Journal of Physics: Conference Series **591** (2015) 012032 doi:10.1088/1742-6596/591/1/012032

Analysis of swarm coefficients in a gas for bi-modal electron energy distribution model

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Abstract

Cross sections for collision between electrons and neutrals in a gas discharge are essential for theoretical and computational developments. They are also required to interpret and analyze the results of experimental studies on swarm parameters namely drift velocity, characteristic energy, and ionization and attachment coefficients. The cross sections and swarm coefficients are interconnected through the most important electron energy distribution function. The traditional method of solving the Boltzmann equation numerically yields the required distribution (EEDF). However there are many situations where a simpler approach is desirable for deriving the energy distribution analytically. Energy distribution in non-uniform electric fields, in crossed electric and magnetic fields, breakdown in mixtures of gases for electrical power or plasma applications, calculation of longitudinal diffusion coefficients are examples.

In other studies the swarm parameters are employed to derive the cross sections in an unfolding procedure that also involves the energy distribution function. Application of Boltzmann solution method, though more rigorous, consumes enormous efforts in time and technical expertise. In an attempt to provide a simpler method the present author has previously suggested a bimodal electron energy distribution in gases. In this paper the author has generalized the idea of bi-modal energy distribution by considering a model gas with representative cross sections and adopted numerical methods for greater accuracy. The parameters considered are the nature of the two distributions, their relative ratio, and the dependence of cross sections on electron energy. A new method for determining the combination of distributions has been shown to be adequate for calculation of swarm parameters. The results for argon are shown to yield very good agreement with available experimental and theoretical values.

Introduction

Interaction between electrons and neutrals in a gas discharge is studied by swarm methods in which the average properties such as drift velocity, diffusion coefficient, ionization and attachment coefficient are measured experimentally. The controlling parameter in these experiments is the reduced electric field, E/N, in which E is the electric field and N is the gas number density. Alternately, various collision cross sections are measured using mono-energetic electrons and the controlling parameter is the electron energy. The beam experiments and the swarm experiments are interlinked by the electron distribution function. Given the collision cross sections as a function of electron energy the swarm coefficients may be calculated at various values of E/N using the electron energy distribution function.

In other studies, the results of beam measurements are employed to calculate the swarm parameters by an unfolding procedure which involves calculation of electron energy distribution functions. Historically, this approach was developed by Frost and Phelps et al. [1] to obtain momentum transfer cross sections at low energies in H_2.

It is appropriate to mention here that swarm properties at low values of E/N (≤ 10 Td, 1 Td = 10^{-21} Vm^2) are relatively easier to measure than cross sections at low electron energies (≤ 1 eV) though the experimental techniques for cross section measurements employing low energy electrons have been mastered since the pioneering work of Phelps and his group [2]. The interrelation ship between the beam measurements, swarm measurements and electron energy distribution is well known. Substitution of the determined electron energy distribution into integral equations, given below, yields the respective swarm quantity. A large number of publications in the literature show very good correspondence between the theoretically calculated swarm parameters, starting from cross sections and experimentally determined values [3]. However, the unfolding procedures for deriving the cross sections from measured swarm parameters are not quite satisfactory.

The motivation for the present work is to examine whether a simpler approach, to the elaborate Boltzmann equation method that requires a specialization on its own, can be devised for the calculation of the four quantities shown in equations (2)-(5). Such an approach would be beneficial where one wishes to use the results of simpler calculations rather than examine detailed collision processes. The present author has previously suggested a bimodal electron energy distribution and demonstrated that accurate values of swarm parameters could be calculated by this method.

Theory

The electron distribution function, F(ε) is calculated using the Monte Carlo procedure or numerically solving the Boltzmann equation.

The swarm parameters are defined by the following relationships:

$$W = -\frac{E}{3N}\left(\frac{2e}{m}\right)^{1/2}\int_0^\infty \frac{\varepsilon}{Q_M(\varepsilon)}\frac{d}{d\varepsilon}\left[\frac{F(\varepsilon}{\varepsilon^{1/2}}\right]d\varepsilon \tag{1}$$

$$D_T = \frac{1}{3N}\int_0^\infty \frac{\varepsilon}{Q_M(\varepsilon)}F(\varepsilon)d\varepsilon \tag{2}$$

$$\frac{\alpha}{N} = \frac{1}{W}\int_{\varepsilon_i}^\infty \varepsilon^{1/2}Q_i(\varepsilon)F(\varepsilon)d\varepsilon \tag{3}$$

And in general [2],

$$k_j = \left(\frac{2}{m}\right)^{1/2}\int_0^\infty \varepsilon Q_{M(j)}(\varepsilon)F(\varepsilon)d\varepsilon \tag{4}$$

In equations (1)-(4) the symbols are defined as follows:

D_T = radial diffusion coefficient (m^2/s); e/m = electron charge to mass ratio; *j* = specific inelastic process, k_j = rate constant for j^{th} process; (m^3/s); Q_i = ionization cross section (m^2); Q_M = momentum transfer cross section (m^2); W = drift velocity (m/s); ε = electron energy (eV), ε_i = ionization threshold energy; ε_j = threshold energy for j^{th} process. Substitution of determined electron energy distribution into equations (1) to (4) yields the respective quantity.

