

Pair production from space- and time-dependent strong fields

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Abstract. The recent development of laser technology and the large number of extreme laser experiments under construction renewed the research related to pair production in strong fields. If the predicted threshold of nonlinear QED is reached, pair production may be observed and the measurements must be compared with appropriate theoretical predictions. However the theoretical side still lacks the understanding of the relevant quantum processes including the effect of laser field parameters on the number and spectrum of created particles. We use the Dirac-Heisenberg-Wigner formalism to investigate the role of the pure electric inhomogeneity on the spectra of created pairs. This simplified model may also be relevant in high energy physics for the description of string fragmentation in the early stages of heavy ion collisions.

1. Introduction

Pair production from vacuum has been long predicted in QED. The first field theoretical calculation by Schwinger established the threshold for the process in constant and homogeneous electric field. Since then it was shown that time dependent field configurations can significantly lower this threshold thus easing the experimental observation. Since then a few other analytical results appeared, but it seems that realistic field configurations can only be addressed by large scale numerical simulations. Specifically the inhomogeneous field configurations only recently become accessible via world-line instanton techniques and Dirac-Heisenberg-Wigner (DHW) formalism. In this paper we use the latter to investigate the role of electric field inhomogeneity on the number of pair produced and the characteristics that can distinguish such a process from a homogeneous one.

In QCD there is an analogous pair production process that happens in heavy ion collisions when the separating quarks create a color flux tube in between each other. As the potential is linearly increasing with the separation distance there is a point when the stored energy exceeds the pair production threshold of a new quark-anti quark pair (or even more composite particles such as diquark-anti diquark pairs) and the separation continues with these new particles. Models based on this idea have proven to be successful in the description of heavy ion spectra but still assume constant and homogeneous fields. As it was shown that the DHW equations have



analogues in QCD and contain the QED case as a special case, our choice of the following toy electric field is not completely arbitrary but motivated by the cross section of a color flux tube in heavy ion collisions.

2. Theoretical Framework

The Dirac-Heisenberg-Wigner formalism provides a relativistic quantum description of pair production in phase space and formulates a Boltzmann-like evolution equation set with one time variable starting from the vacuum initial conditions induced by the prescribed external fields. The derivation for QED is introduced in [1] but we prefer to use the notation of [2]. It is worth noting, that the general equations are challenging to solve because they form a partial differential equation in 7 dimensions, 16 components with non-local differential operators. In the case when only homogeneous but time dependent electric field is considered reduces to a 3 component ordinary differential equation investigated widely in the literature as the Quantum Kinetic equations. For SU(N) theories like QCD the analogous equations are derived in [3] but they have even more complicated structure with more components. However [4] shows that there is a strong Abeilan dominance that justifies the approximation of the more complicated SU(N) models by U(1) pair production.

As we consider QED electron-positron production we solve the 16 component DHW equations. As a well defined departure from homogeneous models we just add a transverse inhomogeneity to the electric field still excluding magnetic fields. To compare the yields in the two cases we need the decomposition of the pair density f in the Quantum Kinetic models. This can be written in terms of the DHW components as: $f = \frac{\epsilon}{4\omega} + \frac{1}{2}$ where ϵ is the phase-space energy density and $\omega = \sqrt{p^2 + m^2}$.

Our choice for the electric field as motivated by the color flux tube cross-section is the following:

$$\vec{E}(x, t) = \frac{\vec{e}_z E_0}{\cosh(t/\tau)^2} \frac{1}{2} \left(1 - \tanh\left(\frac{x+R}{r}\right) \tanh\left(\frac{x-R}{r}\right) \right). \quad (1)$$

It describes an electric field linearly polarized in the z direction but having a transverse plateau-like inhomogeneity in the x direction. The temporal shape is a Sauter pulse that is special because in the homogeneous case it has a known analytical solution for the asymptotic phase space distribution [5], so in the case when $R \gg r$ we expect to reproduce the analytical result near $x = 0$ and can verify the used computational methods.

We solve the DHW equations numerically with 8th order adaptive Runge-Kutta in the time direction and pseudo-spectral collocation methods on Rational Chebyshev basis for the spatial and momentum derivatives [6].

3. Results

On Figure 1. we show the asymptotic pair density f integrated for the spatial coordinate for two different temporal pulse lengths ($\tau = 1\lambda_c/c$ left and $3\lambda_c/c$ right). For short pulses the spectra looks the same as in a homogeneous model but for longer pulses it deviates from the isotropic symmetry and creates particle excess in the backward direction giving the distribution a triangular shape. This deviation may be used as a distinguishing feature in experiments.

The phase-space spectrum can be seen on the on Figure 2. left. While there is the expected bulk pair production from the almost homogeneous field in the spatial range $0 - 4\lambda_c$, a higher amplitude more localized source also manifests at the edge of the plateau field that is attributable to the presence of the large field gradients around that position.

