

Quality Assessment of Vertical Angular Deviations for Photometer Calibration Benches

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Abstract. Lighting, both natural and electric, constitutes one of the most important aspects of the life of human beings, allowing us to see and perform our daily tasks in outdoor and indoor environments. The safety aspects of lighting are self-evident in areas such as road lighting, urban lighting and also indoor lighting. The use of photometers to measure lighting levels requires traceability obtained in accredited laboratories, which must provide an associated uncertainty. It is therefore relevant to study the impact of known uncertainty sources like the vertical angular deviation of photometer calibration benches, in order to define criteria to its quality assessment.

1. Introduction and Motivation

Lighting, both natural and electric, constitutes one of the most important aspects of the life of human beings, allowing us to see and perform our daily tasks in outdoor and indoor environments. The interior spaces of buildings must be designed so as to provide adequate lighting conditions, concerning illumination levels, uniformity and absence of glare, for occupants to perform adequately all visual tasks and activities in conditions of safety and comfort, without eyestrain, and in an efficient way. The safety aspects of lighting are self-evident in areas such as road lighting, including tunnels, urban lighting and also indoor lighting.

The basic quantity used to quantify the lighting environment is the illuminance (E_v), which is defined as the quotient of the luminous flux (lm) incident on a given surface by the area (m^2) of that surface. The S.I. unit of illuminance is the lux (lm/m^2) [1].

Illuminances are usually measured with photosensors, commonly referred as photometers or illuminance meters. Using the photoelectric effect, those photosensors, detect light and convert it into electrical signals obtained from photocurrents developed by sets of photoresistors, photodiodes and photomultipliers connected to other optical elements.

The traceability of these type of instruments can be obtained through calibration in laboratory using different testing techniques, namely, absolute, substitution and monitor approaches (as described in [2]). All these methods use a setup that includes a reference stable light source and a photometric bench and, in some methods, a reference photometer (having properties similar to the photometer to be calibrated).

The setup formed by the geometric bench intends to establish a 1-dimensional line between the centre of a standard light source and the photometer being calibrated, allowing the establishment of a relation between the distance of both elements and the photometer measurements. The experimental setup has



different sources of uncertainty, one of which is the important vertical angular deviation associated with the vertical profile of the bench, creating an angular deviation effect influencing the measurement results. The aim of the proposed work is to study this influence in the uncertainty of the measurement, and use that contribution to establish criteria that can be applied to the quality assessment of a photometer calibration bench at LNEC's Photometry and Colorimetry Unit, with respect to the measurement accuracy required.

The study is based on the example 1.2 of the CIE guide [3] related to the measurement of the effective distance. The experimental example given illustrates the uncertainty evaluation applying the ISO-GUM [4] conventional approach, and its validation with a Monte Carlo alternative approach, considering the existing nonlinearity effects in the mathematical model, for a discussion on the adequacy of current practices in this type of testing.

2. Experimental Procedure and Measurement Model

LNEC's photometry and colorimetry laboratory has a photometric bench where the calibration of photometers is carried out. The method of calibration can be considered an absolute method since it relies on a traceable luminous intensity standard lamp. The basic method of calibration is based on the *inverse square law of illumination* through the variation of the distance to the standard lamp of the photometer to be calibrated. The measured distance differences and the respective illuminance values of the photometer, a "corrected" value for the illuminances (calibration factor) can then be determined (see Figures 1 and 2)

The experimental procedure intends to measure the effective distance between the filament of the reference luminous intensity standard lamp and the aperture of a photometer. It considers a minimum of two positions (d_0 , $d_0 + \Delta d$) on a linear scale of the photometer bench, measuring the photocurrent (or the illuminances) in each of these points (y_0 at d_0 and y_1 at $d_0 + \Delta d$).

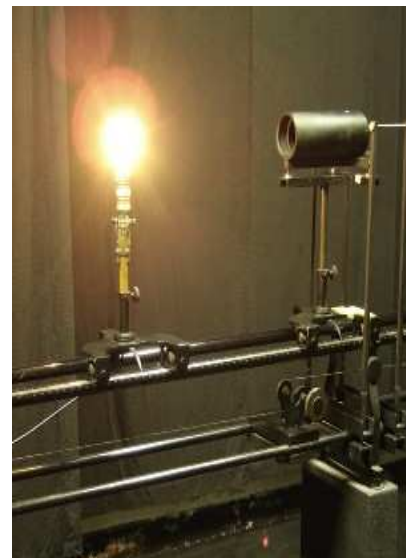


Figure 1 – Illustration of the Photometric Bench used in the calibration of photocells

