

Quantum correlations across horizons

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Abstract. We analyze a simple minisuperspace model for the Schwarzschild-de Sitter spacetime and develop an exact quantization of this geometry. We determine the physical Hilbert space of the model and employ it to study the possible quantum influence of regions of the universe that are not classically accessible.

1. Introduction

In the classical study of spacetimes, the presence of horizons disconnect physically the regions that are separated by them, making any of these regions causally inaccessible from the outside. This is the case, for instance, for event and cosmological horizons, examples on which we will concentrate our attention in this work.

A new feature of our analysis is that, compared with many discussions carried out in the literature about quantum field theory on curved backgrounds (see e.g. [1]), we will construct a canonical quantization of the spacetime itself. So, our conclusions only depend on the quantum behavior of the geometry. For this purpose, we will consider a minisuperspace model for a spherically symmetric spacetime in the presence of a positive cosmological constant, situation described by a Kantowski-Sachs metric. The maximal analytic extension of the classical solutions, which describes Schwarzschild-de Sitter spacetimes, generically have black hole and cosmological horizons that isolate the spacetime region where we would presumably live. We complete a canonical quantization of this model following an extension of the conventional Dirac quantization procedure (see e.g. [2]).



2. Classical solutions

We construct our model starting with a general spherically symmetric metric that depends on two variables, A and b , and on the lapse function N ,

$$\sigma^{-2}ds^2 = -\frac{N(r)^2}{A(r)}dr^2 + A(r)dT^2 + b(r)^2d\Omega_2^2, \quad (1)$$

where all metric variables and coordinates are dimensionless, $d\Omega_2^2$ is the line element on the unit two-sphere, and σ has units of length. Note that this is a Kantowski-Sachs metric with a suitably redefined lapse [3].

Generically, the horizons are located where $A = 0$. The range of the variables A and b are taken to be the whole real line. The change of sign in A is of much importance, because it corresponds to a change in the character of the radial coordinate from timelike to spacelike behavior, or vice versa.

It will prove convenient for our analysis to introduce a new variable $c = Ab$ [4], instead of A , which allows us to simplify the Hilbert-Einstein action, that can be written in terms of the new configuration variables and a positive cosmological constant, Λ , as

$$S = - \int \left(\frac{\dot{b}\dot{c}}{N} + N\dot{B}(b) \right), \quad (2)$$

where the dot stands for the derivative with respect to r , we have defined $\lambda = \sigma^2\Lambda$, and

$$B(b) = \frac{\lambda}{3}b^3 - b, \quad \dot{B}(b) = \partial_b B(b) = \lambda b^2 - 1. \quad (3)$$

The variational principle for this action gives the classical equations of motion, that from the point of view of the metric (1) adopt as solutions the Schwarzschild-de Sitter metric,

$$b(r) = r, \quad A(r) = -1 + \frac{2m}{r} + \frac{\lambda r^2}{3}, \quad (4)$$

where m is an integration constant. Another integration constant appears related with a rescaling of the coordinates r and t , but using the freedom in the definition of these coordinates it can be set equal to one.

The causal structure of these spacetimes depends on the value of m , and in all cases (except for $m = 0$) the solution presents a singularity at $r = 0$. The most interesting case is $0 < m < 1/\sqrt{9\lambda}$, where $A(r) = 0$ has two positive solutions that can be identified as a black hole horizon and a cosmological horizon.

A common feature of all solutions is that “our” spacetime region (between horizons) is characterized by $A < 0$, whereas $A > 0$ characterizes the regions outside.

In order to perform a canonical quantization, we adopt a Hamiltonian formulation of the system. The canonical action can be expressed as

$$S = \int dr \left(\dot{c}p_c + \dot{b}p_b - NC \right), \quad (5)$$

where the canonical conjugate momenta are $p_b = -\dot{c}/N$ and $p_c = -\dot{b}/N$. The variation with respect to the lapse function gives rise to the Hamiltonian constraint, $C = 0$, with

$$C = -p_b p_c + \mathring{B}(b). \quad (6)$$

3. Canonical quantization

3.1. Kinematical space and operator algebra

In order to quantize our system, we will start with the kinematical algebra of phase space functions constructed from the canonical variables b, c, p_b , and p_c (and the unit constant), which is closed under Poisson brackets. As kinematical space we choose the vector space spanned by simultaneous solutions to the equations

$$-i\partial_c \Psi_{hp} = p \Psi_{hp}, \quad [\partial_c \partial_b + \mathring{B}(b)] \Psi_{hp} = h \Psi_{hp}, \quad (7)$$

with h and p being real. Thus, any kinematical state will be a linear combination of these solutions Ψ_{hp} . Explicitly, they take the form [5]

