The influence of nuclear deformations on the exotic cluster decay half-lives

A. Soylu†, O. Bayrak††, S. Evlice†

†Department of Physics, Nigde University, 51240, Nigde, Turkey
††Department of Physics, Akdeniz University, 07058, Antalya, Turkey

Abstract. We systematically study the investigation of the influence of nuclear deformations of the cluster and daughter nuclei on the exotic cluster decay half-lives of heavy nuclei by the WKB method and the Bohr-Sommerfeld quantization condition. Even if the deformations of both cluster and daughter in the half-live values of cluster decays improve the results, considering the deformation of clusters is more efficient than the deformation of daughter for the heavy cluster decay half-live calculations. Moreover, taking into account of angle orientations of daughter and cluster provides a positive contributions to the results as well. The results would be useful for experimental researches in half-lives of exotic decays of some heavy nuclei and radium isotopes.

1. Introduction

After Rose and Jones determined the phenomenon of cluster-radioactivity experimentally [1], a new study field in nuclear physics was opened. Rose and Jones have found out that ^{223}Ra emits a ^{14}C cluster and their experimental results have been confirmed by Gales and co-workers [2] as well as by Price *et al.* [3]. On the theoretical side, Sandulescu *et al.* made the first theoretical description of the cluster decay [4]. The quantum mechanical tunneling calculations may be done either by fission theory to a larger mass asymmetry or by heavier cluster emitted particle from alpha decay theory. Numerous studies have been performed by using different methods and many papers have been published in order to be able to obtain the half-lives of these exotic decay processes [5–13]. The studies of radium isotopes in the field of exotic cluster decays are also important because their position in the trans-lead region in the nuclear chart and its immediate neighbors as one of the decay fragments comes into play when the cluster emission occurs. Therefore, the systematic study on cluster radioactivity from the $^{210-226}$Ra isotopes was done for the first time and $^{210-226}$Ra isotopes were all alpha instable thereby exhibiting alpha radioactivity [14–16]. In order to investigate the influence of the deformations of half lives of exotic decay clusters, some studies have also been done in the literature [17–19].

In this talk, we present the results of the investigation of the influence of nuclear deformations of the cluster and daughter nuclei on the exotic cluster decay half-lives of the nuclei having the mass numbers between 221 and 242 with Woods-Saxon square potential by using WKB and the Bohr-Sommerfeld quantization condition [18]. Furthermore, we have applied the model in order to get the half-lives of cluster decays of $^{210-226}$Ra which are recent interesting topic [20].

2. Model

When we take into account the quadrupole and hexadecapole deformations of both the cluster and the daughter nuclei having the deformations, the effective interaction potential between the daughter and the cluster nuclei can be given the following

$$V_{eff}(r, \theta_c, \theta_d) = V_N(r, \theta_c, \theta_d) + V_C(r, \theta_c, \theta_d) + V_L(r), \tag{1}$$

where $V_N(r, \theta_c, \theta_d)$ is the nuclear potential, $V_C(r, \theta_c, \theta_d)$ is the Coulomb potential and $V_L(r)$ is the centrifugal potential. And, θ_c and θ_d represent the orientation angles of cluster and daughter nuclei, respectively. The nuclear radius is deformed as

$$R(\theta_c, \theta_d) = r_0 A_c^{1/3}(1 + \beta_2^c Y_{20}(\theta_c) + \beta_4^c Y_{40}(\theta_c)) + r_0 A_d^{1/3}(1 + \beta_2^d Y_{20}(\theta_d) + \beta_4^d Y_{40}(\theta_d)), \tag{2}$$

where A_c, β_2^c, β_4^c and $Y_{lm}(\theta_c)$ are the atomic mass, quadrupole and hexadecapole deformation parameters and the spherical harmonics of the cluster nuclei and A_d, β_2^d, β_4^d and $Y_{lm}(\theta_d)$ are the atomic mass, quadrupole and hexadecapole deformation parameters and the spherical harmonics of the daughter, respectively. The repulsive Coulomb potential can be used in the deformed form [21], and the last term in Eq.(1) is Langer modified centrifugal barrier potential. As we have only focused on the favored cluster decay of the radioactive nuclei, $L = 0$ is used in the calculations. As we know the forms of Coulomb and centrifugal potential very well, the only one unknown term in the effective potential in Eq.(1) is the nuclear interaction potential. In order to find out the influence of deformations on the system effectively, we use the deformed Woods-Saxon square potential as follows

$$V_N(r, \theta_c, \theta_d) = -\lambda(\theta_c, \theta_d) \frac{V_0}{\left[1 + exp(\frac{r - R(\theta_c, \theta_d)}{a})\right]^2}, \tag{3}$$

where $\lambda(\theta_c, \theta_d)$, V_0, $R(\theta_c, \theta_d)$ and a are normalization parameter determined by Bohr-Sommerfeld quantization condition, depth of the nuclear potentials, nuclear radius and diffusion parameters, respectively. The detailed information about the calculations can be found in Ref. [18]. Turning points are obtained by the numerical solution of the equation $V_{eff}(r, \theta_c, \theta_d) = Q$. The cluster decay width is given by [22],

