

# Analysis of Low Lying Collective States in the Symplectic Interacting Vector Bosons Model

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**Abstract.** Successful description of both positive and negative parity band configurations and their corresponding band-head's structure is obtained within the framework of the algebraic Interacting Vector Boson Model (IVBM). The analysis of the collective structure of the low lying excited states possessing different parity is carried out together with the results obtained from parabolic distributions of the energies in the space of integer and positive classification parameter (supposedly number of bosons) for large amount of even-even nuclei.

keywords: symplectic boson model, collective bands, energy distribution of states

## 1. Introduction

Contemporary experimental techniques have advanced to the place where information on large sequences of collective states with  $L = 0, 2, 4, 6, \dots$  in a given nucleus is now very rich, particularly in the rare-earth and actinide regions [1]. It is a challenge for the theoretical models to achieve a correct interpretation of the data since the observed collective states should be well organized in bands, revealing their structure and origin. Algebraic models play an important role in this respect by relating the structure of the collective states to basis states of a dynamical symmetry group and its subgroup chains. In this regard, the symplectic model provides a natural and very general framework for investigating the nature of collective excitations in many-body systems [2]. The symplectic model incorporates "elementary" excitations into the structure of the collective states. We exploit a phenomenological Interacting Vector Boson Model (IVBM) that yields a rather accurate description of the low-lying spectra of even-even heavy nuclei [3]. The spectrum generating algebra of the model is that of the  $Sp(12, R)$  group, which has a very rich subgroup structure [3, 4].

Our aim is to study the behavior of the energies of sequences of collective states with fixed angular momentum  $L$  in the spectra of a given even-even nucleus in conjunction with the ordering of these states in collective bands.

This is achieved by employing the algebraic relations between two of the dynamical symmetries of the IVBM, each corresponding to the respective physical interpretation. This gives us from one side possibility to understand the structure of each collective state and from the other side to predict some new unobserved states through their classification.



## 2. Algebraic framework of the symplectic IVBM

We consider  $Sp(12, R)$  – the group of linear canonical transformation in a 12-dimensional phase space – to be the dynamical symmetry group of the model [3, 5]. Its algebra is realized in terms of creation (annihilation) operators  $u_m^+(\alpha)(u_m(\alpha) = (u_m^+(\alpha))^\dagger)$ , of two types of bosons differing by their “pseudospin” projection  $\alpha = p = 1/2$  (proton) and  $\alpha = n = -1/2$  (neutron) in a 3-dimensional oscillator potential with  $m = 0, \pm 1$ . All the bilinear products of the creation and annihilation operators of the two vector bosons generate the noncompact symplectic group  $Sp(12, R)$  [3]. The following correspondence exists between two of the chains of subalgebras of  $sp(12, R)$  [6]:

$$\begin{array}{ccccc} sp(12, R) & \supset & sp(4, R) & \otimes & so(3) \\ \cup & & \cup & & \cap \\ u(6) & \supset & u(2) & \otimes & su(3) . \end{array} \tag{1}$$

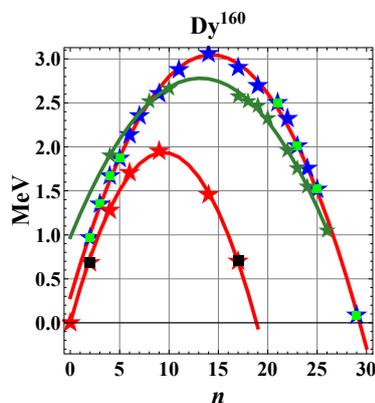
In the first row of (1) a new possible reduction  $sp(12, R) \supset sp(4, R) \otimes so(3)$  of the dynamical symmetry  $sp(12, R)$  [5] is introduced. The generators of the  $sp(4, R)$  are the set of scalar operators with  $L = 0$  from  $sp(12, R)$  generators and as such they commute with the components of the angular momentum  $L_M$  that generate the  $so(3)$  algebra. Hence the eigenvalues of  $L$  are used to characterize the representations of  $sp(4, R)$  and in this way isolate sets of states with given  $L$ . The correspondence (1) is a result of the equivalent pseudospin  $u(2) \supset sp(4, R)$  algebra in both chains, which is complementary to  $su(3) \supset u(6)$ . This permits an investigation into the behavior of low-lying collective states with the same angular momentum  $L$  with respect to the number of excitations  $N$  that build these states [7].

