

The effects of Double Folding Cluster Model Potential on some astrophysical reactions

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Abstract. The Double Folding Cluster Model Potential is constructed using the α - cluster structure of nuclei. It can be derived by folding an $\alpha - \alpha$ interaction with density distributions of α - clusters inside the projectile and target nuclei. This potential has been successfully tested on elastic scattering data of some selected nuclei. In this work, we are interested to investigate the implications of this potential on astrophysical aspects.

1. Introduction

Challenges in determining the accurate nuclear potential motivate the effort in developing various potentials. The most widely used potential is the Woods-Saxon shaped potential given by [1]

$$V_N(r) = \frac{-V_0}{1 + \exp[(r - R_0)/a_0]} + i \frac{-W_0}{1 + \exp[(r - R_W)/a_W]} \quad (1)$$

where the radii $R_0 = r_0(A_P^{1/3} + A_T^{1/3})$ and $R_W = r_W(A_P^{1/3} + A_T^{1/3})$ for the nuclear interaction between a projectile and target nuclei with the mass number A_P and A_T respectively, within their distance of closest approach, r . The Akyüz-Winther (A-W) parameterization [1] is often used to determine the parameter for the real part, i.e. the surface diffuseness parameter, a_0 , the potential depth, V_0 and the radius parameter, r_0 . The double folding potential has become one of the popular microscopic models to calculate the real part of the optical potential. Apart from that, there is also the Double Folding Cluster (DFC) model potential which is based on the $\alpha - \alpha$ interaction folded with the density distributions of α - clusters inside the projectile and target nuclei introduced by Azab et al. [2]. For this potential, the nuclei are assumed to be composed of an integer number m of α particles, i.e., $A = 4m$.

Several studies have shown that the DFC potential has successfully reproduced the differential cross-section of elastic scattering data for a few reactions [2, 3, 4, 5]. Meanwhile, this potential could describe the broad features of the fusion, S-factor and elastic-scattering angular data



simultaneously as reported by Kocak et al. [6]. Based on these reports, we are interested to further the study on this type of potential on the $^{12}\text{C}+^{12}\text{C}$, $^{12}\text{C}+^{16}\text{O}$ and $^{16}\text{O}+^{16}\text{O}$ reactions. Since these three reactions are important in stellar evolution and nucleosynthesis, this investigation emphasizes on the effects of this potential on astrophysical aspects.

2. Calculation

We present here a short description of the main formula used to calculate the DFC model potential while the detailed description can be found in Azab et al [2]. The DFC can be formulated as an effective $\alpha - \alpha$ interaction, $v_{\alpha\alpha}$ folded with the α - cluster distributions ρ_{cP} and ρ_{cT} for the projectile and target nuclei respectively

$$V_{DFC}(\vec{r}) = \int \int \rho_{cP}(\vec{r}_P) \rho_{cT}(\vec{r}_T) v_{\alpha\alpha}(\vec{s}) d\vec{r}_T d\vec{r}_P. \quad (2)$$

The vector $\vec{s} = |\vec{R} + \vec{r}_T - \vec{r}_P|$ while the $\alpha - \alpha$ potential given by $v_{\alpha\alpha} = -122.6225 \exp(-0.22r^2)$ is taken from Buck et al. [7].

If ρ_c is the α - cluster distribution function inside the nucleus, then the nuclear matter density distribution function of the nucleus, ρ_M can be related to that of the α -particle nucleus, ρ_α as

$$\rho_M(\vec{r}) = \int \rho_c(\vec{r}') \rho_\alpha(|\vec{r} - \vec{r}'|) d\vec{r}'. \quad (3)$$

The matter density distribution of both projectile and target nuclei which can be written in a modified form of the Gaussian shape and the corresponding α density is given by [2]

$$\rho_M(\vec{r}) = \rho_{0M}(1 + \omega r^2) \exp(-\beta r^2) \quad (4)$$

$$\rho_\alpha(\vec{r}) = \rho_{0\alpha} \exp(-\lambda r^2). \quad (5)$$

From Eq.(4) and Eq.(5), the α -cluster distribution function ρ_c can be obtained by using the Fourier transform [8], on Eq.(3) as

$$\rho_c(\vec{r}') = \rho_{0c}(1 + \mu r'^2) \exp(-\xi r'^2) \quad (6)$$

where

$$\eta = \lambda - \beta, \quad \xi = \beta\lambda/\eta, \quad \mu = \frac{2\omega\lambda^2}{\eta(2\eta - 3\omega)} \quad (7)$$

while ρ_{0M} , $\rho_{0\alpha}$ and ρ_{0c} can be obtained from the normalization condition. The parameters used in this work for all nuclei involved are given in Table 1.

Table 1: The nuclear density parameters.

Nucleus	ω (fm ⁻²)	$\beta(\lambda)$ (fm ⁻²)	$\langle r^2 \rangle^{1/2}$ (fm)	Ref.
^4He	0	0.7024	1.461	[8]
^{12}C	0.4988	0.3741	2.407	[9]
^{16}O	0.6457	0.3228	2.640	[9]

The DFC potential is then used to replace the real part in the nuclear potential, $V_N(r)$ to calculate the fusion cross section

$$V_N(r) = N_R V_{DFC(M)}(r) + i \frac{-W_0}{1 + \exp[(r - R_W)/a_W]} \quad (8)$$

with N_R is the normalization constant that can be varied to fit the experimental data. The value $W_0 = 50$ MeV, $r_W = 1.0$ fm, and $a_W = 0.4$ fm are chosen for the imaginary potential parameters to make sure that any differences arise only from the real part of the potential. We only consider the average description of the data by neglecting the resonant oscillation since the main purpose of this work is not to investigate the fusion oscillation especially in the $^{12}\text{C} + ^{12}\text{C}$ and $^{12}\text{C} + ^{16}\text{O}$ reactions.