$$\Psi(b, c) = \int_{\mathbb{R}} dh \int_{\mathbb{R}} dp \tilde{\Psi}(h, p) e^{ipc + i[B(b) - bh]/p}, \quad (8)$$

where $\tilde{\Psi}(h, p)$ is a distribution. Actually we have introduced two kinematical representations that we can use: the metric (b, c) -representation and the (h, p) -representation. In the (h, p) -representation, basic operators act as

$$\begin{aligned} \hat{b} &= -ip\partial_h, & \hat{c} &= i\partial_p + \frac{B(-ip\partial_h)}{p^2} + \frac{i\partial_h h}{p}, \\ \hat{p}_b &= \frac{\mathring{B}(-ip\partial_h) - h}{p}, & \hat{p}_c &= p, \end{aligned} \quad (9)$$

as one can check by direct application of these operators on the kinematical states (8).

It is convenient to introduce an inner product in the kinematical space on which the operators \hat{b} , \hat{c} , \hat{p}_c , and \hat{p}_b are self-adjoint, namely:

$$(\Psi_1, \Psi_2) = \int_{\mathbb{R}} dh \int_{\mathbb{R}} dp \tilde{\Psi}_1(h, p)^* \tilde{\Psi}_2(h, p), \quad (10)$$

where the symbol $*$ denotes complex conjugation. Then, the kinematical Hilbert space is $L^2(\mathbb{R}^2, dh dp)$.

Finally, the Hamiltonian constraint can be represented in this kinematical space by the operators

$$\hat{C} = \partial_b \partial_c + \mathring{B}(b), \quad \hat{C} = h, \quad (11)$$

in the metric (b, c) -representation and in the (h, p) -representation, respectively.

3.2. Physical Hilbert space

The space of solutions can be obtained by imposing the constraint

$$\hat{C}\tilde{\Phi}(h, p) = 0. \quad (12)$$

Then, solutions turn out to be

$$\tilde{\Phi}(h, p) = \frac{1}{\sqrt{2\pi}} \delta(h) \phi(p), \quad (13)$$

where $\phi(p)$ is an arbitrary distribution. In this physical vector space, the operators \hat{p} and \hat{q} (which commute with the constraint \hat{C}) are represented as $\hat{p} = p$, and $\hat{q} = i\partial_p$.

We choose an inner product in this space of physical states so that the observables \hat{p} and \hat{q} be self-adjoint:

$$\langle \Phi_1, \Phi_2 \rangle = \int_{\mathbb{R}} dp \phi_1(p)^* \phi_2(p). \quad (14)$$

Hence the physical Hilbert space of quantum states for our system is $L^2(\mathbb{R}, dp)$. We will refer to this representation as the p -representation.

These physical states can also be written in terms of the metric variables b and c :

$$\Phi(b, c) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} dp \phi(p) e^{i[pc + B(b)/p]}. \quad (15)$$

The inverse of this transformation is

$$\phi(p) = \frac{1}{\sqrt{2\pi}} e^{-iB(b)/p} \int_{\mathbb{R}} dc \Phi(b, c) e^{-ipc}, \quad (16)$$

where we can see that the physical states $\Phi(b, c)$ depend on b only through $B(b)$.

This transformation allows us to write the inner product in terms of the metric variables on configuration sections of fixed value of b :

$$\langle \Phi_1, \Phi_2 \rangle = \int_{\mathbb{R}} dc \Phi_1(b, c)^* \Phi_2(b, c). \quad (17)$$

From the independence on b of the inner product, it immediately follows that we actually have not only one, but a whole family of p_b -representations labeled by b . This resembles a kind of transformation from the Heisenberg picture, $\phi(p)$, to the b -Schrödinger picture, $\phi(b, p)$, since

$$\phi(b, p) = \hat{U}(b) \phi(p) = e^{iB(b)/p} \phi(p), \quad (18)$$

where, for each value of b , we can define the observables $\hat{\pi}_b = \hat{p}$ and $\hat{c}_b = i\partial_p$.

We can adopt two alternative viewpoints analogous to the Schrödinger and Heisenberg pictures of standard quantum mechanics. From the first point of view, we can consider that the p_b -representations give the evolution of the p -representation, hence, the family of representations provide the corresponding Schrödinger “dynamics” in the parameter b .

From the second point of view, we can choose a fixed p_b -representation for a given value of b , and represent our family of observables in a kind of Heisenberg picture (see e.g. [6]). The family of observables corresponding to c at different values b_0 of b would be given by

$$\hat{c}_b^0 = \hat{U}^\dagger(b, b_0) \hat{c}_b \hat{U}(b, b_0) = i\partial_p + \frac{B(b_0) - B(b)}{p^2}, \quad (19)$$

where $\hat{U}(b, b_0) = e^{i[B(b) - B(b_0)]\hat{p}^{-1}}$. This observable gives, in the p_b -representation, the value of c when $b = b_0$.

