Journal of Physics: Conference Series **595** (2015) 012001 doi:10.1088/1742-6596/595/1/012001

On choosing layer profiles in atmospheric tomography

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Abstract.

In control of the AO system of an ELT, most algorithms for atmospheric tomography are based on assuming atmospheric turbulence to occur only within a certain number of horizontal layers. The number of these layers is in general significantly smaller than the number of turbulent layers in the real atmosphere. We investigate the question, how the choice of the number and heights of the hypothetical layers (as parameters of the reconstruction algorithm) influences the resulting quality of the tomographic reconstruction.

1. Introduction

In this article we investigate the influence of a chosen reconstruction layer profile, especially the heights of its layers, on the quality of the solution to the atmospheric tomography problem which arises e.g. during operation of an ELT in the operating mode LTAO. The tests were performed in the context of the project "Mathematical algorithms and software for E-ELT Adaptive Optics". We utilize the Kaczmarz algorithm (see [2]) for tomographic reconstruction and use the simulation tool MOST for evaluation. We will describe a simple *compression* algorithm for generating an appropriate reconstruction layer profile from a given atmosphere model in Section 2 and a simple method for subsequent *optimization* in Section 3. The resulting profiles are compared numerically with respect to the center Strehl ratio in Section 4, finally the importance of informations about the atmosphere is investigated in Section 5 via comparison to 'ignorant' reconstruction profiles that are constructed without any knowledge about the atmosphere model.

The contents of this article are essentially extracted from the author's PhD thesis [1], which is still work in progress.

2. Compression of an atmospheric turbulence profile

Before we can formulate the compression algorithm, we have to define *Voronoi intervals*, which are needed for determining disjoint intervals around given points on an axis: Let $0 \leq h_1 < \cdots < h_n$ be a sequence of ascending real numbers. Then the Voronoi intervals $I_1, \ldots, I_n$ are defined as

$$I_1 := \left[0, \frac{h_1 + h_2}{2}\right]; \forall 1 < k < n : I_k := \left[\frac{h_{k-1} + h_k}{2}, \frac{h_k + h_{k+1}}{2}\right]; I_n := \left[\frac{h_{n-1} + h_n}{2}, 2h_n\right].$$

We now assume an N-layer model of atmospheric turbulence given in form of heights $H_1, \ldots, H_N$ and C_n^2-values $C_1, \ldots, C_N$ (in addition, wind speeds and directions are given, but we do not use

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this information explicitly). In order to derive an appropriate reconstruction layer profile P with a chosen number of layers L, we suggest the following algorithm:

(i) Set the temporary number of layers $\tilde{L} := L$

(ii) Create a geometric sequence $h_2, \ldots, h_{\tilde{L}}$ with $h_2 = \min\{H_n \neq 0\}$ and $h_{\tilde{L}} = H_N$ and add a ground layer $h_1 := 0$.

(iii) For each $k = 1, \ldots, \tilde{L}$, set up a Voronoi interval I_k around h_k and set $c_k := \sum_{n:\, H_n \in I_k} C_n$.

(iv) If the number of values $c_k \neq 0$ is less than L, set $\tilde{L} := \tilde{L} + 1$ and repeat from step (ii).

(v) Select those indices k, for which $c_k \neq 0$, resulting in an L-layer profile P.

Some examples for the compression algorithm can be seen in Figure 1: Starting from a 26-layer model for the atmosphere provided by ESO, the compression algorithm was performed for 6, 10, 15 and 20 layers. Note that in all plots the C_n^2-values are scaled by dividing through the length of the corresponding Voronoi-interval in order to represent something looking more like a continuous function, which makes the similarities more visible.