## 3. Energy distribution of low-lying collective bands and states

Because of the correspondence (1) and the relation between the  $SU(3)$  and  $SU(2)$  second order Casimir operators, the Hamiltonian  $H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2$  and bases  $|[N]_6; (\lambda, \mu); K, L, M; T_0\rangle \equiv |(N, T); K, L, M; T_0\rangle$  are equivalent in both chains and the eigenvalues of the Hamiltonian  $H$

$$E((N, T); KLM; T_0) = aN + bN^2 + \alpha_3 T(T + 1) + \alpha_1 T_0^2 + \beta_3 L(L + 1). \tag{2}$$

can be used in both cases for evaluation of the energy distribution of states with a fixed  $L$  [7] and for calculation of the energies of states from a given band [4]. In the first case, since  $L$  is fixed the energies of the collective states depend on the number of phonons (vector bosons)  $N$  and is obviously parabolic. All the rest of the quantum numbers defining the states  $T, T_0$ , and  $L$  are expressed in terms of  $N$  by means of standard reduction procedures [6]. This result confirms



**Figure 1.** Empirical distribution of the experimentally observed energies of the  $0^+$  (red curve with red stars)  $2^+$  (green curve) and  $3^+$  (pink curve with blue stars) states in  $^{160}Dy$ , with respect to the integer classification parameter  $n$  related to the number of bosons, that built the respective states [7].

the empirical investigation of the states with fixed angular momentum [8]. The set of  $N_{L_i}$  with minimal value of  $\chi^2$  in a multistep fitting procedure determines the distribution of the  $L_i^+$  states energies (the parameters of the Hamiltonian) with respect to the number of bosons  $N_{L_i}$  that build the states, which is illustrated as an example in fig. 1. Applying this approach two new low-lying excited  $0^+$  states in the  $^{160}\text{Dy}$  nucleus were experimentally observed as a result of the theoretical prediction by the parabolic energy distribution of  $0^+$  states [9].

Next in the most important application of the  $U(6) \subset Sp(12, R)$  limit of the theory we exploit the possibility it affords for describing both even and odd parity bands up to very high angular momentum [4]. This application is related in the first place to the proper identification of the experimentally observed bands with the sequences of basis states from the even and odd representation of  $Sp(12, R)$ . Here we use the following identification of the experimentally observed ground, excited ( $K^\pi = 0^+$ ),  $\beta^-$ , ( $K^\pi = 2^+$ ),  $\gamma^-$  and octupole  $K^\pi = 0^-$  bands. Using

Experimental states $K^\pi$	$SU(3)$ Basis states
Ground states band $K^\pi = 0^+$ , $\mu_0 = 0$ $\beta^-$ -bands $\mu_0 = \lambda_0 \neq 0$ -even, $L = 0, 2, 4, \dots$	$(\lambda, \mu) = (\mu_0 + 2i, \mu_0), i = 0, 1, 2, \dots,$ $L = 2i, N = N_0 + L, T = (N_0 + 3L)/2$
$\gamma^-$ band $K^\pi = 2^+$ , $\mu_0 = K = 2,$ $L = 2, 3, 4, \dots$	$(\lambda, \mu) = (\mu_0 + 2i, \mu_0), i = 0, 1, 2, \dots,$ $L = \mu_0 + i, N = N_0 + 2L - 4, T = (L - 1)$
Octupole band $K^\pi = 0^-, \mu_0 = 0,$ $L = 1, 3, 5, \dots$	$(\lambda, \mu) = (\mu_0 + 2i, \mu_0), i = 0, 1, 2, \dots, L = \mu_0 + 2i$ $N = N_0 + L - 1, T = \frac{1}{2}(N_0 + L - 1)$

the above assignments of the experimentally observed collective bands to sequences of  $SU(3)$  basis states and the relations between the quantum numbers labeling the representations of the subalgebras in the reduction scheme (1) we evaluate the parameters of the Hamiltonian  $H$  by fitting the energies in (2) to the experimentally observed states in the considered ground,  $\beta^-$ ,  $\gamma^-$  and octupole bands in a given nucleus [4]. In most cases we obtain a very good agreement with the experiment up to very high spins [10]. An example is given on the left panel of fig. 2.

