

# Nuclear Astrophysics with the Trojan Horse Method

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**Abstract.** In stars nuclear reactions take place at physical conditions that make very hard their measurements in terrestrial laboratories. Indeed in astrophysical environments nuclear reactions between charged nuclei occur at energies much lower than the Coulomb barrier and the corresponding cross section values lie in the nano or picobarn regime, that makes their experimental determination extremely difficult. This is due to the very small barrier Coulomb penetration factor, which produces an exponential fall off of the cross section as a function of energy. Additionally, the presence of the electron screening needs to be properly taken into account when dealing with cross section measurements at low-energies. The Trojan Horse Method (THM) represents an independent experimental technique, allowing one to measure astrophysical  $S(E)$ -factor bared from both Coulomb penetration and electron screening effects. The main advantages and the most recent results are here shown and discussed.

## 1. Introduction

Because of their crucial role in understanding the first phases of the Universe history and the subsequent stellar evolution. The main bare nucleus cross section measurements of the reaction between charged particles at the Gamow energy ( $E_G$ ) should be known with an accuracy better than 10% [1]. The effective cross section in the stellar plasma  $\sigma_{pl}(E)$ , is related to the bare nucleus cross section  $\sigma_b(E)$  and to the stellar electron screening enhancement factor  $f_{pl}$  by the relation

$$\sigma_{pl}(E) = \sigma_b(E)f_{pl}(E) = \sigma_b(E) \cdot \exp(\pi\eta U_{pl}/E) \quad (1)$$

where  $U_{pl}$  is the plasma potential energy and  $\eta$  the Sommerfeld parameter. If  $\sigma_b(E)$  is measured at the ultralow energies  $E_G$  and  $f_{pl}$  is estimated within the framework of the Debye-Hückel theory, it is possible to estimate the  $\sigma_{pl}(E)$  from equation (1).

In neutron-induced reactions, the lack of a Coulomb barrier and the typical  $\sigma_b(E) \sim 1/E^{1/2}$  energy dependence lead to a precise knowledge of  $\sigma_b(E_G)$  for many cases. Whereas the Coulomb barrier of height  $E_C$  in charged-particle induced reactions cause an exponential decrease of the cross section  $\sigma_b(E)$  at  $E > E_C$ ,  $\sigma_b(E) \sim \exp(-2\pi\eta)$ , leading to a low-energy limit of direct measurements of  $\sigma_b(E)$ , which are typically much larger than  $E_G$ .

Owing to the strong Coulomb suppression, the behavior of the cross section at  $E_G$  is usually extrapolated from the higher energies by using the definition of the smoother astrophysical factor  $S(E)$ :

$$S_b(E) = E\sigma_b(E)\exp(2\pi\eta) \quad (2)$$

where  $\exp(2\pi\eta)$  is the inverse of the Gamow factor, which removes the dominant energy dependence of  $\sigma(E)$  due to the barrier penetrability.



Recently the availability of high-current low-energy accelerators, such as that at the underground Laboratori Nazionali del Gran Sasso [2], together with the improved target production and detection techniques have allowed us to perform  $\sigma_b(E)$  measurements down to (or at least close to)  $E_G$ . Therefore no  $\sigma_b(E)$  extrapolation would be needed anymore for these reactions. However, measurements in laboratory are affected by the electron screening effects, which leads to an exponential increase of the measured [3, 4] value of the cross section  $\sigma_s(E)$  [or equivalently of the astrophysical factor  $S_s(E)$ ] with decreasing energy relative to the case of bare nuclei. This can be described by an enhancement factor defined by the relation

$$f_{lab}(E) = \sigma_s(E)/\sigma_b(E) = \exp(\pi\eta U_e/E) \quad (3)$$

Therefore even if it is possible to measure cross sections in the Gamow energy range, the bare cross section  $\sigma_b$  is extracted by extrapolating the direct data from higher energies where a negligible electron screening contribution is expected.

Thus, alternative methods for determining bare nucleus cross sections of astrophysical interest are needed. In this contest a number of indirect methods, e.g. the Coulomb dissociation [5, 6], the ANC (asymptotic normalization coefficient) method ([7] and references therein), and the Trojan-horse method (THM) were developed [8, 9, 10, 11, 12, 13]. The THM has already been applied several times to reactions connected with fundamental astrophysical problems. In this paper we will show the result of the measurement of the astrophysical S(E)-factor in the case of application of THM to the measurement of the two-body reactions  $^{10}\text{B}(p, \alpha_o)^7\text{Be}$  [14] which are a reaction involved in the depletion of light nuclei.