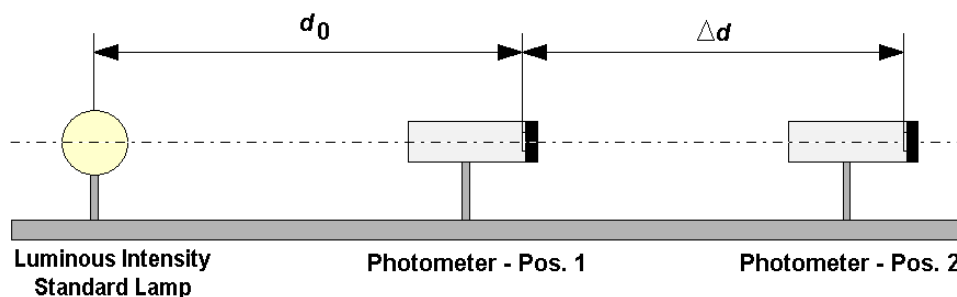


Figure 2 – Setup of the effective distance measurement for photometer calibration purposes.

The theoretical approach considers that the luminous intensity I (in cd), is constant, thus leading to following equation:

$$I = E_0 \cdot d_0^2 = E_1 \cdot (d_0 + \Delta d)^2 \quad (1)$$

with the photocurrent values y_0 and y_1 the output quantities related to E at d_0 and $(d_0 + \Delta d)$. The mathematical model that yields the relation between input and output quantities is:

$$d_0 = \Delta d / \left(\sqrt{\frac{y_0}{y_1}} - 1 \right) \quad (2)$$

This simplified model, however, does not state clearly some of the quantities that are known sources of uncertainty, due to the modelling (stray-light effect, non-lambertian angular distribution, instability of the source and “field of view” limitation of the photometer aperture) and due to the experimental setup conditions (vertical and horizontal angular deviations due to the photometer bench and temperature and humidity effects during testing).

The focus of this study is on the impact that the vertical angular deviation of the photometer bench, θ , has in measurement results, especially those related to uncertainty, with the intention of defining criteria for the assessment of the bench experimental setup.

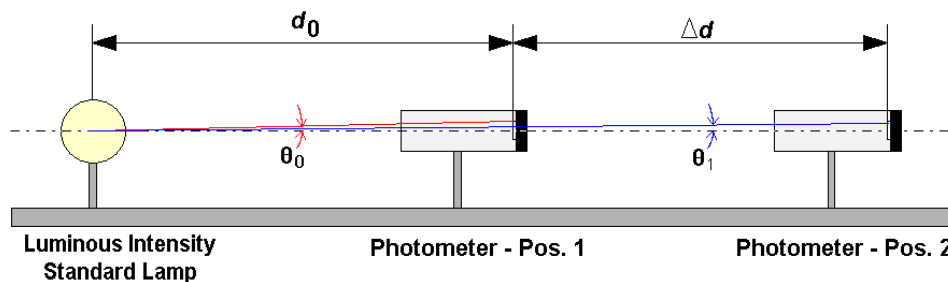


Figure 3 – Angular deviations of the calibration setup.

To study this problem, equation (2) is modified to include the vertical angle variation, θ , between the two distances (θ is the difference between θ_0 and θ_1) yielding the following uncertainty mathematical model:

$$d_0 = \left(\frac{\Delta d}{\cos(\theta)} \right) / \left(\sqrt{\frac{y_0}{y_1}} - 1 \right) \quad (3)$$

The uncertainty analysis (without vertical angular deviation effect) is provided by [3,5] for a case study, using the ISO-GUM approach. The evaluation of uncertainty performed in this study use the data of that example and evaluates the measurement uncertainty using also a Monte Carlo Method (MCM), considering that the non-linear effects due to the mathematical model might be significant.

3. Measurement Uncertainty of Effective Distance using GUM and Monte Carlo Methods

The case studied was based on an experimental example proposed in [3], including the measurement of the effective distance between the filament of a luminous intensity lamp and the aperture of a photometer and the evaluation of its measurement uncertainty using classical GUM approach, being the results compared with those resulting from the use of a Monte Carlo method approach first using the same mathematical model (2) and secondly using model (3) which includes the contribution of the vertical angular deviation effect, being evaluated its impact for different angular uncertainty values aiming to establish basic knowledge to develop quality control criteria.