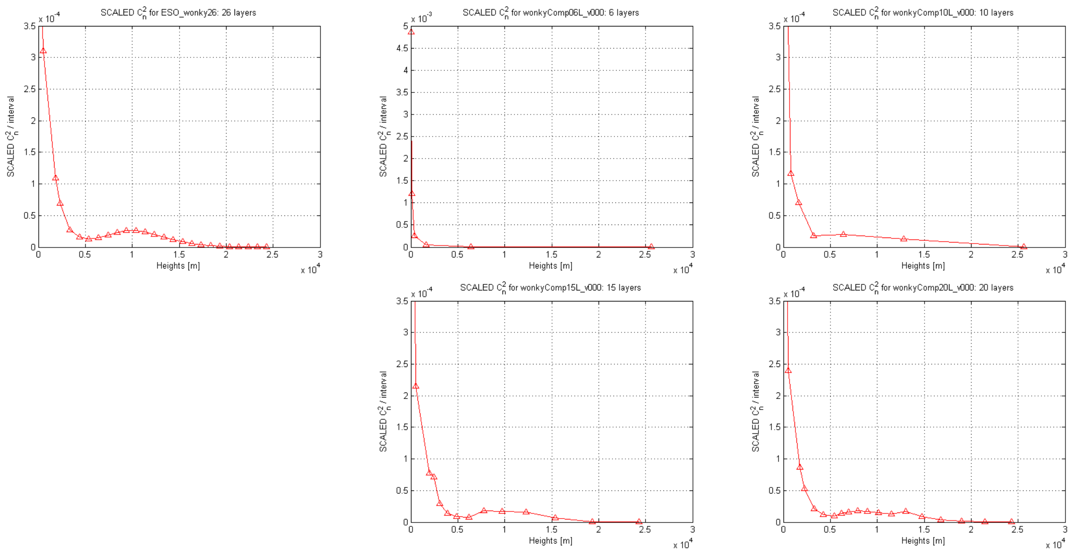

Figure 1. Full atmosphere model (top left) and compressed reconstruction layer profiles for 6, 10, 15 and 20 layers. The absolute values C_n^2 are divided by the lengths of the corresponding Voronoi-intervals.

3. Optimization of reconstruction layer profiles

During test runs in MOST, we used a very simple method for optimizing the profiles gained by compression of the atmosphere model. It is essentially a heuristic line search algorithm along canonical search directions. For application of the following algorithm, in each optimization step the Kaczmarz method was run in parallel for the same atmosphere dynamics, but with different reconstruction layer profiles over several hundreds of system time steps.

(i) For all $k = 2, \ldots, L$, try a new profile with h_k increased by a fixed amount (e.g. 20%) and choose k^* such that variation of h_{k^*} has most significant impact.

(ii) Try several values for h_{k^*}, choose the optimum and assign it to h_{k^*}.

(iii) Optimize one value c_{j^*} analogously to steps (i),(ii), keeping the complete profile scaled to $\sum_{j=1}^{L} c_j = 1$.

(iv) If overall quality could be increased, perform another iteration beginning with step (i).

An example for performing step (i) can be seen in Figure 2: The same 26-layer atmosphere model is used as for Figure 1, and the compressed 6-layer profile as starting value. The curves show the the performance of the compressed profile (blue) and profiles modified by increasing the heights of layers 2 (green), 3 (red), 4 (cyan), 5 (magenta) and 6 (yellow) about 20%. The evolution of the SE center Strehl ratio clearly indicates that variation of the height of layer number 5 (magenta curve) has most direct impact. Hence we examine more variations of this layer in step (ii): h_5 is raised by 10, 20, 30, 40, 60 and 80 %, the results are shown in Figure 3 and suggest to increase h_5 by 50%.