Further from the same assignments we obtain the following dependencies of the energies of the considered collective bands on the numbers of phonon excitations building their band head configurations  $N_{0\beta}, N_{0\gamma}$  and  $N_{0oct}$ , where  $N_{0gr} = 0$ .

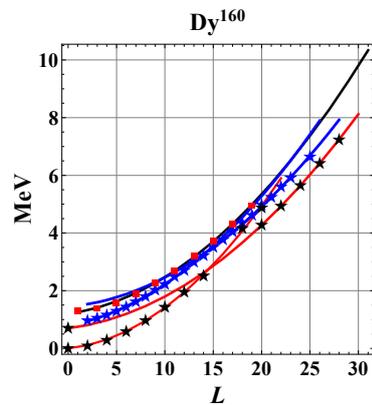
$$E_{gr} = aL + bL^2 + \beta_3 L(L + 1), \quad L = N \tag{3}$$

$$E_\beta = a(L + N_{0\beta}) + b(L + N_{0\beta})^2 + \alpha_3 \frac{1}{2} \left( \frac{1}{2} (L + N_{0\beta}) + 1 \right) (L + N_{0\beta}) + \beta_3 L(L + 1) + \alpha_1 \tag{4}$$

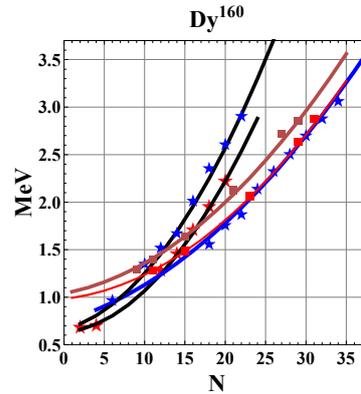
$$E_\gamma = a(N_{0\gamma} + 2L - 4) + b(N_{0\gamma} + 2L - 4)^2 + \frac{\alpha_3}{2} (L - 1)(2L - 1) + \beta_3 L(L + 1) + a_1 \tag{5}$$

$$E_{oct} = a(L + N_{0oct} - 1) + b(L + N_{0oct} - 1)^2 + \frac{\alpha_3}{2} \left( \frac{1}{2} (L + N_{0oct} - 1) + 1 \right) (L + N_{0oct} - 1) + \beta_3 L(L + 1) + a_1 \tag{6}$$

Then introducing the obtained parameters in the expressions (3),(4),(5) and (6) we evaluate the dependence of the states with fixed angular momentum  $L$  on the numbers  $N_{0\beta}, N_{0\gamma}$  and  $N_{0oct}$ , which define the numbers of phonon excitations that build the band head configurations. See the right panel of fig.3. This dependence defines the energy distribution of the band



**Figure 2.** Fit of the theoretical (curves) to the experimental (symbols) energies of the ground and  $\beta$ - (red)  $\gamma$ - (blue) and octupole (black) bands of the  $^{160}\text{Dy}$ .



**Figure 3.** Energy distribution of the  $0^+$  and  $2^+$  states of the  $\beta$ -bands (black curves),  $2^+$  states of the  $\gamma$ -bands (blue curve),  $1^-$  states (red curve)  $3^-$  states (brown curve) of the octupole bands in the  $^{160}\text{Dy}$ .

head configurations (fig.3) in conjunction with the respective bands' evolution (fig.2) and respectively determines the positions of the bands with respect to each other in the nuclear spectra. The experimental states with the considered  $L$  that don't lie on the obtained curves should be considered as having some other type of collective nature that is not included in the IVBM assumptions. The energies corresponding to values of  $N_{0\beta}$ ,  $N_{0\gamma}$  and  $N_{0oct}$  for which an experimental state is not observed could be considered as predictions for new collective states.

In conclusion we could summarize that in the framework of the IVBM with the present physical assignment of the basis states to the experimentally observed low laying collective bands, their properties for the heavy even-even nuclei from the regions of the rare-earth and the actinides are very well reproduced and interpreted in a rather simple, but physically meaningful way. It is important to proceed systematically in this way, in order to relate the observed phenomena to the microscopic and geometrical structure of the nuclei. The employed group-theoretical approach is a rather convenient tool to achieve these goals.

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