Load Error in Metrology – A General Approach

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Abstract. The behaviour of a measurement processes is shaped by three types of measurement errors, by *transfer errors*, *disturbance errors* and *load errors*. The following sections will provide for the first time a general and consistent treatment of the load error in Metrology: An expanded model of the measurement process will reveal the significance of this rarely treated error. Besides, there are many comparable loading phenomena in different fields outside Metrology; some concepts may with good reason be transferred and adopted. It will be shown that the main concern has to be focused on backward structures because of physical loading effects, which interdict the commonly assumed nonreactive relations in metrological structures. These backward relations recommend the use of *linear fractional representations* (LFR) as models, with the *Redheffer star product* as a valuable operator. Given such general structures, load error corrections can be derived for dedicated applications.

1. Introduction

Important topics in Metrology are measurement errors and uncertainties. While *measurement uncertainties* keep being covered in numerous publications, *measurement errors* are widely ignored. This situation seems to change. Measurement errors experience some sort of rehabilitation as well as an alignment with self-evident error concepts in other fields: "The approach based on true value and error was questioned as being based on unknowable quantities, i.e. idealized concepts. The very terms were almost banned from the literature, and whoever dared to use them was considered suspiciously as a supporter of old ideas." [1].

On the other hand, a large number of error types have survived in workshops, labs, offices, institutions and lecture halls, though often without being based on sound theoretical definitions. It has been shown recently that for physical measurement processes, three and only three types of measurement errors exist, *transfer errors, disturbance errors* and *load errors* [2]. All other common errors are subtypes of these three main error types. Of course, model errors are active too, but they reside on a different analytical level.

Considering these three error types, the load errors are the least familiar ones. Usually they are covered casually, if at all, and only in order to avoid momentary, specific inconveniences. A general consideration and treatment does not exist yet.

As a start the following sections will repeat the definition of the three main error types and concentrate on *load errors* later on. In a *forward analysis* we describe errors and error processes as distinct items.

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The strategic tools will be taken from Signal and System Theory, which is supported by versatile software for analysis and simulation.
One of several notable features will be the avoidance of the usual simplifying nonreactive behaviour between quantities of interest. This means that the single conventional *block* in a *signal relation diagram* will not be appropriate anymore to describe the relation between two signals. Note that the topic does not just concern *signals*, but also *systems* as *sources of signals* due to their insufficient *loadability* or inappropriate generalised *impedance* respectively. If we consider the domain of processes [3], the *removal of mass, energy, impulse and information* loads sources and thus changes their *state*. At first, the whole issue seems to become needlessly complicated. But as soon as the structure of all relations is compiled systematically, in a signal relation diagram for example, the result will turn out to be clear, comprehensible and implementable. In some fields the concept of generalised quadrupole and multipole representations is predominant.
So, the recurrent demand for a coherent consideration of *process* P and *measurement process* M, both linked in the *process under measurement* PUM (Figure 1), gets a mandatory relevance in the analysis and discussion of load errors and their compensation.

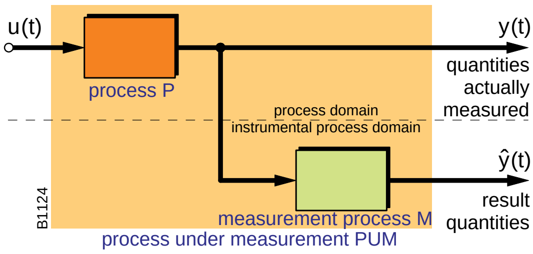

Figure 1. Process P and measurement process M linked in the process under measurement PUM

