

On cosmic quantum tunneling from “nothing”

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Abstract. We extend to a general Λ -Friedmann-Lemaître-Robertson-Walker (Λ FLRW) a previous result by Vilenkin and others according to which a closed de Sitter universe could be created from “nothing”. More specifically, our main result is that only the closed Λ FLRW universe (but not the open and flat ones) could be created from a corresponding *instanton*, that is, from the corresponding solution with signature +4 of the Einstein field equations. Before getting this result the suitable corresponding instantons are calculated. The result is in accordance with previous results by another authors obtained by different methods.

1. Introduction

In four seminal papers [1, 2, 3, 4] Vilenkin has estimated the non zero probability of quantically creating from “nothing” a closed de Sitter universe, here “nothing” meaning a state without time. More specifically, an Euclidean differentiable manifold (one with +4 signature) instead of a Lorentzian one (one with +2 signature). In the present paper an Euclidean manifold which is a solution of the Einstein field equations will be called an *instanton*.

An expression for the above creating probability, P , in the semiclassical approximation is [3, 4]:

$$P \propto \exp(-|S_E|),$$

with S_E the action of a suitable instanton associated to the Lorentzian metric whose creation probability we want to calculate.

In the present paper, we extend the Vilenkin results to the general Friedmann-Lemaître-Robertson-Walker universe with a cosmological constant, Λ , that is a Λ FLRW universe. To this end, we follow a general procedure that covers the treatment given by Vilenkin to his particular case.

Our result is that the closed Λ FLRW model is quantically creatable, but the open non flat and the flat ones are not. This is in agreement with the result in [5], whose authors do not follow the quantum creation from “nothing” procedure *à la* Vilenkin used for us, but one where the FLRW universe comes from some initial static space-time configuration.

2. The general procedure for the quantum creation of a universe: its two prescriptions

Let $T_{\alpha\beta}$ be the energy-momentum tensor in General Relativity, and $g_{\alpha\beta}$ a solution of the corresponding Einstein field equations ($\alpha, \beta, \dots = 0, 1, 2, 3$, signature +2). In order to provide a general proposal for the creation of a given universe, we advance the following two prescriptions.



First prescription:

We define the instanton “energy-momentum” tensor $T_{\alpha\beta}^E$ as the one obtained putting in $T_{\alpha\beta}$ an instanton metric $g_{\alpha\beta}^E$ associated to the corresponding Lorentzian one. This associated instanton metric, $g_{\alpha\beta}^E$, is a suitable Euclidean (signature +4) solution of the Einstein field equations according to the following

Second prescription:

Given $g_{\alpha\beta}$, its quantum creation probability, P , can be estimated from $P \propto \exp(-|S_E|)$, by associating to $g_{\alpha\beta}$ some $g_{\alpha\beta}^E$ satisfying the Darmois continuity conditions. For a singular Lorentzian metric to be created these Darmois conditions are defined in the following way:

First, let it be a space-like 3-surface, Σ_3 , where $g_{\alpha\beta}$ is singular, and let us consider the first, ${}^3g_{ij}$, and the second, K_{ij} , fundamental forms, whose meaning respectively is ${}^3g_{ij}$, the restriction of $g_{\alpha\beta}$ on Σ_3 , and $K_{ij} = (\nabla_\mu n_\nu) \gamma_i^\mu \gamma_j^\nu$. Here, n_μ is the unit normal 4-vector to Σ_3 and γ_β^α the projector on Σ_3 .

Then, for the associated instanton metric $g_{\alpha\beta}^E$, we similarly define its first and second fundamental instanton forms: ${}^3g_{ij}^E, K_{ij}^E$. Then, the imposed Darmois conditions are the vanishing of ${}^3g_{ij} - {}^3g_{ij}^E$, and $K_{ij} - K_{ij}^E$, through Σ_3 , that we will write in a concise way:

$$[{}^3g_{ij}] = 0, \quad [K_{ij}] = 0.$$

3. A particular canonical Einstein equations for Λ FLRW metrics, including instantons

The Einstein equations for a Λ FLRW metric and its instantons

$$ds^2 = \epsilon du^2 + a^2 \left(\frac{d\rho^2}{1 + \epsilon k \rho^2} + \rho^2 d\sigma^2 \right), \quad d\sigma^2 = d\theta^2 + \sin^2 \theta d\phi^2$$

(a , the cosmic expansion factor, $\epsilon = -1$ for the Lorentzian solutions, and $\epsilon = +1$ for the instanton solutions) can be written

$$\frac{\dot{a}^2}{a^2} = \frac{\kappa}{3} \mu - \frac{k}{a^2} - \frac{\epsilon}{3} \Lambda, \quad \frac{\ddot{a}}{a} = -\frac{\kappa}{6} (\mu - 3\epsilon p) - \frac{\epsilon}{3} \Lambda, \quad (k = +1, 0, -1) \quad (1)$$

all in accordance with Ellis [6].