Indeed the THM is currently an alternative technique for measuring the cross section of reactions induced by neutrons [15].

## 2. The Trojan Horse method

The basic idea underlying indirect methods is the use of nuclear reaction theory to link the cross section of astrophysical importance to the one of a different process, easier to be studied with present-day facilities. Indirect techniques have been developed over the past several decades to determine reaction rates that cannot be measured in the laboratory.

The THM is a powerful indirect technique that allows one to determine the bare nucleus astrophysical S-factor for rearrangement reactions. The THM involves obtaining the cross section of the binary process  $x + B \rightarrow C + D$  at astrophysical energies by measuring the Trojan Horse (TH) reaction (the two-body to three-body process ( $2 \rightarrow 3$  particles))  $A+B \rightarrow C + D+S$  in quasi-free (QF) kinematics regime, where the "Trojan Horse" particle,  $A = (Sx)$ , which has a dominant cluster structure, is accelerated at energies above the Coulomb barrier.

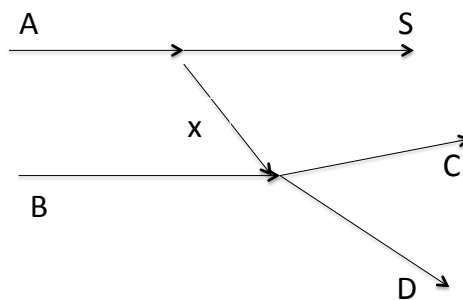
After penetrating the barrier, the TH nucleus  $A$  undergoes a breakup leaving particle  $x$ (participant) to interact with target  $B$  while projectile  $S$ (spectator) flies away. From the measured cross section of TH reaction, the energy dependence of the binary sub-process is determined [1]

The advantage of the THM is that the extracted cross section of the binary sub-process does not contain the Coulomb barrier factor and electron screening does not play a role in the cross section. Consequently, the TH cross section can be used to determine the energy dependence of the astrophysical factor,  $S(E_{xB})$ , of the binary process down to zero value of relative kinetic energy  $E_{xB}$  of particles  $x$  and  $B$ , without distortion due to the electron screening [9, 10].

At low energies, where electron screening becomes important, the comparison between the astrophysical factor determined through the THM and the direct result allows to estimate of the screening potential.

The THM has been successfully employed to determine the bare nucleus cross section of reactions between charged particles at sub-Coulomb energies. Furthermore, the equations

presented in the literature are obtained for an s-wave bound state of  $A = (Sx)$ , and therefore, a limited number of "virtual target or projectile" might be used for the application of THM. In addition, the THM cannot be applied to radiative capture reactions  $(p, \gamma)$ ,  $(n, \gamma)$ ,  $(\alpha, \gamma)$ . The THM has already been applied several time [10, 12, 16, 17, 18] to the rate determination of nuclear reactions of the type  $(p, \alpha)$  and sometimes of the type  $(\alpha, n)$ . The description of the quasi-free reaction (*QFR*) is very simple in the Impulse Approximation (*IA*) and can be represented by the Feynman diagram in figure 1. In the *IA* approximation the three body



**Figure 1.** Feynman diagram representing the quasi-free  $B(A, CD)S$  reaction.

reaction cross section is proportional to the cross section of the binary reaction [19]. Following the Plane Wave Impulse Approximation (PWIA) the three body reaction can be factorized into two terms corresponding to the vertices of figure 1 and is given by:

$$\frac{d^3\sigma}{d\Omega_C d\Omega_D dE_C} = (KF) \cdot |\phi(p_{xS})|^2 \cdot \left[ \frac{d\sigma}{d\Omega} \right]^{HOES} \quad (4)$$

where:

- (i)  $\left[ \frac{d\sigma}{d\Omega} \right]^{HOES}$  is the half-off-energy-shell differential cross section for the binary  $B(x, C)D$  reaction at the center of mass energy  $E_{CM}$  given in post-collision prescription (*PCP*).
- (ii)  $KF$  is a kinematical factor containing the final state phase space factor that is a function of the masses, momenta, and angles of the outgoing particles;
- (iii)  $\phi^2(k_{xS})$  is the Fourier transform of the radial wave function for the  $\chi(r_{xS})$  inter-cluster motion usually described in terms of Hankel, Eckart and Hulthen function depending on the  $xS$  system properties.