An additional abnormality arises from the fact that the so called load error does not appear in a measurement result $\hat{\mathbf{y}}$(t) directly, but only indirectly by the impact on the quantity to be measured *within the process* P, which means, on the measurand $\mathbf{y}$(t) itself. This inconvenience makes this error type almost untraceable. Whether a load error is acceptable or unacceptable, whether it has to be corrected eventually, this does not concern the following investigations and statements [4]. They tend to develop the concept of a holistic framework by creating a common structure and thus fostering strategies for the correction of load errors.
It is worth mentioning here that we only talk about *errors* and not about *uncertainties* in the result quantities. Of course, once we have defined and analysed the load errors in a *forward analysis*, further investigations on our *uncertain knowledge* about the errors, as well as the parameters in question, may follow in a *backward analysis* concerning *error uncertainties*. They will then be included in the general uncertainty budget for a final measurement result according to the Guide to the Expression of Uncertainty in Measurement (GUM) [5]. *Errors are independent of uncertainties, but uncertainties are not independent of errors*.
In the following sections we assume multivariate, linear dynamic systems. Without loss of generality, we express our statements in the time domain. They are valid in the frequency domain as well. So, the describing transfer function matrices **G** may be *temporal transfer function matrices* **G**(t) or *spectral transfer function matrices* **G**(s) [6].

2. General Error Structure

How does nonideality actually arise, or more precisely, which deviation from a stated ideal behaviour may occur? This question is treated by means of Signal and System Theory [2].

Firstly, what does the model of an ideal, multivariable measurement process M look like at all? Normally, we state the *ideal* (*nominal*) *transfer function matrix* $\mathbf{G}_{nom}$ of a measurement system, which relates one set (vector) of output signals to one set (vector) of input signals, to be equal the identity matrix **I**. Of course, it may adopt any other reasonable structure and parameter set.
Secondly, if we have got the *nonideal transfer function matrix* **G**, the deviation between ideal and nonideal behaviour leads directly to the *error definitions* of interest.
But, what makes a transfer function matrix **G** nonideal? We assume that a nonideal system will be disturbed via additional inputs and will disturb its surroundings by additional outputs. So the nonideal system will have two sets (vectors) of input signals and two sets (vectors) of output signals (Figure 2).

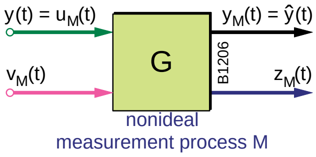

Figure 2. Generalised model of the nonideal measurement process M with disturbance quantities $v_M(t)$ and load quantities $z_M(t)$.

This structure reveals three error sources at the utmost:
- Irregularities *within* the measurement process M between the input quantities $\mathbf{u}_M(t)$ and the output quantities $\mathbf{y}_M(t)$. This leads to *transfer errors*.
- Influences *from* the environment via the disturbance quantities $\mathbf{v}_M(t)$ at the input *on* the measurement process M. This leads to *disturbance errors*.
- Influences *from* the measurement process M via the *load quantities* $z_M(t)$ at the output *on* any environment. This leads to *load errors*.

This generalised model of the nonideal measurement process M is a multivariate system, which is normally assumed to be linear, time-invariant (LTI) and dynamic (Figure 3), but which can readily become nonlinear and time-variant (NLTV).

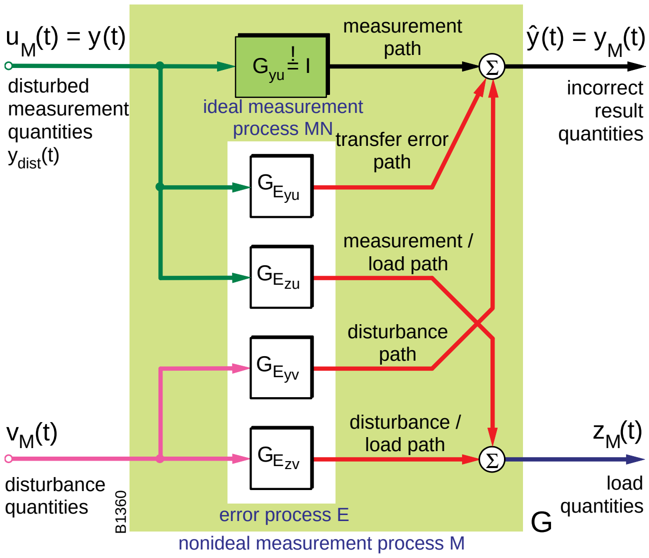

Figure 3. In-depth model of the nonideal measurement process M, consisting of the defined ideal (nominal) measurement process MN and the summarising error process E.