$$\Gamma = PF \frac{\hbar^2}{4\mu} S, \tag{4}$$

where P is the preformation factor, F is the normalization factor and S is the transition probability of cluster nuclei, respectively. The detailed information about the calculations can be found in Ref. [18]. Then the half life of the cluster decay is calculated by [23, 24]

$$T_{1/2} = \hbar \frac{\ln 2}{\Gamma}. \tag{5}$$

3. The Results

We have calculated the heavy-cluster half-lives of the nuclei having the mass numbers $221 \leq$ A ≤ 242 with WKB method and Bohr-Sommerfeld quantization condition [18]. Comparison between the experimental results and the calculated heavy decay half-lives $\log_{10} T_{1/2}$ for ^{14}C decay from various parent nuclei for $0^0 - 0^0$ can be shown in Fig. 1. Moreover, the obtained and experimental values of half-lives with deformation of cluster and daughter nuclei for 0^0-0^0, 0^0-30^0, 0^0-60^0, 0^0-90^0 and over all orientation angles for deformation parameters are seen in Fig. 2. It should be noted that the deformation parameters of the cluster and daughter nuclei which have deformations are taken from refs. [23, 25].
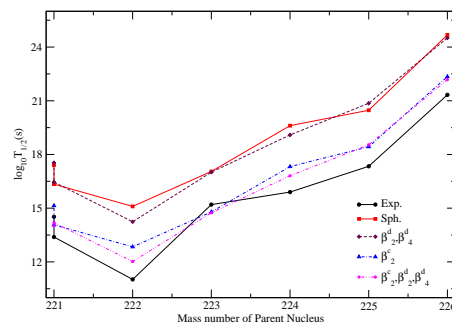
Figure 1. Comparison between the experimental results and the calculated heavy decay half-lives $\log_{10} T_{1/2}$ for ^{14}C decay from various parent nuclei for $0^0 - 0^0$ [18].
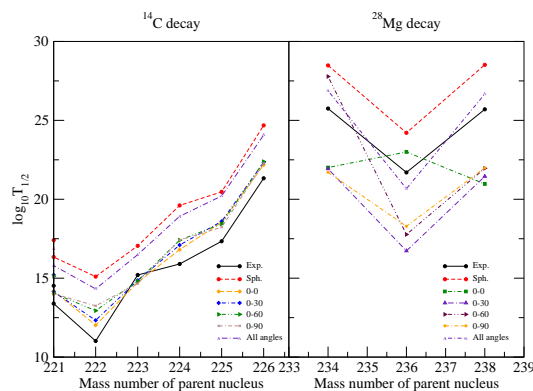

Figure 2. Comparison between the experimental results and the calculated heavy decay half-lives $\log_{10} T_{1/2}$ for ^{14}C and ^{28}Mg decays from various parent nuclei for $0^0 - 0^0$, $0^0 - 30^0$, $0^0 - 60^0$, $0^0 - 90^0$ orientation angles and over all angles [18].

We have also calculated the half-lives of ^{4}He, ^{8}Be, ^{12}C, ^{14}C, ^{16}O, ^{18}O, ^{20}O, ^{22}O cluster decays of $^{210-226}$Ra in order to address whether there is any effect of the deformations on the heavy-cluster half-lives of radium isotopes [20]. The $\log_{10} T_{1/2}$ with Woods-Saxon square and experimental values for **a)** spherical case (sph), **b)** only daughter deformation case **c)** only cluster deformation case **d)** with both daughter and cluster deformations case for ^{4}He, ^{8}Be, ^{12}C, ^{14}C, ^{16}O, ^{18}O, ^{20}O, ^{22}O cluster decays of $^{210-226}$Ra are seen in Fig 3. As seen in these figures, while in the spherical case there are large differences between experimental values and the calculated values for ^{14}C decays, taking into account the deformation effects decreases these differences. In other words, considering such kind of deformation effect improves the results effectively and it makes them closer to the experimental values.

