

Confidence in the Mass Hierarchy Determination from Reactor Neutrino Experiments

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Abstract. In the next decade, a number of experiments will attempt to determine the neutrino mass hierarchy. We will discuss some of the challenges that these experiments will face, focusing on the reactor neutrino experiments at intermediate baselines. Here, for example, even a small nonlinear energy response can severely affect the sensitivity to the hierarchy. Using MC simulations, we will show the probability of successfully determining the hierarchy for some realistic locations of the detector. We will also show that, since the two hierarchies are non-nested hypothesis, the statistic $\Delta\chi^2$ does not follow a one-degree-of-freedom χ^2 distribution and hence the sensitivity to the hierarchy cannot be estimated by taking the square root of the expected $\Delta\chi^2$; we will present the correct formula for the sensitivity.

1. Introduction

The recent measurement of θ_{13} revealed that it is significantly larger than the expected. One of the consequences of this discovery is that now it is practical to determine the neutrino mass hierarchy using reactor neutrino experiments at intermediate baselines. We will present some of the challenges that this kind of experiment will face, discussing possible ways to increase the sensitivity to the hierarchy. More in detail, we will present our results on the interference effect, which happens when the detector receives neutrinos from two or more nuclear reactors, on the non-linear energy response of the detector and we will discuss about the statistical distribution of $\Delta\chi^2$ in the hierarchy determination.

2. Interference

Usually a detector receives neutrino from different nuclear reactors; the different baselines mean that the oscillation probabilities are out of phase, triggering an interference effect [1]. If ΔL is of the order of 1 km the 1-3 oscillations are affected and the sensitivity to the hierarchy can be drastically reduced; if the difference between the baselines is larger (>10 km) the probability of determine the right hierarchy is not so compromised, but it could affect the precise measurement of θ_{12} , which is another of the goals of this kind of experiment.

Using Monte Carlo simulations we studied the effect of the interference for different values of ΔL and we compared the probability of determine the right hierarchy for several realistic locations of the detector [2, 3]. For the JUNO detector, we proposed a new detector site, DongKeng, between the TaiShan and YangJiang reactor complexes. Our Monte Carlo study revealed that it leads to a much better sensitivity to the hierarchy than the old Daya Bay site.



3. Statistical distribution of $\Delta\chi^2$

The determination of the hierarchy is basically the determination of the sign of $\Delta m_{31}^2 = m_3^2 - m_1^2$: if it is positive the hierarchy is called normal, otherwise it is inverted.

Usually the Wilks' theorem ensures that the $\Delta\chi^2$ follows a statistical one-degree-of-freedom χ^2 distribution; however in the case of the hierarchy the two hypothesis are non-nested, *i.e.* one is not a subset of the other. In this case the Wilks' theorem cannot be applied, as pointed out first by [4], and the $\Delta\chi^2$ follow a Gaussian distribution with

$$\mu = \overline{\Delta\chi^2} \quad \sigma = 2\sqrt{\overline{\Delta\chi^2}} \quad (1)$$

where $\overline{\Delta\chi^2}$ is the expected value of $\Delta\chi^2$. We extended this result [5], showing that it is true also if pull parameters are used, and we tested it with Monte Carlo simulations.

3.1. Definitions of Sensitivity to the Mass Hierarchy

In the case of a null hypothesis nested in a general hypothesis, Wilks' theorem yields a frequentist measure of the confidence in the null hypothesis: this number can be obtained simply taking the square root of $\Delta\chi^2$. The neutrino mass hierarchies are disjoint, not nested hypotheses and so Wilks' theorem does not apply. We are instead interested in the sensitivity to the hierarchy, which is the Bayesian probability that the hierarchy predicted by a binary classification test is the true hierarchy. We present simple formulas for the mean and median sensitivities of a χ^2 test to the hierarchy. Although the sensitivity is a probability, we will use the error function to express them as a number of σ 's

- The **mean sensitivity** answers to the question: “**which is the probability that a certain experiment will find the correct hierarchy?**”. The number s_c of σ 's in this case reads

$$s_c = \sqrt{\overline{\Delta\chi^2}}/2 \quad (2)$$

- However, it could be more interesting to consider the sensitivity that can be obtain from a “typical” experiment. The **median sensitivity** answers to the question: “**which is the probability that the median experiment will give us the correct hierarchy?**” (we define the median experiment as the one in which $\Delta\chi^2 = \overline{\Delta\chi^2}$). Assuming symmetric Bayesian priors the number s_v of σ 's is

$$s_v = \sqrt{2}\text{erf}^{-1} \left(\frac{1 - e^{-\overline{\Delta\chi^2}/2}}{1 + e^{-\overline{\Delta\chi^2}/2}} \right) \quad (3)$$

In Fig. 1 you can find these two quantities plotted for different values of $\overline{\Delta\chi^2}$.