The experimental procedure described in [3] considers a source as luminous intensity standard (having nearly Lambertian angular distribution) illuminating a photometer at a distance d_0 . In a setup similar to the one shown in Fig. 2, both the offset-corrected photometer signal, y_0 , and y_1 , obtained after the distance dislocation of Δd , are the average values of sets of 10 independent readings. In these base conditions, equation (2) gives estimates of the effective distance d_0 . The experimental data is presented in Table 1.

Table 1 – Input quantities estimates and measurement uncertainty probabilistic information

Quantity	Estimate	Standard uncertainty	PDF* info
Δd	0,500 0 m	0,000 5 m	Type B - Gaussian, $\nu^{**} = \infty$, no correlation.
y_0	8,000 0 V	0,000 8 V	Type A - Gaussian, $\nu^{**} = 9$, no correlation.
y_1	6,000 0 V	0,000 6 V	Type A - Gaussian, $\nu^{**} = 9$, no correlation.

* Probability Distribution Function ** Degrees of Freedom (DoF)

In order to apply GUM approach, again considering the mathematical model (2), sensitivity coefficients need also to be considered, with expressions and estimated values listed below.

$$c_1 = \frac{\partial d_0}{\partial \Delta d} = \frac{1}{\sqrt{\frac{y_0}{y_1} - 1}} = 6,464 \quad (4)$$

$$c_2 = \frac{\partial d_0}{\partial y_0} = \frac{\Delta d}{2 \cdot \sqrt{\frac{y_0}{y_1}} \cdot (y_0 + y_1) - 4y_0} = -1,508 \frac{\text{m}}{\text{V}} \quad (5)$$

$$c_3 = \frac{\partial d_0}{\partial y_1} = \frac{\Delta d \cdot \sqrt{\frac{y_0}{y_1}}}{2y_1 \cdot \left(\sqrt{\frac{y_0}{y_1}} - 1 \right)} = 2,010 \frac{\text{m}}{\text{V}} \quad (6)$$

Table 2 summarizes all this information as proposed by GUM [4] leading to the standard measurement uncertainty of the effective distance, also found in the same Table.

Table 2 – Uncertainty budget table related to the effective distance measurement

Quantity X_i	Symbol	Value	Absolute Standard uncertainty	Type	DoF	Absolute sensitivity c_i	Absolute contribution $u_i(y)$	Relative contribution $w_i(y)$
Dislocation / m	Δd	0,500 0	0,000 5	B	∞	+ 6,464 1	+ 0,003 2	+ 0,001 0
Initial photometer signal / V	y_0	8,000 0	0,000 8	A	9	- 1,507 8	- 0,001 2	- 0,000 4
Dislocated photometer signal / V	y_1	6,000 0	0,000 6	A	9	+ 2,010 4	+ 0,001 2	+ 0,000 4
Effective distance / m	d_0	3,232 1			365		0,003 6	0,001 1

The values of the degrees of freedom for the several input quantities were considered using the Welch-Satterthwaite formula [6,7], to obtain the related coverage factor [2] that should multiply the standard measurement uncertainty in order to evaluate the expanded measurement uncertainty of the effective distance estimate.

$$\nu_{\text{eff}} = \frac{u^4(y)}{\sum_{i=1}^n \frac{u_i^4(y)}{\nu_i}} = \frac{0,0036^4}{\frac{0,0012^4}{9} + \frac{0,0012^4}{9}} = 365 \quad (7)$$

$$k(\nu_{\text{eff}} = 365) = 2,00 \quad (8)$$

$$U_{95}(d_0) = k \cdot u(d_0) = 2,00 \cdot 0,0036 \text{ m} = 0,0072 \text{ m} \quad (8)$$

To evaluate the effect in uncertainty due to non-linear behaviour of the mathematical model used (2) a Monte Carlo method simulation was performed, considering that it would allow to validate the results obtained using the GUM approach. The numerical simulation developed used the same experimental estimates of the input quantities and the probabilistic information shown in Table 1, for numerical sequences obtained from 10^6 runs each, using MatLab software validated routines (namely to generate pseudo-random numbers sequences with known PDFs and to order the output numerical sequences) and also using the procedure described in [8] to obtain probability intervals from the output PDF.