4. Conclusion

Even if the deformations of both cluster and daughter in the half-live values of exotic cluster decays improve the results, considering the deformation of clusters is more efficient than the deformations of daughter in the heavy cluster decay half-live calculations for heavy nuclei and especially for radium isotopes. Considering the orientations of daughter and cluster in the half-lives calculations improves the results clearly so it would be important to take into account such kind of effects in the theoretical calculations. Searching dominant orientations in the half-lives calculations and finding out the orientation angle is the most compatible one with experimental data would be important to explain the observable in the theoretical models. The model would
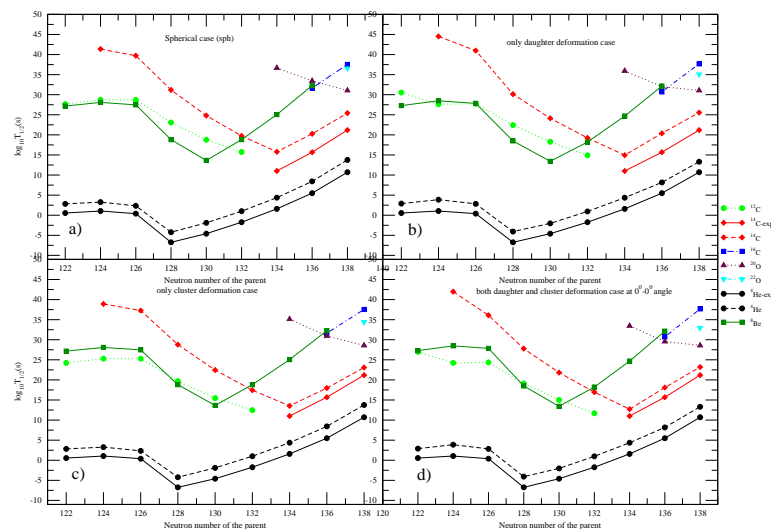
Figure 3. The calculated $\log_{10} T_{1/2}$ versus neutron number of the parent nuclei with Woods-Saxon square and experimental values for **a)** Spherical case (sph), **b)** only daughter deformation case, **c)** only cluster deformation case, **d)** both daughter and cluster deformation case for $0^0 - 0^0$ angles for ^{4}He, ^{8}Be, ^{12}C, ^{14}C, ^{16}O, ^{18}O, ^{20}O, ^{22}O cluster decays of $^{210-226}$Ra [20].

be applied to make some predictions on the unknown experimental half lives of possible cluster decays of Bismuth, Cerium, Barium isotopes as future studies.

5. Acknowledgments

[1] Rose H J, Jones G A 1984 *Nature* **307** 245.
[2] Gales S, Hourani E, Hussonnois M, Schapira J P, Stab L, Vergnes M 1984 *Phys. Rev. Lett.* **53** 759.
[3] Price P B, Stevenson J D, Barwick S W, Ravn H L 1985 *Phys. Rev. Lett.* **54** 297.
[4] Sandulescu A, Poenaru D N, Greiner W 1980 *Sov. J. Part. Nucl.* **11** 528.
[5] Sheng Z, Ni D, Ren Z 2011 *J. Phys. G: Nucl. Part. Phys.* **38** 055103.
[6] Buck B, Merchant A C 1989 *J. Phys. G: Nucl. Part. Phys.* **15** 615.
[7] Ibrahim T T, Perez S M, Wyngaardt S M, Buck B, Merchant A C 2012 *Phys. Rev. C* **85** 044313.
[8] Poenaru D N, Gherghescu R A, Greiner W 2011 *Phys. Rev. Lett.* **107** 062503.
[9] Ren Z, Xu C, Wang Z 2004 *Phys. Rev. C* **70** 034304.
[10] Buck B, Merchant A C, Perez S M 1999 *Nucl. Phys. A* **657** 267.
[11] Santhosh K P, Joseph A 2000 *Pramana J. Phys.* **55** 375.
[12] Santhosh K P, Joseph A 2000 *Proc. Int. Nat. Symp. Nucl. Phys. (India)* **43B** 296.
[13] Santhosh K P, Biju R K, Joseph A 2008 *J. Phys. G: Nucl. Part. Phys.* **35**, 085102.
[14] Santhosh K P *et al.* 2009 *Proc. Nat. Symp. Nucl. Phys. (India)* **1** 33.
[15] Santhosh K P *et al.* 2011 *Prc. DAE Symp. Nucl. Phys. (India)* **56** 298.
[16] Santhosh K P, Sahadevan S, Priyanka B, Unnikrishnan M S 2012 *Nuclear Physics A* **882** 49.
[17] Sawhney G and Sharma M K, Gupta R K 2011 *Phys. Rev. C* **83** 064610.
[18] Soylu A, Sert Y, Bayrak O, and Boztosun I 2012 *Eur. Phys. J. A* **48** 128.
[19] Santhosh K P, Joseph A 2002 *Pramana, Indian Academy of Sciences* Vol. **59** 679.
[20] Soylu A, and Evlice S 2014 Deformations Effects on Exotic Decays of Radium Isotopes unpublished.
[21] Brink D M and Takigawa N 1977 *Nucl. Phys. A* **279** 159.
[22] Buck B, Merchant A C, and Perez S M 1994 *Phys. Rev. Lett.* **72** 1326.
[23] Denisov V Y and Khudenko A A 2009 *At. Data Nucl. Data Tables* **95** 815.
[24] Xu C and Ren Z 2006 *Phys. Rev. C* **74** 014304.
[25] Belgya T, *et al.* 2006 RIPL-2. IAEA-TECDOC-1506 (IAEA, Vienna, 2006). Available online at http://www-nds.iaea.org/RIPL-2/.