4. Non-Linear Energy Response

Since the positions of the peaks must be determined with great precision, even a small non-linear energy response of the detector can significantly affect the precision of the experiment [6]. We consider three different models of non-linearity [7]: one where ΔE is proportional to E^2 , one in which it decreases exponentially and one in which the non-linearity is tuned to mimic the behavior of the inverted hierarchy (we call it the “worst-case” model). We calculated the expected $\Delta\chi^2$ using these models: the results can be found in Fig. 2. We also calculated the same quantity for some realistic locations of the JUNO and RENO 50 detectors, using the worst-case model where the energy shift ΔE was rescaled by a factor 0 (*i.e.* no non-linearity), 1/3, 2/3 and 1, respectively: the results can be found in Tab. 1. It is clear that a second detector can significantly reduce the systematic error due to the non-linear response. Moreover, with two detectors it is possible to measure with good precision the leptonic CP violation with the addition of a cyclotron complex [8].

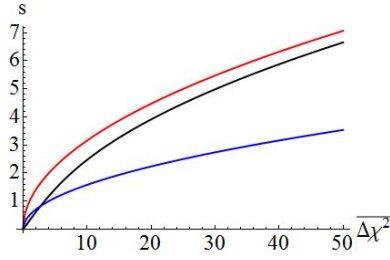


Figure 1. s_c (blue curve) and s_v (black curve). For comparison, also $\sqrt{\Delta\chi^2}$ is shown (red curve).

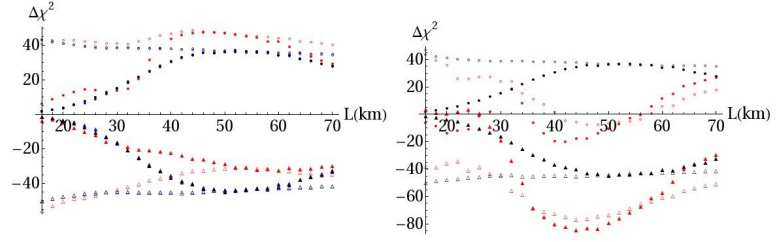


Figure 2. Black: No non-linearity, Blue: quadratic model, Red Panel 1: exponential model, Red Panel 2 :worst-case model. Empty circles (triangle): 1 20kt detector and normal (inverted) hierarchy; Filled circles (triangles): 2 10kt detector, one at 55km and normal (inverted) hierarchy. 6 years running.

Table 1. Expected $\Delta\chi^2$ for specific locations of the detectors

Site	NH/IH:0	NH/IH:1/3	NH/IH:2/3	NH/IH:1
DongKeng (JUNO)	14.1/-17.0	8.2/-21.5	2.2/-24.6	-2.4/26.3
+LuGuJing	13.2/-16.2	7.8/-21.4	3.1/-25.6	-2.3/-27.6
+ZiLuoShan	13.5/-16.1	13.9/-15.3	13.7/-15.8	14.2/-14.2
GuemSeong (RENO50)	6.2/-7.7	3.3/-10.0	0.2/-12.1	-2.5/-13.0
+Jangamsan	5.6/-6.6	5.3/-7.0	3.8/-9.4	4.7/-9.8
Munmyeong (RENO50)	11.8/-13.6	6.4/-17.5	1.6/-21.3	-2.9/-23.2
+Buncheon-ri	11.5/-13.6	9.4/-16.4	7.8/-17.3	3.2/-20.2
Munsusan (RENO50)	9.4/-11.7	5.9/-14.4	2.3/-16.8	-1.9/-18.3
+Buncheon-ri	10.3/-12.0	8.6/-14.6	6.9/-15.3	4.6/-16.6

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