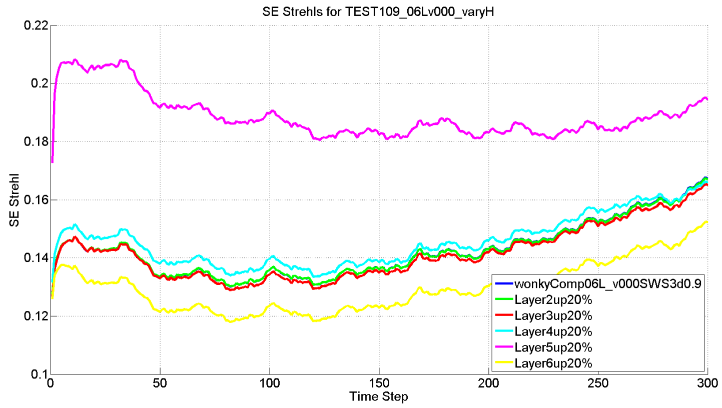

Figure 2. Step (i) of optimization: The 6 layer profile, all heights separately raised by 20% - clearly, raising layer $k^* = 5$ has the largest impact on the reconstruction quality.

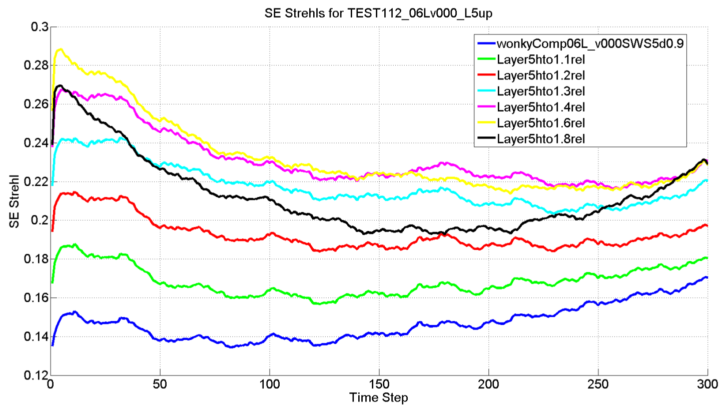

Figure 3. Step (ii) of optimization: Layer 5 is raised by several amounts

Step (iii) does not bring any significant benefit in this situation - in general it seemed throughout all these tests that variation of the heights h_k has much more impact on the quality than variation of the C_n^2-values c_j. However, we show the successive quality gain for the 10 layer profile, where optimization seems finished (settlement in a maximum) after 4 iterations - these can be seen from Figure 4: The blue curve shows the SE Strehl ratio evolution for the compressed 10 layer profile, the successively increasing quality is shown by the green, red, cyan and magenta curves. Quality gained by usage of the full 26 layer profile is shown for comparison, using 2 (yellow) and 5 (black) Kaczmarz iterations.

4. Comparison of the resulting profiles

The results with the 10 layer profiles indicate that it is not possible to reach the quality of the full 26 layer profile by usage of 10 layers. The experiments were repeated with 15 and

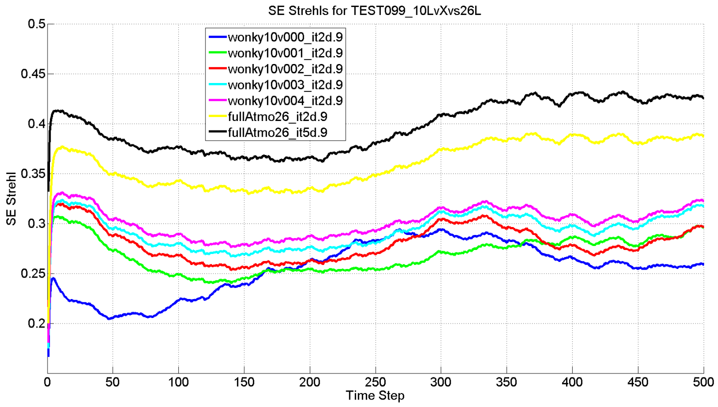

Figure 4. Original compressed 10 layer profile (blue) vs. optimization iterates 1-4. Yellow and black curves: Usage of the full 26-layer profile from the atmosphere model, for 2 and 5 Kaczmarz iterations, respectively.