The applicability of the pole approximation is limited to small momentum  $p_{xS}$ . Namely, the region where the pole diagram is expected to be predominant in the reaction mechanism was suggested to be:

$$0 \leq p_{xS} \leq k_{xS} \quad (5)$$

being  $p_{xS}$  the momentum, in the half off energy shell (*HOES*), of the cluster  $x$  when it interacts with the particle  $B$  and  $k_{xS}$  the momentum in the half off energy shell (*HOES*) defined by the relation

$$k_{xS} = \sqrt{2\mu_{xS}B_{xS}} \quad (6)$$

where  $\mu_{xS}$  is the reduced mass and  $B_{xS}$  is the binding energy of the system  $xS$ , and  $Q_{xS}$  is the  $Q$ -value for the virtual decay. In order to select the data to be analysed, which correspond to

the region where  $QF$  mechanism is dominant, coincidence events for momentum  $S$ - spectator (defined by the equation (5)) were considered in the standard analysis. The shape of  $\phi^2(k_{xS})$  can be related to angular momentum and the radial wave function of the  $xS$  component of the wave function. The equation (4) can be applied only to specific cases in which involved reactions with the presence of related contributions to a single  $l_t h$  partial wave (for example  $l_0$ ).

### 2.1. Incident energy prescriptions.

The incident energy  $E_{AB}$  of the projectile must be chosen in such a way that the bombarding energy  $E_{AB}$  overcomes the Coulomb barrier in the entry channel:  $E_{AB} > (E_{AB})^{Coul.Barr.}$ . Thus, particle  $x$  is brought into the nuclear interaction zone to induce the reaction of interest  $x + B \rightarrow C + D$ . The  $QF$  kinematical conditions must be chosen in such a way that the  $E_{xB}$  relative energy can span the astrophysical region of interest below the Coulomb barrier:  $E_{xB} < (E_{xB})^{Coul.Barr.}$  [8]

In post collision prescription  $E_{xB} = E_{CD} - Q_{2b} = E_{cm}$  where the symbol  $E_{CM}$  is often used instead of  $E_{x-B}$ ,  $Q_{2b}$  is the  $Q$ -value of the  $B + x \rightarrow C + D$  two-body reaction and  $E_{CD}$  is the relative energy between the outgoing particles  $C$  and  $D$ . The approach based on the idea that the initial projectile velocity is compensated for by the binding energy of particle  $x$  inside  $A$ :  $E_{cm} = E_{x-B} - B_{xS}$  with  $E_{x-B}$  center of mass system. Thus the relative energy of the fragments in the initial channel  $x + B$  of the binary reaction can be very low and even negative. In contrast, this condition is difficult or impossible to achieve in direct measurements due to the Coulomb barrier.

## 3. Data analysis

### 3.1. Extraction of two-body cross section from the measured three-body reaction

In the analysis the two body cross section is derived by dividing the selected three-body coincidence yield ( $Y_d$ ) by the result of the Monte Carlo calculation of the product of kinematical factor time the interval of experimental momentum distribution

$$\left[ \frac{d\sigma}{d\Omega} \right]^{HOES} \propto \left[ \frac{Y_l}{(KF)\phi^2(p_{xS})} \right] \quad (7)$$

where  $\phi^2(p_{xS})$  is referred to the range of momentum distribution corresponding to the  $p_{xS}$  range that can be considered equation (5). We stress that this relation is a limit of the pole approximation applicability. This cross section can be normalized to the binary reaction obtaining the absolute value in the energy range  $\Delta(E_{CM})$  of the excitation function above the Coulomb barrier.

In the THM application the reaction  $x + B \rightarrow C + D$  is induced inside short-range nuclear field the penetration probability of the Coulomb barrier has to be introduced to compare the THM cross section with the direct data (taken from literature) from literature in the energy region below the Coulomb barrier

$$P_l(k_{xB}r_{xB}) = \frac{k_{xB}r_{xB}}{F_l^2(k_{xB}r_{xB})G_l^2(k_{xB}r_{xB})} \quad (8)$$

where  $F_l$  and  $G_l$  are the regular and irregular Coulomb wave functions,  $k_{xB}$  and  $r_{xB}$  the  $x$ - $B$  relative wave number and interaction radius, respectively. In this case the two body cross section correspond to the nuclear contribution to the cross section and the TH two body cross section is defined by the relation

$$\left[ \frac{d\sigma}{d\Omega} \right]^{THM} \propto \left[ \frac{Y_l}{(KF)\phi^2(p_{xS})} \right] [P_l(k_{xB}r_{xB})] \quad (9)$$

and the quantities to be compared for the validity test are

$$\left[ \frac{d\sigma}{d\Omega} \right]^{THM} \propto \left[ \frac{d\sigma}{d\Omega} \right]^{OES} \quad (10)$$

