Utilization of Software Tools for Uncertainty Calculation in Measurement Science Education

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Abstract. Despite its importance, uncertainty is often neglected by practitioners in the design of system even in safety critical applications. Thus, problems arising from uncertainty may only be identified late in the design process and thus lead to additional costs. Although there exists numerous tools to support uncertainty calculation, reasons for limited usage in early design phases may be low awareness of the existence of the tools and insufficient training in the practical application. We present a teaching philosophy that addresses uncertainty from the very beginning of teaching measurement science, in particular with respect to the utilization of software tools. The developed teaching material is based on the GUM method and makes use of uncertainty toolboxes in the simulation environment. Based on examples in measurement science education we discuss advantages and disadvantages of the proposed teaching philosophy and include feedback from students.

1. Introduction and Motivation

Measurements are required for many tasks and the quality of measurement results has a major impact on the overall outcome. Surprisingly, the concept of uncertainty is comparatively young and was proposed for the first time in 1984 [1]. The idea was to overcome some of the limitations that are associated with the previously used term error [2]. With respect to metrology, the uncertainty reflects the fact that measurements can only provide incomplete knowledge and that a measurement is only useful when the lack of knowledge is somehow quantified. This is particularly true with respect to safety and reliability. Consider, for example, a monitoring system that should validate that a certain parameter lies within a certain interval. However, if the measurement uncertainty of the monitoring system becomes larger than the interval to be monitored, then the monitoring system can *never* be used to validate that the parameter is actually *within* the interval; it can only be used to validate that the parameter (with high probability) resides *outside* of the interval. This may not be apparent for a user or even for a developer of such a system, in particular considering that the engineer may not be an expert in stochastic and uncertainty quantification. Therefore, it seems to be reasonable to provide a method that is commonly accepted by practitioners and experts, can easily be applied for a wide range of problems and still provides good results (yet they may not be optimal in a theoretical sense).

In 1977, as it was recognized that there exists a lack of international consensus on the expression of uncertainty in measurement, the world's highest authority in metrology, the Comit

International des Poids et Mesures (CIPM), requested the Bureau International des Poids et Mesures (BIPM) to address the problem in conjunction with the national standards laboratories and to make a recommendation. An effort that finally led to the definition of the Guide to the Expression of Uncertainty in Measurement (GUM) [2]. According to the GUM, the ideal method should be universal (applicable to all kinds of measurements and to all types of input data used in measurements), internally consistent (directly derivable from the components that contribute to it), and transferable (possibility to directly use the uncertainty evaluated for one result as a component in evaluating the uncertainty of another measurement in which the first result is used).

As a result of these requirements, the GUM treats all uncertainty contributions identically, more or less as if the distributions were Gaussian and the relations were linear. This is in accordance with one of the initial requirements for such a recommendation: The approach has to be universal. However, the GUM working group was aware that there are limitations of the original GUM method and supplementary [2, 3, 4] suggested to use Monte Carlo sampling in certain cases. A recent survey [5] on current research activities in the field of measurement uncertainty reports that most recent work addresses the GUM (Guide to the Expression of Uncertainty in Measurement. Consequently, the present paper focuses on this approach, which has a wide acceptance within the field of metrology.

The tools we use for education are Metas.UncLib Matlab toolbox[6], which implements the GUM tree method [7] and also a toolbox for Matlab that is developed in our group. For the basic operations as shown in this paper and as are required for bachelor level measurement science education, both toolboxes are similar in usage and functionality.

2. Teaching Concept

The course in question is on Measurement Science, Sensors and Actuators and is intended for students of information technology in the third year. Consequently, the students are familiar with basic concepts of electrical engineering, have some experience with measurement devices and also have background from a mathematically oriented course on stochastic. Therefore, they are familiar with the concept of random variables, Ohms law and electrical networks. Thus we decided to actually start the course with the concept of measurement uncertainty before we introduce the SI system. Consequently, the discussion on "good" definitions for base units can also be based on the uncertainty concept. Furthermore, traceability is also directly linked to this discussion. This should provide a holistic view of how measurement science work and that knowing the uncertainty is as important as knowing the estimate of some parameter in question.

Our proposed introduction to the concept of uncertainty is illustrated in Table 1. Starting from an interpretation of measurements as realization of random variables that provide some information about the parameter in question we introduce the original GUM including terms such as standard uncertainty, combined uncertainty and determination by means of Taylor series expansion.