20 layer profiles - in these cases the optimization routine could not significantly increase the reconstruction quality, i.e., compression alone seems to produce optimal profiles. A summary of all results for the 26 layer atmosphere is shown in Figure 5:

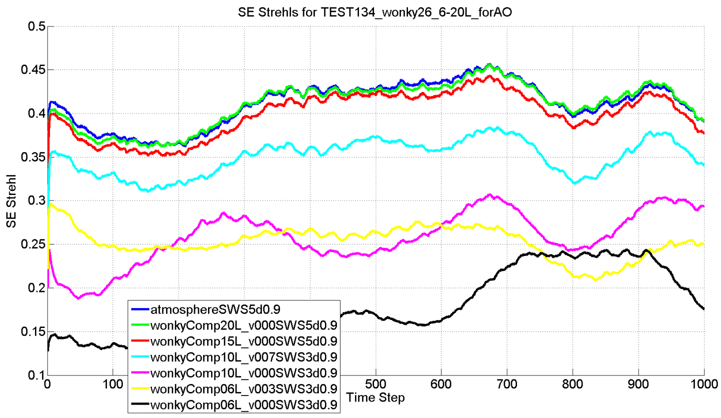

Figure 5. Performance of compressed and optimized profiles for 6, 10, 15 and 20 layers, using a 26 layer atmosphere model

The black curve results from the compressed 6 layer profile, the yellow curve from the same profile optimized by 3 iterations. The magenta and cyan curves result from the 10 layer profiles, original compressed version and after 7 optimization steps, respectively. Center Strehl ratios for the compressed 15 and 20 layer profiles are shown by the red and green curve, the blue curve results from reconstruction on the complete 26 layer profile. The curves give evidence for the following conclusions:

- For a sufficiently large number of reconstruction layers (in our case ≥ 15), usage of the compressed profile is sufficient and no optimization is needed.
- The corresponding quality in that case is not significantly less than reconstruction on the full profile.
- When not enough (in our case ≤ 10) reconstruction layers are applied, the quality of the full 26 layer profile cannot be reached, but still optimization can yield a significant increase of quality (Gain from the black curve to the yellow one for 6 layers, from magenta to cyan for 10 layers).
- For instance, the 6 layer profile after 3 optimization steps yields results comparable to the compressed 10 layer profile without optimization (yellow and magenta curve). Thus, in this case optimization reduces the amount of data to be processed without significant loss of quality.

In the following section we will briefly summarize some more optimization tests based on initial profiles that are constructed without any knowledge of the atmosphere model.

5. Layer optimization without knowledge of the atmosphere

In order to avoid too much computational time, the subsequent tests were performed on smaller amounts of data: We used a 9 layer atmosphere model provided by ESO (shown in Figure 6) and tried 'ignorant' reconstruction profiles with 3, 5 and 7 layers. These reconstruction layer profiles were simply chosen as arithmetic sequences for the heights, $c_1 = 1/2$ and all other c_k equal and such that the sum over all C_n^2-values is 1. Note that this arbitrary setting does not use any information about the atmosphere.

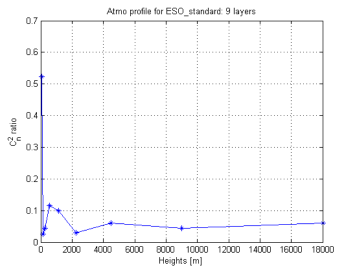

Figure 6. The 9 layer atmosphere profile defined by ESO: absolute values C_n^2 over layer height h.

The results for these profiles and compressed versions, as well as optimized versions of both, can be seen from the plots in Figures 7 and 8, all of them showing the atmosphere model used as reconstruction layer profile as blue curve for comparison:

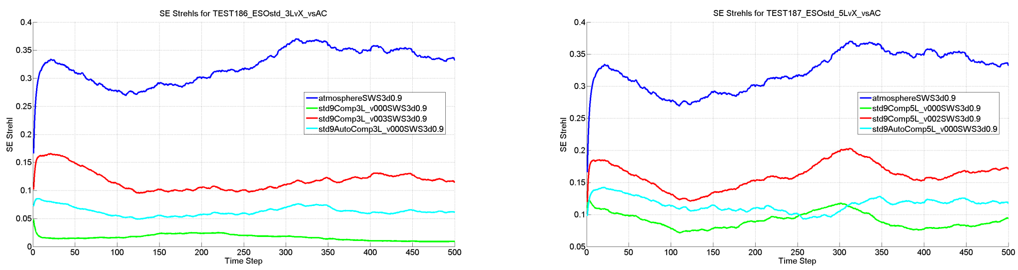

Figure 7. Comparison of different optimization iterates vs. automatic compression for 3 and 5 layers