#### 4. Astrophysical S-factor

The THM provides an independent bare nucleus cross section measurement,  $\sigma_b(E)$  (or equivalently the bare nucleus astrophysical factor  $S_b(E)$ ). Thus, it is possible to extract information on the electron screening potential [Spitaleri2011] by comparing  $S_b(E)$  with the one obtained from shielded nuclei. Once the two-body cross section has been extracted, the THM astrophysical S(E)-factor can be obtained according to:

$$[S_b(E)]^{THM} = E[\sigma_b(E)]^{THM} \exp(2\pi\eta) \quad (11)$$

where  $\sigma_b(E)^{THM}$  is the bare nucleus cross section obtained via the THM by the Eq. (12). Thus, the energy dependence of  $[S_b(E)]^{THM}$  should show the same trend of the one derived by the direct measurements, excepted in ultra low energy range  $\Delta E_{ES}$  where the two data sets should differ due to the effects of electron screening:

$$[S_b(E)]^{THM} \propto [S_b(E)]^{OES} \quad (12)$$

where  $E > \Delta E_{ES}$ . If needed, the value for the electron screening potential  $U_e$  can then be obtained by comparing the two data sets. Clearly, this procedure does not allow us to extract the absolute value of the astrophysical S-factor. However, the absolute scale for  $S_b(E)^{THM}$  can be obtained by normalization of the THM data to the direct  $S(E)^{OES}$  ones obtained at energies  $E^*$  where the electron screening effects are negligible.

$$N_{abs.value} = \frac{[S_b(E^*)]^{OES}}{[S_b(E^*)]^{THM}} \quad (13)$$

#### 5. Example of THM application: $^{10}\text{B}(p, \alpha_0)^7\text{Be}$ reaction $E = 24.5$ MeV

Fig. 1 shows the TH  $^{10}\text{B}(p, \alpha_0)^7\text{Be}$  S(E) factor obtained in an experiment performed at Tandem of the Laboratori Nazionali del Sud- INFN, Catania [14].

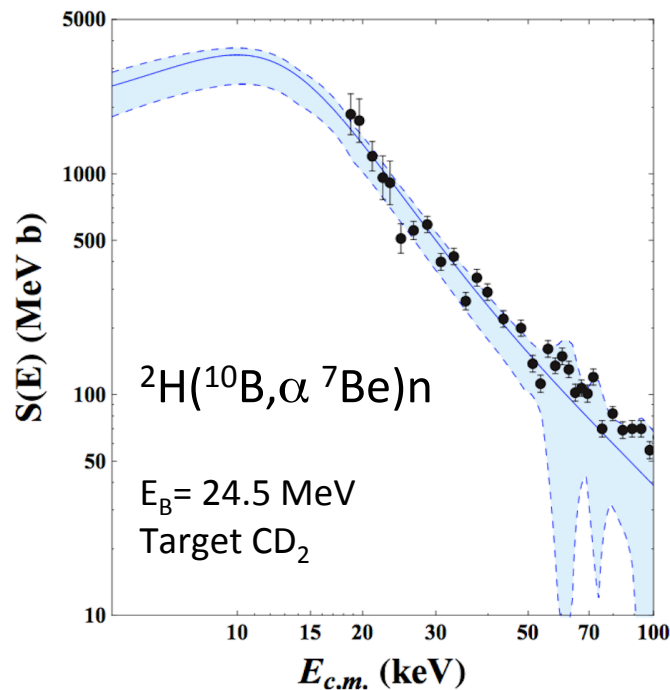
The data are reported at infinite resolution, together with their upper and lower limits as given by the corresponding uncertainties. The THM data are compared with the low-energy direct data [21]. While at energies lower than 30 keV direct data are strongly influenced by electron screening effects. The TH S(E) factor describes the typical bare-nucleus behavior.

#### 6. Conclusions

In summary, the THM has proved to be an extremely versatile and powerful technique to investigate both reactions of astrophysical interest at the extreme of sub-Coulomb energies and purely nuclear phenomena otherwise difficult to study by a direct approach. Important results have already been obtained in both areas and new applications of the method to further cases of interest (e.g., reactions involving neutrons and/or reactions with Radioactive Ion Beams), are currently underway.

#### 7. Acknowledgments

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**Figure 2.** The TH  $^{10}\text{B}(p, \alpha_0)^7\text{Be}$  astrophysical  $S(E)$  factor compared with the low-energy direct data [14].

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