In this detailed signal relation diagram the internal *decomposition* assumes a parallel connection of the *nominal*, *ideal measurement process* MN and the *error process* E, The error process is the source of the three defined error types. Here already, it is obvious that the contribution to load errors does not affect the measurement result $\hat{\mathbf{y}}(t)$ directly.

It has to be mentioned, that besides this *parallel connection* of ideal system and error system, there is also the possibility of the *series connection* and the *feedback connection* [2]. However, the two latter show certain disadvantages.

3. Thermal Example

Load quantities $\mathbf{z}_M(t)$ impact the environment. In measurement procedures, especially the process P of interest is loaded. A plastic integrated circuit (IC) as process P may serve as a simple example (Figure 4), whose temperature $\vartheta_2(t)$ should be measured by a metallic temperature sensor process S as part of the measurement process M. Under measurement, thermal energy will flow from the hot IC to the cold sensor process S. The temperature of interest $\vartheta_2(t)$ will decrease accordingly and the sensor acquires a temperature, which is lower than it would be under normal, unconnected conditions.

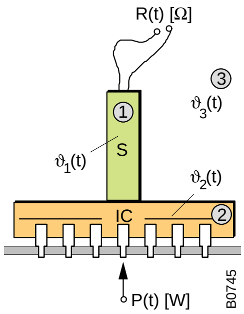

Figure 4. Sensor process S loads process IC thermally during a measurement procedure.

4. Load Error Structure

The dominant background is the physical *connection on*, or *insertion in*, a process P of a given measurement process M (Figure 5). Thereby the load quantities $\mathbf{z}_M(t)$ of the measurement process M become the disturbance quantities $\mathbf{v}(t)$ of process P, which corrupt the quantities to be measured $\mathbf{y}_{dist}(t)$ more or less.

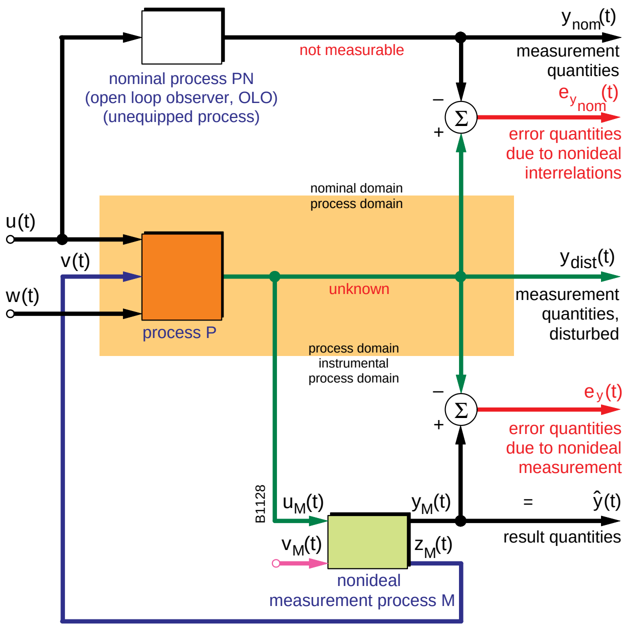

Figure 5. General structure of the loading impact of a nonideal measurement process M on a process P to be measured.

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This impact leads to systematic load errors. Again, they often remain undetected, because they obviously become effective outside the actual measurement process M.
The main reason for load errors is mass and / or energy removal from process P, which goes parallel with the withdrawal and transfer of information during the measurement procedure. This means that impedance mismatch, or generally speaking, the match of *capability* and *necessity* between *sources* and *sinks*, is an issue.
Besides, resisting elements (impedances) combined with storing elements (capacities) become responsible for dynamic effects and therefore for *dynamic load errors*. Mathematical modelling of the interconnected processes with load quantities and load errors reveals and quantifies such effects: The measurement path (chain) is not nonreactive anymore. Feedback paths appear in different configurations. We get extended series connections (Figure 6).