The two plots in Figure 7 show results for the 3 layer profile (left) and the 5 layer profile (right), respectively. In both cases, the green curves show the center SE Strehl ratio evolution for the 'ignorant' profiles, the red curves result from these profiles after sufficiently many optimization steps (left: 3, right: 2). The cyan curves show the performance of the profiles resulting from compression of the given atmosphere model - in both cases, optimization did not yield any increase of quality. This means that the compressed profiles seem to be stuck in a local optimum and optimization cannot reach the global one, whereas optimization of the 'ignorant' profiles can reach higher quality. Hence, in this cases the profiles generated without any knowledge of the atmosphere turn out to be better suited as starting values for the optimization routine.

Figure 8 shows corresponding tests for the 7 layer profile, where the situation is different: The green curve shows the results from the 'ignorant' profile (i.e. without knowledge of the atmosphere), the red curve after sufficiently many (in this case 2) optimization steps - a slight

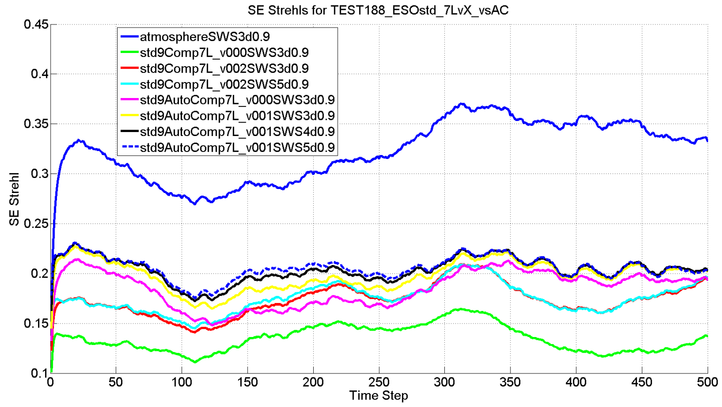

Figure 8. Comparison of different optimization iterates vs. automatic compression for 7 layers

improvement could only be reached by increasing the number of Kaczmarz iterations from 3 to 5 (cyan curve). In contrast to the 3 and 5 layer profiles, the compressed 7 layer profile performs immediately better (magenta curve) and can still be improved by one optimization step (yellow) and subsequently increasing the number of Kaczmarz iterations from 3 to 4 (black) and 5 (dashed blue).

These results indicate the following conclusions:

- For a very small number of reconstruction layers (in our case 3 or 5), there is no guaranty that a reconstruction profile resulting from compression of the atmosphere model is a good starting value for optimization - arbitrarily chosen profiles can perform better.
- For a larger number of reconstruction layers (in our case 7), it definitely pays off to use compression, i.e., to exploit as much information about the atmosphere as possible.
- Although 7 is not much less than 9, performance of the 'best possible' 7 layer profile is significantly worse than usage of the full 9 layer atmosphere model, as is evident from Figure 8 (dashed blue vs. solid blue curve).

This definitely shows that it seems recommendable to use as much reconstruction layers as possible and spend enough time in optimization, exploiting as much information about the atmosphere as possible. Note that this clearly indicates a preference to tomographic reconstruction algorithms which can be run in parallel with respect to the reconstruction layers, as is the case e.g. for the Kaczmarz algorithm.

References

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[2] R. Ramlau and M. Rosensteiner. An efficient solution to the atmospheric turbulence tomography problem using Kaczmarz iteration. *Inverse Problems*, 28(9):095004, 2012.