Table 3 – GUM vs. MCM comparative results

Quantity X_i	Symbol	Approach	Estimated value / m	Expanded measurement uncertainty (95% confidence) / m
Effective distance / m	d_0	GUM	3,232 1	0,007 2
		MCM	3,232 0	0,007 2

The values presented shows that GUM approach is suitable to evaluate effective distance measurement uncertainty in the conditions of the example [3] (all input PDFs are Gaussian), which cannot be generalized if input quantities with different PDF had been considered. The comparison study also showed that the output PDF obtained using MCM has a Gaussian shape (Fig. 4), which differs from the trapezoidal PDF presented in [3]

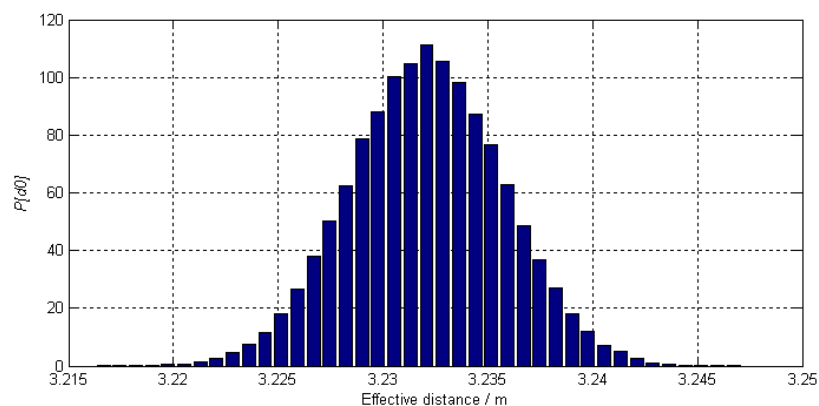


Figure 4 – Effective distance output PDF obtained from MCM study.

4. Analysis of the Effect of Vertical Angular Deviation in the Effective Distance Uncertainty

The second part of the study was intended to evaluate the effect of vertical angular deviation input quantity, in the effective distance output combined uncertainty. Different intervals of uncertainty of vertical angular deviation (from 1° to 5° – using Gaussian PDFs) were tried, now using equation (3). The evaluation of the measurement uncertainty intervals was based on a MCM approach using the same data of the previous analysis. The following table resumes the conditions and results obtained.

Table 4 – Measurement estimates of effective distance and related uncertainty considering different vertical angular deviation uncertainty intervals

Quantity X_i	Symbol	$u(\theta)$ / °	Estimated value / m	Expanded measurement uncertainty (95% confidence) / m
Effective distance / m	d_0	1	3,233	0,007
		3	3,236	0,014
		5	3,245	0,034

One main advantage of using MCM is the access to the output PDF information which, in this case, is particularly informative. In fact, looking for the PDFs presented in the following Figures, is possible to see that although the vertical angle deviation (θ) has Gaussian PDF, the distributions of $\cos(\theta)$ – Figures on the left – are highly asymmetric leading to the need of knowing how they influence output PDFs (Figures on the right).

In fact, Table 4 shows a deviation of the estimates and an increase of uncertainty due to this quantity, reaching about 1% of the output estimate. This evaluation allows the user to establish tolerance limits for the vertical angular deviation observed on the experimental bench metrological characterization, according to the tolerances required for the experimental setup (usually, from 2% to 5 %).

