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle - G_{Ad}^{(0)}((t, \vec{x}), (t, \vec{x}'))]$. One then gets

$$C_{\ell, N=0}^{SW} = \frac{2\pi T_0^2}{25} \int_{-1}^1 d \cos \theta P_\ell(\cos \theta) [\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle - G_{Ad}^{(0)}((t, \vec{x}), (t, \vec{x}'))] < \infty. \quad (14)$$

Evaluating this well-defined integral at late times yields the result [we define the new parameter $\bar{r}_L = r_L a H(1 - \epsilon)$, with $|\vec{x}| = r_L$ the comoving radial coordinate of the last scattering surface]

$$C_{\ell, N=0}^{SW} = \frac{8\pi T_0^2}{25} \frac{4\pi G}{\epsilon} \frac{H^2(1 - \epsilon)^2}{16\pi^2} \frac{\Gamma(3 - n)\Gamma(\ell + \frac{n-1}{2})}{\Gamma(\ell + 2 - \frac{n-1}{2})} \bar{r}_L^{1-n}, \quad (15)$$

where we have used $\nu - \frac{3}{2} = \frac{1-n}{2}$, and n represents the scalar index of inflation $n = 1 - 4\epsilon - 2\delta + O(\epsilon, \delta)^2$. This is in exact agreement with the well-known standard result, obtained with the momentum-space power spectrum. It is worth to remark that the same result can be obtained if one consider the large-scale behavior [i.e. the late-time one during inflation $a|\vec{x} - \vec{x}'| \gg H^{-1}$] of the two-point function

$$\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle \sim \frac{\Gamma(\frac{3}{2} - \nu) \Gamma(\nu)}{4\pi^2 z^2 \eta^2 \sqrt{\pi}} \left(\frac{\Delta x}{-\eta} \right)^{2(\nu-3/2)}. \quad (16)$$

and use this expression in (13). This is indeed the standard approach, as reported in many textbooks [5]. Therefore, it is actually equivalent to the calculation of $C_{\ell, N=0}^{SW}$ with the renormalized two-point function at zero adiabatic order.

Even though this procedure provides a finite result, what we have really encountered in (12) is the typical quadratic and logarithmic short-distance behavior of a quantum field in a curved

background. So, as argued before, it seems natural to remove both divergences, not only the leading one. Accordingly, it is natural to propose the following identification

$$\langle \Delta T(\vec{n}) \Delta T(\vec{n}') \rangle_{SW} = \frac{T_0^2}{25} [\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle - G_{Ad}^{(2)}((t, \vec{x}), (t, \vec{x}'))] , \quad (17)$$

instead of $\langle \Delta T(\vec{n}) \Delta T(\vec{n}') \rangle_{SW} = \frac{T_0^2}{25} [\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle - G_{Ad}^{(0)}((t, \vec{x}), (t, \vec{x}'))]$, as used in (14) to obtain (15).

The subtraction term at second adiabatic order is found to be

$$G_{Ad}^{(2)}((t, \vec{x}), (t, \vec{x}')) = \frac{GH^2(1-\epsilon)^2}{4\pi\epsilon} \left\{ \frac{4}{\Delta \bar{x}^2} + \left(\frac{1}{4} - \nu^2 \right) \log \frac{\Delta \bar{x}^2}{4} + \frac{2-\epsilon}{3(1-\epsilon)^2} + \left(\frac{1}{4} - \nu^2 \right) \left[2\gamma + \log \frac{\mu^2}{H^2(1-\epsilon)^2} \right] \right\} , \quad (18)$$

where μ is a renormalization scale, and γ is the Euler constant. Now we should obtain a suitable expression for the two-point function. A good way to deal with this is to expand expression (11) as a power series of the “slow-roll” parameter ν around $\nu = 3/2$, and stay at first order (for details see [16]). The result reads:

$$\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle \approx \frac{GH^2(1-\epsilon)^2}{4\pi\epsilon} \left\{ \frac{4}{\Delta \bar{x}^2} - 1 + \frac{2}{(3/2-\nu)} \left(\frac{\Delta \bar{x}^2}{4} \right)^{\nu-3/2} \right\} \quad (19)$$

We can now proceed to do the subtraction. The renormalized two-point function then reads

$$\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle - G_{Ad}^{(2)}((t, \vec{x}), (t, \vec{x}')) \approx \frac{GH^2(1-\epsilon)^2}{4\pi\epsilon} \left\{ \frac{2}{(3/2-\nu)} \left(\frac{\Delta \bar{x}^2}{4} \right)^{\nu-3/2} + 2 \log \Delta \bar{x}^2 - \frac{5}{3} + 4\gamma + 2 \log \frac{\mu^2}{H^2} \right\} . \quad (20)$$

We now proceed to compute the corresponding angular power spectrum from the renormalized two-point function and for the Sachs-Wolfe regime

$$C_{\ell, N=2}^{SW} = \frac{2\pi T_0^2}{25} \int_{-1}^1 d \cos \theta P_\ell(\cos \theta) [\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle - G_{Ad}^{(2)}((t, \vec{x}), (t, \vec{x}'))] . \quad (21)$$

To evaluate the logarithmic contributions of (20) to (21) we take into account that $\int_{-1}^1 dy \log(1-y) P_\ell(y) = -2/\ell(\ell+1)$, $\ell = 1, 2, \dots$. The final result is very well approximated by the following analytical expression:

$$C_{\ell, N=2}^{SW} \approx \frac{4\pi G}{\epsilon} \frac{8\pi T_0^2}{25} \frac{H^2(1-\epsilon)^2 \bar{r}_L^{1-n}}{16\pi^2} \left\{ \frac{\Gamma(\ell + \frac{n-1}{2})}{\Gamma(\ell + 2 - \frac{n-1}{2})} - \frac{\bar{r}_L^{n-1}}{\ell(\ell+1)} \right\} . \quad (22)$$

The above expression is valid for $\ell \geq 1$, as for $\ell = 0$ there would be present all the constant contributions from the renormalized two-point function (20), including the one depending on the renormalization scale. In fact, the renormalization scale may be fixed by imposing the natural condition $C_{0, N=2}^{SW} = 0$.

Notice that the first term in (22) comes from the renormalization procedure at zero adiabatic order, and reproduces the standard result (15). The second one comes from the purely second adiabatic order subtraction term, but it shows scale invariance as well. Therefore, Eq. (22) is also consistent with observations [6]. The first term is time-independent, while the second term

depends slightly on time through the quantity $\bar{r}_L^{n-1} \equiv \alpha$. This parameter can be regarded as a phenomenological parameter, varying in the range $1 > \alpha > 0$. In the limiting case $\alpha \rightarrow 0$ one recovers the standard prediction, and this would correspond to the subtraction terms evaluated after the end of inflation. If the subtraction terms are evaluated a few e -foldings after the horizon exit of the scale r_L , the parameter α approaches 1 and the physical significance of the correction increases.

3. Conclusions

We have briefly reported a critical overview on the renormalization of cosmological observables. Firstly, we have analyzed two-point correlators and self-correlators of scalar primordial perturbations in quasi-de Sitter spacetime backgrounds. Then, the evaluation of the multipole coefficients C_ℓ has been specially scrutinised. We have stressed the fact that the standard result in the literature is equivalent to a renormalization of the two-point function at zero adiabatic order. We have argued that renormalization at second adiabatic order is more suitable from a physical point of view, it is necessary to ensure UV finiteness of direct physical observables such as temperature correlations. This may change significantly the predictions of inflation, provided the renormalization subtraction terms are evaluated a few e -foldings after the first horizon crossing of the scale r_L .

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