It will be useful for our study to define a family of bases formed by eigenstates of the self-adjoint operator \hat{c}_b^0 in the p_b -representation with real eigenvalues c^0 ,

$$\phi_{c^0}(b, p) = \frac{1}{\sqrt{2\pi}} e^{-ipc^0 - i[B(b_0) - B(b)]/p}. \quad (20)$$

In this family of representations, the identity operator can be written as [5]

$$\mathbb{1}(b, p; b, p') = \int_{\mathbb{R}} dc^0 \phi_{c^0}(b, p) \phi_{c^0}(b, p')^*. \quad (21)$$

Clearly, it is possible to decompose this identity operator in the sum of two orthogonal projectors, one for positive eigenvalues of c^0 , \hat{P}_+^0 , and the other for negative eigenvalues, \hat{P}_-^0 ,

$$\mathbb{1} = \hat{P}_+^0 + \hat{P}_-^0. \quad (22)$$

The superindex 0 indicates the dependence of the projection operators on the value of b_0 where c is evaluated.

4. Quantization and horizons

If we concentrate our attention on certain positive value b_0 of b , and there we observe only the region with negative values of c , we need to restrict our analysis just to states with null projection under \hat{P}_+^0 . This restriction corresponds classically to considering only our region of the universe at the given “instant b_0 of the dynamical variable b ”.

The question about whether we can forget about the regions beyond the horizons quantum mechanically, as it occurs classically, is related now with the issue of whether this restriction to our region of the universe is robust dynamically. In our scheme, this is equivalent to ask whether observations of the values of c at different values of b are compatible, something which requires that the two considered observables \hat{c}_b^0 and \hat{c}_b^1 commute. A direct calculation shows that this is not the case, and therefore the family of considered observables are not mutually compatible. Then, the two projections (at the different values b_0 and b_1) would differ, and the restriction to our region of the universe between horizons would not be stable, in the sense that the projection at b_0 on negative values of c^0 would generally have a non-vanishing projection at b_1 on positive values of c^1 , and vice versa. The restriction to the interior of the horizons would depend on the value of b , and would therefore be unstable under evolution in this variable [5].

We have seen that the dynamics in b mixes the projections, and thus it is not consistent to describe states as direct sum of positive and negative c^0 -states. In consequence, general physical states rather belong to the tensor product of the projection subspaces for a particular instant of b :

$$\mathcal{H}^0 = \mathcal{H}_+^0 \otimes \mathcal{H}_-^0. \quad (23)$$

In this formalism, any observable \hat{O} can then be decomposed in four operators between both projections:

$$\hat{O}_{\pm\pm}^0 : \mathcal{H}_{\pm}^0 \rightarrow \mathcal{H}_{\pm}^0, \quad \hat{O}_{\pm\mp}^0 : \mathcal{H}_{\pm}^0 \rightarrow \mathcal{H}_{\mp}^0, \quad (24)$$

defined as $\hat{O}_{\pm\pm}^0 = \hat{P}_{\pm}^0 \hat{O} \hat{P}_{\pm}^0$ and $\hat{O}_{\pm\mp}^0 = \hat{P}_{\pm}^0 \hat{O} \hat{P}_{\mp}^0$.

The operators $\hat{O}_{\pm\mp}^0$ mix the two subspaces corresponding to the considered projections, \mathcal{H}_{\pm}^0 , and lead to correlations between them. This is the case of \hat{c}_b^1 , as we have seen. Finally, note that if \hat{O} is a unitary observable, for instance $\exp(i\hat{c}_b^1)$, the existence of the two mixing components indicates that unitarity is not respected in each of the subspaces \mathcal{H}_{\pm}^0 separately.

5. Conclusion

We have analyzed a Kantowski-Sachs minisuperspace model of a spacetime with a positive cosmological constant, whose classical solutions are Schwarzschild-de Sitter universes. We have argued that quantum mechanics applied to the whole spacetime generically introduces quantum correlations between different classically disconnected regions (separated by horizons), so that we can not restrict ourselves to the observed classical region when we consider the spacetime quantum mechanically. Then, the physical structure is consistent only if we consider the whole spacetime [5]. This is explicitly shown by checking that unitarity is preserved only when the whole spacetime is taken into account. Another way to see that fact is because the two subspaces in which we have decomposed the physical Hilbert space of the system, corresponding to states with support either between or beyond the horizons, are not stable under the action of unitary operators that describe a natural concept of evolution on physical states. Therefore, these states are better conceived as belonging to the tensor product of both subspaces, which are not separable but entangled.

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