3. Results

The fusion cross sections for $^{12}\text{C} + ^{12}\text{C}$, $^{12}\text{C} + ^{16}\text{O}$ and $^{16}\text{O} + ^{16}\text{O}$ reactions have been calculated using the DFC potential. The normalization constant can be adjusted to obtain the best fitting by minimizing the chi-square of the fitting. The results are compared to the A-W potential and tabulated in Table 2. The results show that the DFC potential could produce a fitting as good as the A-W potential or even better.

Table 2: The normalization constant, N_R and the value of chi-square χ^2 of the fitting for A-W and DFC potentials.

Reaction	Potential	N_R	χ^2
$^{12}\text{C} + ^{12}\text{C}$	A-W	-	54.17
	DFC	1.00	42.83
		1.17	59.41
$^{12}\text{C} + ^{16}\text{O}$	A-W	-	2.03
	DFC	1.00	2.52
		1.09	2.00
$^{16}\text{O} + ^{16}\text{O}$	A-W	-	1.63
	DFC	1.00	6.98
		1.35	1.42

The modified astrophysical S-factor can be defined as $S^*(E) = \sigma(E)E \exp(2\pi\eta - gE)$ where $2\pi\eta = 0.9896Z_1Z_2(\mu/E)^{1/2}$ and $g = 0.46\text{MeV}^{-1}$ [10]. The $S^*(E)$ with $\sigma(E)$ from the DFC potential is compared to the one calculated using A-W potential and depicted in Fig. 1 for $^{12}\text{C} + ^{12}\text{C}$ reaction. The fusion cross section obtained in this calculation is then used to calculate the thermonuclear reaction rates using the standard formalism [10]

$$\langle\sigma v\rangle = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E)E \exp\left(-\frac{E}{kT}\right) dE. \quad (9)$$

The ratio of the reaction rates obtained from this work to the rates from Caughlan and Fowler's compilation 1988 (CF88) [11] is then calculated and depicted in Fig. 2. It is shown that the reaction rates obtained from the DFC potential seems lower than the CF88 at very low temperature.

The temperature that is typical of core carbon burning govern by the $^{12}\text{C} + ^{12}\text{C}$ reaction is within the location of the Gamow peak which is $T \approx 0.85$ GK. For the oxygen burning where the $^{16}\text{O} + ^{16}\text{O}$ fusion reaction occurs, the typical temperature is in the range of $T = 1.5 - 2.7$ GK, depending on the stellar mass. Although the rates predicted from the DFC potential give large differences at very low temperature, the ratio of the rates is approaching unity at least in the range of the Gamow temperature where the reactions mostly occur.

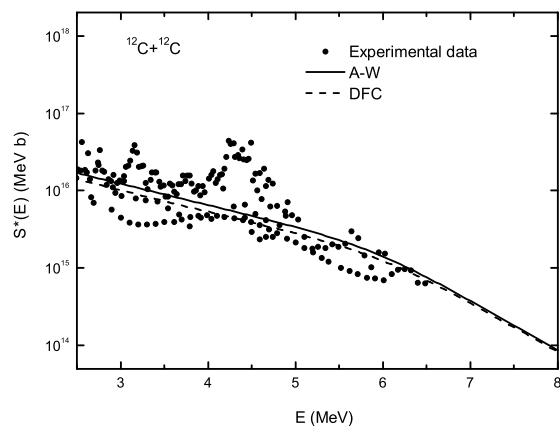


Figure 1: The $S^*(E)$ calculated using $\sigma(E)$ from the DFC potential and the A-W potential for $^{12}\text{C}+^{12}\text{C}$ reaction. Experimental data are from [12].

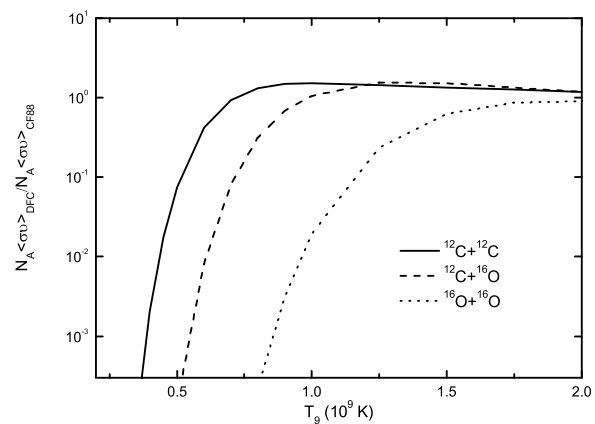


Figure 2: The ratio of the reaction rates calculated using the DFC potential to the CF88 rates for $^{12}\text{C}+^{12}\text{C}$, $^{12}\text{C}+^{16}\text{O}$ and $^{16}\text{O}+^{16}\text{O}$ reactions.

4. Conclusion

The DFC potential can be an alternative potential since it could reproduce the experimental fusion cross section data very well. However, the prediction on the astrophysical aspects gives some significant differences. Hence, further investigation on this potential is required.

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