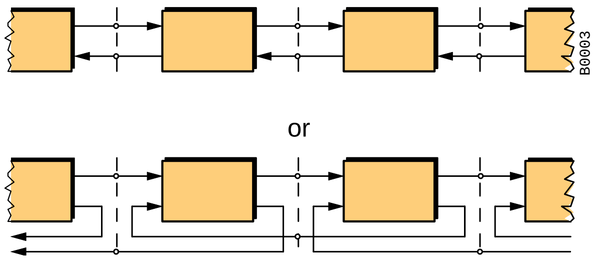

Figure 6. Generalised structures of feedback relations with loading effects.

It has to be amended that the product of *forward* (intensive) *quantities* and *backward* (extensive) *quantities* between two blocks mark *generalised* (signal) *power*. Signal and System Theory handles such structures elegantly by *Linear Fractional Representation* (LFR), in which the so-called *Redheffer Star Product* [7] plays a particular role.

5. Electrical Example

The connection of an electrical source Q to an electrical sink L serves as another obvious and impressive example. In measurement these two processes could represent a thermocouple and an indicating millivolt meter, which is described by the following model (Figure 7).

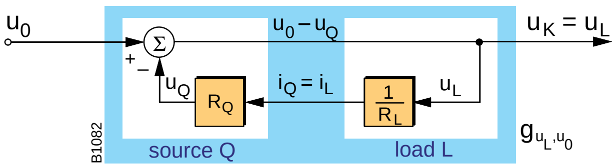

Figure 7. An unmatched series connection of two electrical systems with back-load effect.

The transfer equation of this series connection, which describes the relation between input voltage u_0 and output voltage u_K is well known:

$$u_K = u_L = \frac{1}{1+\dfrac{R_Q}{R_L}} u_0 = g_{u_L,u_0} u_0 \quad [\mathrm{V}]$$

Apparently, the quantity u_0 to be measured, drops to u_0 u_Q because of the load (sink) impedance R_L and the output (source) impedance R_Q. Unfortunately, the primary measure for an improvement of the nonideal measurement process, a source impedance $R_Q = 0$ and / or a load impedance $R_L = \infty$, is not realisable. However, the second measure, the implementation of a reconstruction process R will be *the* solution, as soon as the model is given. Another similar process is the voltage divider with its nonlinear characteristic caused by loading due to the similar impedance mismatch.

6. General Structure

The following general structure provides a basis for the analysis of loading effects. Unfortunately, the main obstacle of a widespread application is the usual lack of a suitable qualitative and quantitative model (structure, parameter) for the process P as well as for the nonideal measurement process M. Therefore trial and error strategies usually prevail.