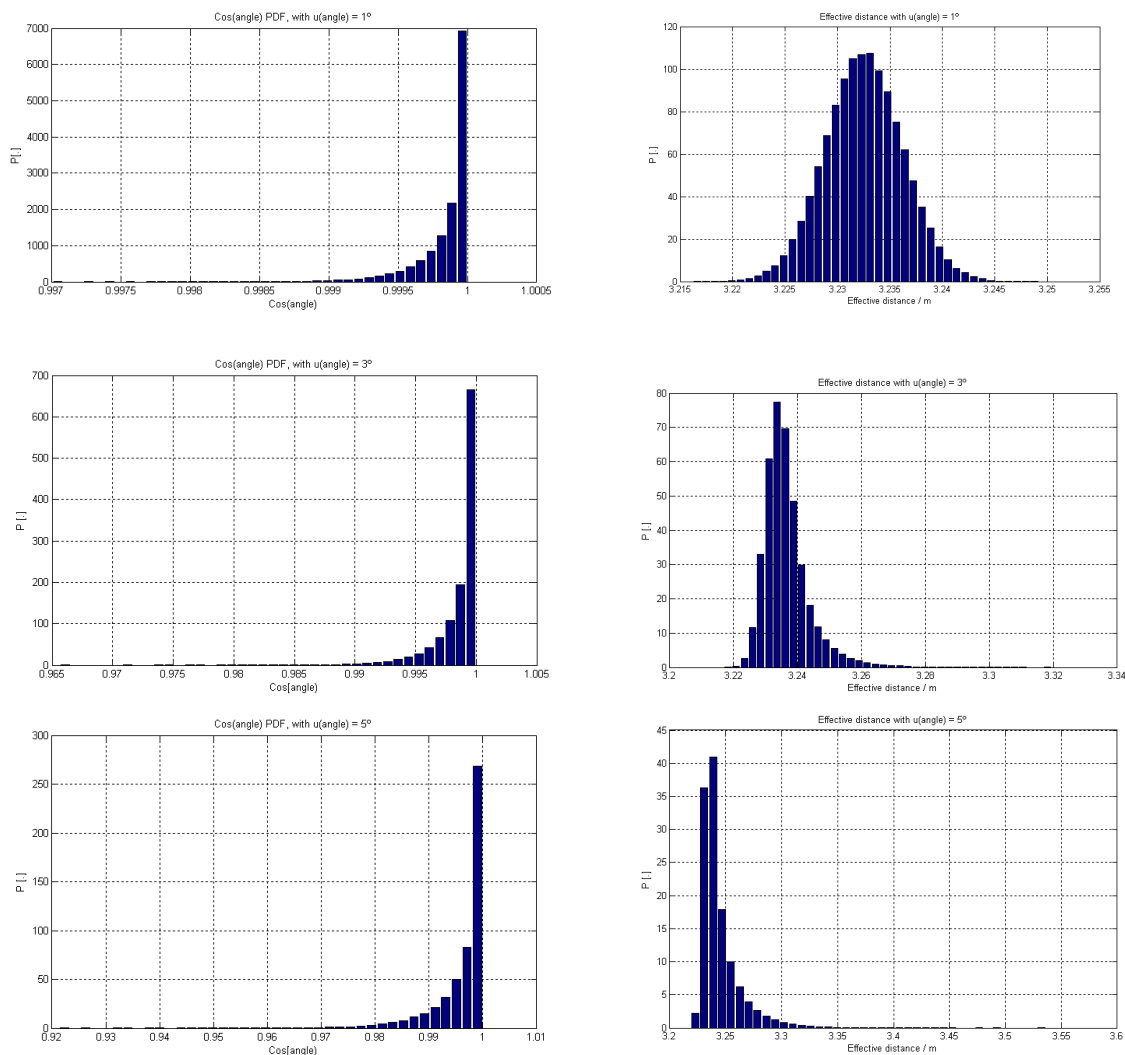


Figure 5 – Input PDFs of $\cos(\theta)$ and related output PDFs (effective distance) obtained for vertical angle deviation (θ) measurement uncertainties of 1°, 3° and 5°.

5. Conclusions

The progress of metrology techniques to study known measurement problems permits the access to relevant information that improves our knowledge of the quality of measurement. In the present case, regarding the calibration of photometers by the evaluation of effective distance, the output measurement quantity can be strongly affected by the setup that supports the measurement - the experimental bench - being, therefore, relevant to establish quality assessment criteria suitable to the requirements and tolerances to perform this task with confidence.

From the several geometric influence quantities that must be considered, the vertical angular deviation is certainly a very important one, which motivated this work. The first study aimed to use an MCM approach to validate measurement uncertainty results obtained using the GUM conventional approach, considering that the mathematical model used could be affected by its non-linear nature, with the conclusion of suitability on the latter for input quantities having Gaussian PDFs (according with the example given in [3]).

The second study aimed to evaluate how measurement uncertainty due to vertical angular deviation could affect both estimate and measurement uncertainty of the output quantity. In this case, the results shows an asymmetric influence not relevant for lower uncertainty values of this input quantity ($u(\theta)=1^\circ$) but to be considered for higher uncertainty values (reaching 1% of uncertainty of the output quantity). These results imply the need for the user to promote a geometrical characterization of the calibration bench in order to evaluate deviations along the longitudinal support for the displacement of the photometer to be calibrated.

Finally, it is advisable that, for calibration purposes, the user should establish criteria for the quality assessment of vertical angular deviations for photometer calibration benches, based on the uncertainty evaluation and the expected accuracy of illuminance and effective distance measurements.

This study is intended to be further developed in order to establish a broader analysis of the uncertainty contributions due to other geometrical and environmental quantities aiming to establish a generalized quality assessment of this type of testing setups.

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