The first practical example is the determination of a resistance value and corresponding combined standard uncertainty from the measurement of voltage and current with respective rectangular distribution of uncertainty using Ohm's law

$$R = \frac{U}{I} \tag{1}$$

as the measurement model and the corresponding equation for uncorrelated input quantities and linearization

$$u_c^2(R) = \left(\frac{\partial R}{\partial I}\right)^2 u^2(I) + \left(\frac{\partial R}{\partial U}\right)^2 u^2(U) \tag{2}$$

Section	Content	Educational Objective
1	Introduction to Uncertainty	Understand measurements as realizations of random variables
2	Introduction to original GUM, Error Propagation by means of Taylor Series	Know GUM and it's application to measurement equations
3	Example Resistor Measurement (Voltage/Current)	Practical experience with the original GUM
4	Consideration of systematic errors	Understand importance of measurement model
5	Example Wheatstone Bridge - Simple	Identify pitfalls in approach according to section 2
6	Example Wheatstone Bride - More Accurate	Practice, learn that a correct consideration of uncertainties may be time consuming if done "by hand"
7	Example Wheatstone Bridge - Tool Based	Understand the concepts of "uncertain" as a datatype
8	Example Wheatstone Bridge - Analysis of Sources of Uncertainty	Understand that tools can ease the analysis of measurement chains
9	Example Wheatstone Bridge - Influence of Parameters	Understand that parameters that do not directly contribute to the uncertainty may do so through other parameters.
10	Discussion	Understand that uncertainty must always be considered in measurement. Awareness that tools are available that simplifies most of the calculations. Awareness that the tools and GUM have limitations

Table 1. Steps of the proposed introduction to the concept of uncertainty.

As a second step, systematic errors due to the inner resistance of the measurement instruments are included in the measurement model.

After this simple example we apply the method to the Wheatstone Bridge circuit as another common method to evaluate resistance. There we also introduce software tools for the evaluation of uncertainty. This is described in the next section.

3. The Wheatstone Bridge with Uncertainties

Bridge circuits for the determination of unknown impedances and as realization of the compensation method are important building blocks in measurement science and thus usually treated in introductory courses. We use the Wheatstone Bridge to emphasize how tools for uncertainty calculation may change the way how the material is presented to students. Figure 1 shows the circuit of a Wheatstone Bridge.

First, we start from the classical result for the equation to determinate the value of the unknown resistor R_1, i.e.

$$R_1 = R_2 \frac{R_3}{R_4} \tag{3}$$

and directly apply the GUM method to this, leading to

$$u_C{}^2(R_1) = \frac{\delta R_1}{\delta R_2}^2 u^2(R_2) + \frac{\delta R_1}{\delta R_3}^2 u^2(R_3) + \frac{\delta R_1}{\delta R_4}^2 u^2(R_4) \tag{4}$$

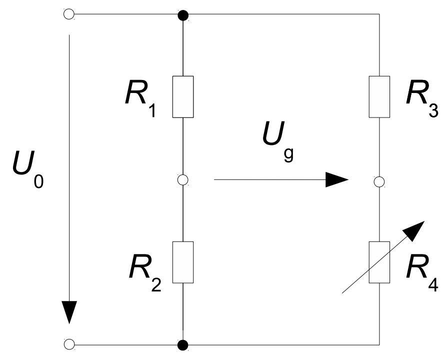

Figure 1. Example for a Wheatstone Bridge.

and let the students do an interpretation of the results. The aim is to point out that apparently some important influences are missing, E.g., it seems that the choice of U_0 and the accuracy of the instrument that measures U_g would not be important with respect to the uncertainty. This is obviously not correct. This leads to a more detailed analysis, showing that it may not be sufficient to only consider a measurement equation to fully determine the standard uncertainty.

The students are instructed to derive a more complete measurement model such as

$$R_1 = \frac{U_0 R_2 (R_3 + R_4)}{U_0 R_4 + U_g (R_3 + R_4)} - R_2 \tag{5}$$

Here we emphasize that a value that is measured to be zero may still be significantly different from zero and must thus not be omitted for uncertainty considerations. In the next step the measurement equation is derived with respect to U_0, U_g, R_2, R_3, R_4, leading to the coefficients $C_{R2} = \frac{\delta R_1}{\delta R_2}$, $C_{R3} = \frac{\delta R_1}{\delta R_3}$ and so on. The resulting uncertainty of the resistor R_1 can be calculated as follows:

$$u_C{}^2(R_1) = C_{U0}^2 u^2(U_0) + C_{Ug}^2 u^2(U_g) + C_{R2}^2 u^2(R_2) + C_{R3}^2 u^2(R_3) + C_{R4}^2 u^2(R_4) \tag{6}$$

with the uncertainty $u(X)$ of the respective input quantities and the coefficients calculated previously.

Following this analysis we introduce toolboxes for Matlab which use a datatype "uncertain". With this we perform the same calculation as above, but numerically and step by step as shown in Figure 2 emphasizing that it does not require any additional effort to obtain the combined standard uncertainty but providing the standard uncertainty for the input quantities. It is also shown that the toolbox could also be applied to equation (3) but leading to an incorrect result (Fig. 3).