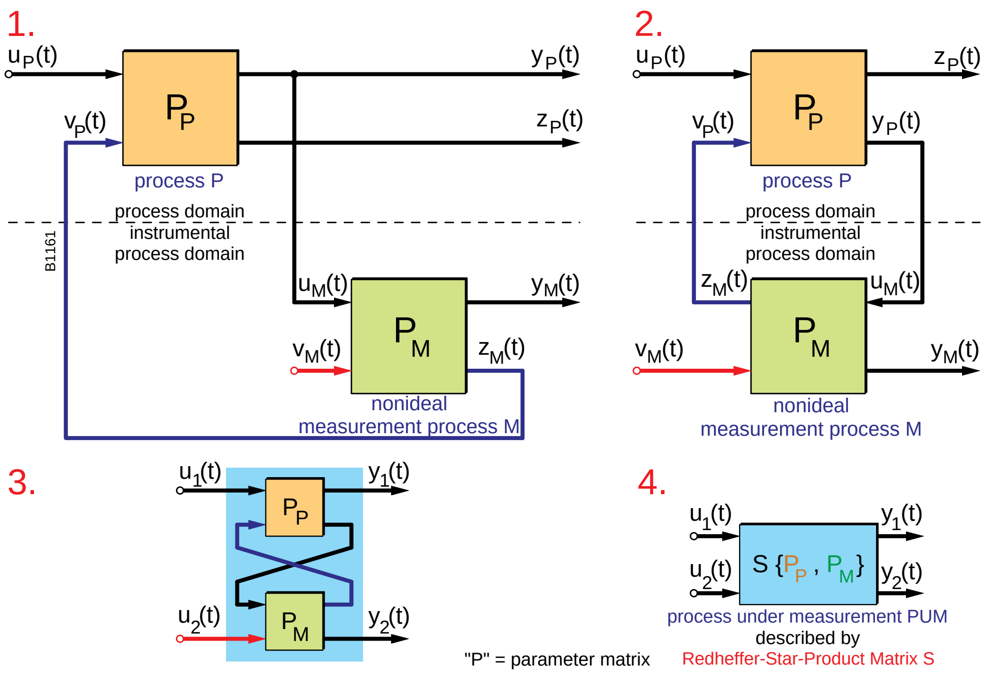

Figure 8. Reduction from step 1. to step 4. of the specific measurement structure to a general, universally valid structure.

This general structure of the *process under measurement* PUM, here a linear, dynamic system of two sets of input signals and two sets of output signals, can be implemented, analysed and discussed conveniently by common software products. The so-called Redheffer Star Product in the Linear Fractional Representation framework [7] serves as a ready-to-use function.

7. Reconstruction Process

Since we are normally unable to improve given processes and measurement processes according to specific demands physically, we design a correction of load errors by implementing a reconstruction process R, connected in series with the nonideal process under measurement. The model of this reconstruction process will be the inverse model of the nonideal process, as can be imagined intuitively. This concept is generally valid, though several conditions concerning the inversion operations must be fulfilled.

Preconditions for a successful realisation of a reconstruction process is the availability of models. On the one hand, we need the individual models of the process P and the measurement process M, each unconnected. On the other hand, we need the model of the process under measurement (PUM), both sub-processes interconnected. They all have to be developed by analytical and / or empirical identification (calibration).

8. Summary

The main reasons for unwanted load effects are physical dependencies between processes and measurement processes. They are seldom recognised and therefore not taken into consideration. Here, sources and structures of load errors have been investigated from the point of view of Signal and System Theory. Patterns of relevant relations become simple when approached systematically. They can be converted into the presented, universally valid structure, supported by dedicated algorithms.

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References

[1] Bich, W.: "From Errors to Probability Functions. Evaluation of the Concept of Measurement Uncertainty". IEEE Transactions on Instrumentation and Measurement (TIM), 8 (2012), p. 2153

[2] Ruhm, K. H.: "The Significant Role of Errors in Metrology – Justification by System Theory" XX IMEKO World Congress (2012), Busan, Korea; http://imeko2012.kriss.re.kr/sub02/tc/down/SW_Error%20in%20Metrology.pdf

[3] Ruhm, K. H.: "Process and System – A Dual Definition, Revisited with Consequences in Metrology" 13th IMEKO TC1-TC7 Joint Symposium (2010), London http://iopscience.iop.org/1742-6596/238/1/012037/pdf/1742-6596_238_1_012037.pdf

[4] Profos, P.: "Messfehler", Teubner (1983)

[5] BIPM: Guide to the expression of uncertainty in measurement (GUM) (2008) http://www.bipm.org/utils/common/documents/jcgm/JCGM_100_2008_E.pdf

[6] Ruhm, K. H.: "Concept and Term «Transfer» Concerning Systems" http://www.mmm.ethz.ch/dok01/d0000536.pdf

[7] Redheffer, R. M.: "On a Certain Linear Fractional Transformation" Journal of Mathematical Physics, 39 (1960), 269 – 286