Assuming that the cross section is independent of energy Morse, Allis and Lamar [4] derived an analytical equation to the electron energy distribution according to:

$$F(\varepsilon) = G\frac{\varepsilon^{1/2}}{\bar{\varepsilon}^{3/2}}\exp\left[-H\left(\frac{\varepsilon}{\bar{\varepsilon}}\right)^{2(p+1)}\right] \tag{5}$$

where $\bar{\varepsilon}$ is the mean energy, p is a constant having value of -1 < p and no upper limit seems to exist though at large values the distribution tends to be so small that it ceases to have any practical significance. G and H are functions given by:

$$G = 2(p+1)\left[\Gamma\frac{5}{4(p+1)}\right]^{3/2} \times \left[\Gamma\frac{3}{4(p+1)}\right]^{-5/2} \tag{6}$$

$$H = \left[\Gamma\frac{5}{4(p+1)} \times \frac{1}{\Gamma\dfrac{3}{4(p+1)}}\right]^{2(p+1)} \tag{7}$$

There is now a renewed interest in the problem because of the fact that gas discharges are being used in a number of different areas involving specialists from various disciplines, for example plasma medicine. Towards this end a bimodal energy distribution is proposed, expressed as,

$$G(\varepsilon) = (1-\gamma)F_1(\varepsilon) + \gamma F_2(\varepsilon) \tag{8}$$

Where γ is a constant, $0 \le \gamma \le 1$, F_1 and F_2 are individual distributions respectively and G is the combined distribution, which is normalized according to

$$\int_0^\infty F(\varepsilon)\,d\varepsilon = 1 \tag{9}$$

In this paper the author has generalized the idea of bi-modal energy distribution by considering a model gas with representative cross sections and adopted numerical methods to calculate the swarm parameters using equations (1)−(3) for greater accuracy. The parameters considered are the nature of the two distributions, their relative ratio, and the dependence of cross section on electron energy. A new method for determining the combination of distributions has been shown. The results for argon are shown to yield excellent agreement without the need for scaling factors for cross sections.

Results for argon and discussion

Application of the theory to argon gives extremely satisfactory results for as shown in Figures 1−2 for W, D (Not shown) and the first ionization coefficient α/N respectively. Comparisons with published data for W, D and E/N are very satisfactory.

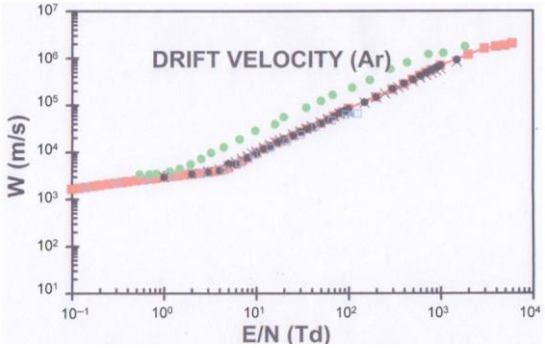

Figure 1. Drift velocity as a function of E/N. (Δ) Kücükarpaci and Lucas [3]; (□) Nakamura and Kurachi [3]; both these sets of symbols are obscured by other points; (●) Lisovskiy et al. [3]; (■) Raju [2, 3]; (×) Boltzmann method; (●) This paper (2014).

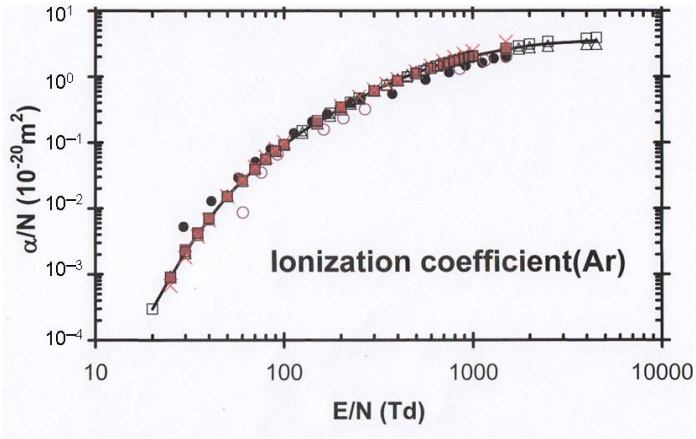

Figure 2. Density reduced ionization coefficients as a function of E/N for argon. (□) Kruithoff [3]-Experimental; (○) Davies and Milne [3]-Experimental; (●) Lakshminarasimha and Lucas [3]-Experimental; (Δ) Phelps and Petrović [3]-analytical expression; (——) Raju [3]; (×) Boltzmann code [5]; (■) This paper (2014)-Theory.

Conclusions

A bi-modal distribution method for calculating the electron energy distribution has been developed and shown to yield extremely satisfactory results for argon. The method is much simpler and does not require detailed knowledge of collision processes. The semi-empirical energy distribution function is independent of the momentum transfer cross section though its influence is reflected through the mean energy. An interactive spread sheet gives the results with no need for an iterative procedure and it is expected that the method will be tested for other gases to verify its general applicability. The EEDF can be expressed analytically which is a major advantage in many applications.

Acknowledgement

The results in the paper for Boltzmann distribution are obtained by using the freeware www.Bolsig. Com.

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