4. Matching Λ FRW universes with their instantons

The Darmois continuity conditions $[{}^3g_{ij}] = 0, [K_{ij}] = 0$, through the singular space-like 3-surface $u = t = 0$, now give

$$[a] = 0, \quad [\dot{a}] = 0, \quad [\epsilon k] = 0.$$

These conditions are satisfied in the particular case considered by Vilenkin [1, 2]: the quantum creation from “nothing” of a closed de Sitter universe. Thus, our procedure to create any Λ FLRW model extends this previous result.

In particular, the continuity condition $[\epsilon k] = 0$ associates the closed instanton:

$$ds_E^2 = du^2 + a_E^2(u) \left(\frac{d\rho^2}{1 - \rho^2} + \rho^2 d\sigma^2 \right), \quad E \text{ from “Euclidean”,}$$

to the closed Λ FRW universe

$$ds^2 = -dt^2 + a^2(t) \left(\frac{d\rho^2}{1 - \rho^2} + \rho^2 d\sigma^2 \right),$$

where a_E and a satisfy Eqs. (1), for $k = -1$, $\epsilon = +1$ in the a_E case, and for $k = +1$, $\epsilon = -1$, in the a case, and with $a_E|_{u \rightarrow 0} = a|_{t \rightarrow 0}$, $\dot{a}_E|_{u \rightarrow 0} = \dot{a}|_{t \rightarrow 0}$ from the other two Darmois conditions $[a] = [\dot{a}] = 0$.

Since these two limiting conditions, near $u = t = 0$, because of the radiation dominance for the Λ FRW universe, we obtain for the approximate instanton behavior:

$$\mu_E(u) \simeq \mu(u), \quad a_E \simeq a_{E_e} \sqrt{\frac{u}{u_e}},$$

a_{E_e} standing for the value of a_E in some “instant”, u_e , of the “energy” instanton phase.

5. Quantum creation of a closed Λ FRW universe

We try a particular instanton model by imposing in a natural way that $\mu_E(u) = \mu(u)$, $\forall u$. On the other hand the instanton action S_E in evident notation is

$$S_E = \frac{1}{2\kappa} \int R_E \sqrt{g_E} d^4x + \int L_E \sqrt{g_E} d^4x.$$

Further, extending to instanton metrics a procedure by Hawking and Ellis [7], we obtain $L_E = \mu$, and then

$$\frac{1}{2\kappa} R_E + L_E = \frac{1}{2}(\mu - 3p_E) + 2 \frac{\Lambda}{\kappa}.$$

For a closed FLRW model, the calculation of this integral over $\rho \in [0, 1]$ gives a finite value. Then, in this model the creation probability becomes finite. Thus the closed Λ FRW universe becomes quantically creatable.

Differently, it can be seen that the open non flat and flat models are not quantically creatable, and so, as quoted above, we recover similar results by Atkatz and Pagels [5] obtained by a different method.

These results remain true for an arbitrary (out of the above unavoidable condition $\lim_{u \rightarrow 0} \mu_E(u) = \lim_{u \rightarrow 0} \mu(u)$) μ_E function, provided that for $a_E \rightarrow a_{E_m}$ we have

$$\frac{\dot{a}_E}{a_E} \propto (a_{E_m} - a_E)^n, \quad 0 \leq n < 1,$$

where a_{E_m} is the minimal positive value of a_E such that $\dot{a}_E = 0$.

6. Final considerations

The closed and flat Λ FLRW models have vanishing *intrinsic* 4-momenta [8] (in particular, have vanishing *intrinsic* energy), while the open non flat one has a minus infinite intrinsic energy. But, the flat Λ FLRW model, perturbed in the standard inflationary scenario, has an infinite *intrinsic* 4-momenta [9]. Thus the suggestion by Fomin [10] and Tryon [11], according to which a quantically creatable universe should have vanishing energy, becomes reinforced by our present result on the quantum creatability of the closed Λ FLRW model but not of the open and flat ones.

An enlarged and detailed treatment of the topic covered by the present article will be published elsewhere.

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