To study the scaling of total number of pairs produced with field parameters it is enough to

integrate the spectra in the coordinate and longitudinal momentum direction :

$$n_L = \int_{-\infty}^{\infty} dx \frac{dp_z}{2\pi} f(x, p_x = 0, p_y = 0, p_z) \quad (2)$$

because the spectra peaks at zero transverse momentum. Figure 2. *right* shows the difference of the homogeneous and inhomogeneous models for different temporal pulse width. Again, we see, that for shorter pulses they are almost identical but for pulses with larger temporal extent more particles are produced in the final state. The field inhomogeneity acts as another channel for pair production that dominates the total yield over $\tau = 1\lambda_c/c$.

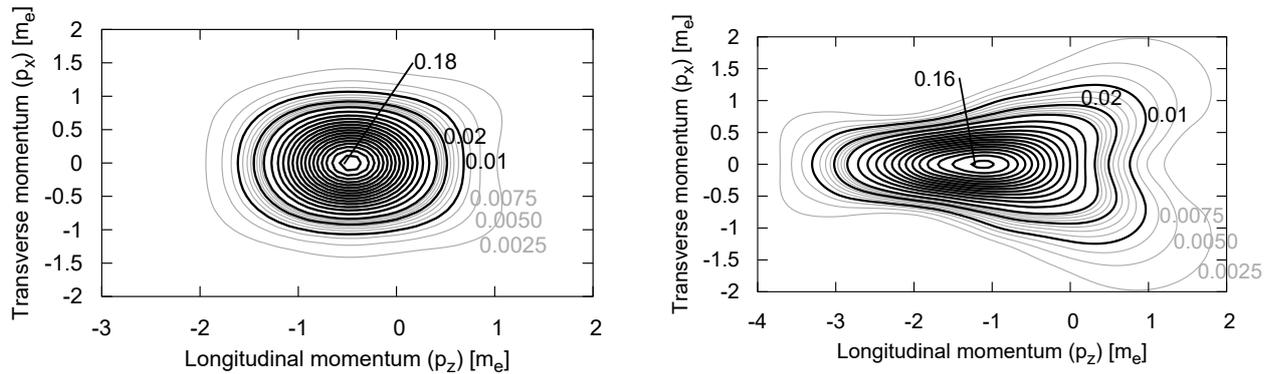


Figure 1. Asymptotic pair density spectra ($f(p_x, p_z)$) integrated for the spatial coordinate x shown for $\tau = 1\lambda_c/c$ left and $3\lambda_c/c$ right. The other field parameters are: $E_0 = 0.75E_{cr}$, $R = 4\lambda_c$, $r = 0.5\lambda_c$.

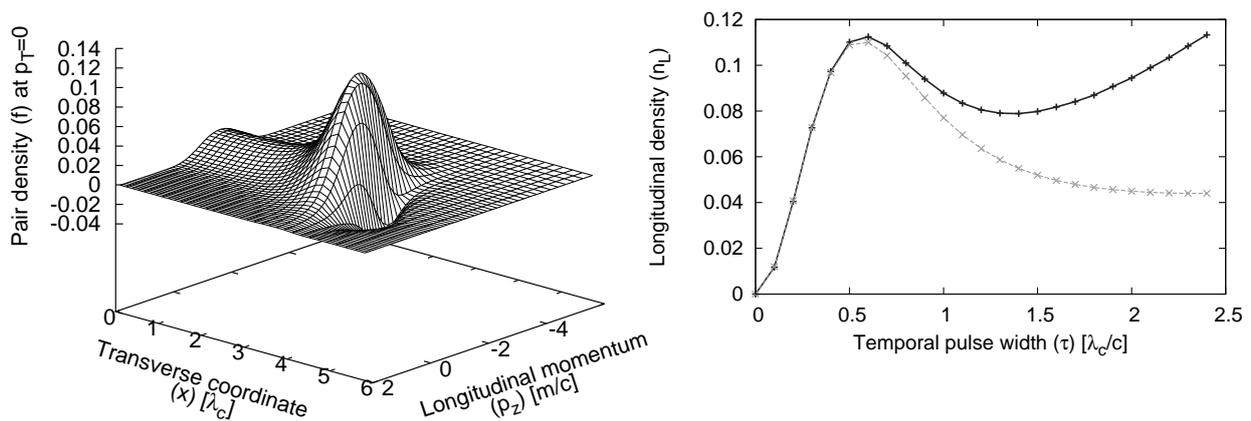


Figure 2. *Left:* phase-space view of the asymptotic pair density with parameters as in Fig. 1. except $\tau = 2\lambda_c/c$. Note that the E field has the steepest gradient at $x = 4\lambda_c$. *Right:* Comparison of the particle yield in the inhomogeneous model (solid black line) with the homogeneous reference (dashed grey) for varying temporal pulse width (τ).

4. Conclusion

We investigated the effect of a transverse spatial electric field inhomogeneity on pair production yields and spectra in the Dirac-Heisenberg-Wigner formalism. This field configuration is an Abelian analogue of the QCD color string but the results are as well relevant to extreme laser collider experiments. We found that close to the Compton scales significant part of pair production is happening near the edges of the field and results in characteristic deviations from the homogeneous configuration that might be detectable by measuring momentum distribution of pairs around the interaction point. More details can be found in [7].

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