The reason - failing to correctly consider the uncertainty of the voltage measurement (the imbalance of the bridge) - is otherwise often not obvious for students. Consequently, several rules for use of uncertain measurement results but also for the derivation of measurement equations (including their simplification)can be derived. This allows obtaining the well known result according to equation (3) but clearly highlights that other parameters that not occur in this equation contribute to the uncertainty.

However, the benefit is not just the automatic calculation of the standard uncertainty. Additionally, toolboxes also provide means to determine the contribution of the uncertain input variables to the combined uncertainty of the result. This is shown in Figure 4. In the present example the main contribution to the uncertainty comes from the uncertainty of the voltage measurement, which is in practice not truly zero.

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Value Standard Uncertainty

```
>> U0=unc(5,0.1);R2=unc(1000,5);R3=unc(1000,5);
>> R4=unc(1001,1);Ug=unc(0,0.01);
>> I4=U/(R3+R4);
>> U4=I4*R4;
>> U2=U4+Ug;
>> I2=U2/R2;
>> R1=(U0-U2)/I2,

R1 =

   (999.001 ± 10.713)
```

Figure 2. Calculation of the measurement result including the standard uncertainty of the unknown resistor R_1 using Metas.UncLib toolbox for Matlab [6].

```
>> R2*R3/R4

ans =

    (999.001 ± 7.13415)
```

Figure 3. Incorrect determination of the standard uncertainty of R_1 due to direct application of the GUM to the classical solution according to equation (3).

```
>> get_jacobi(R1)

ans =

    0.0000    4.9950    4.9950   -0.9980   -7.992(
      (U0)      (R2)      (R3)      (R4)      (Ug)
```

Figure 4. Determination of the contributions of the various sources of uncertainty to the standard uncertainty of R_1 using Matlab and an uncertainty toolbox

In this example the contribution of U_0 to the uncertainty is close to zero. By reducing U_0 to half of its original value we show that although its contribution to the standard uncertainty is still negligible we still see an increase of the combined standard uncertainty as other contributions increase. Here we aim to emphasize such interdependencies and how they are easily studied with the tools.

4. Discussion of Advantages and Disadvantages and Student Feedback

We consider toolboxes for determination of standard uncertainty just as an additional feature of an electronic calculator. Similarly as nonlinear functions such as division or logarithm are difficult to calculate without the help of electronic calculators, the results are readily available without any effort. This implies that most of the time it is fully sufficient to understand what a function e.g. logarithm does but not necessarily how it is actually calculated numerically (although the concept should be known). As with the electronic calculator in general, a frequently observed disadvantage is a loss of the 'feeling' whether a result is feasible or not. We consider toolboxes for determination of standard uncertainty just as an additional feature of an electronic calculator. Similarly as nonlinear functions such as division or logarithm are difficult to calculate without the help of electronic calculators, the results are readily available without any effort. This implies that most of the time it is fully sufficient to understand what a function e.g. logarithm does but not necessarily how it is actually calculated numerically (although the concept should be known). As with the electronic calculator in general, a frequently observed disadvantage is a loss of the 'feeling' whether a result is feasible or not. Our aim with respect to uncertainty is to put the focus on the fact that measurement results are actually realizations of random variables and that the GUM tries to describe the probability density function using the standard uncertainty. This is in contrast e.g. with the electronic evaluation of the logarithm, where the result is highly accurate. This is not necessarily the case for the described tools, in particular when only linearization according to the original GUM is used. In particular revisions of the GUM such as projected in [8]. With the use of the tools students may not be aware of the limitations of these tools as they are used to fully rely on the results from electronic calculators.

The positive students' feedback reasons from the initial introduction of the uncertainties, which are directly coupled to the variables. This allows copying the uncertainties given in the specification directly to the variable itself. Since the students are familiar with Matlab, the barrier of using the tool is pretty low. They can work in the known environment without any additional training. Nevertheless, the most important positive point is that the students' attention can be unglued from the already known mathematical calculations and guided towards the measurement model construction and the analysis of the contribution of the various sources of uncertainty. Especially, the simple access of the contribution of the various sources allows to immediately highlight the main contributors to the uncertainty. The students get a feeling on what shall be tried to improve first. On the other side some minor negative feedback was given, e.g. on naming of commands such as get_jacobi, which may not be intuitive for the students. Additionally, more experienced students objected that the provided number of digits for the results (e.g. 999.001 ± 10.713) is somewhat in contradiction with the actual accuracy. We aim to consider such aspects in the improvement of our tool.

5. Conclusion

We propose an approach for considering uncertainty in measurement science education from the very beginning using special tools. The approach does not offer different or even better results; it is currently simply based on the original GUM method. However, it offers an alternative philosophy for the workflow. We believe that this approach may help to increase the awareness with respect to uncertainty, has a high practical applicability in particular with respect to safety engineering and may thus lead to consideration of uncertainty in early design phases.

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